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CIRCUIT ANALYSIS OF A-C POWER SYSTEMS

VOLUME I

Symmetrical and Related Components

By

EDITH CLARKE

PROFESSOR ELECTRICAL ENGINEERING

UNIVERSITY OF TEXAS

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PREFACE

This book is a compilation of notes and lectures given over a period of years to members of the Central Station Engineering Department of the General Electric Company in Schenectady, New York. Beginning in 1928, the notes were revised and extended for new groups of men entering the department, practical problems in power system performance with numerical solutions being added from time to time as they were presented by operating engineers. As the notes were helpful to members of the department and others receiving them, it was suggested that they be put in book form. In 1932, with Professor H. W. Bibber as co-author, a book on symmetrical components was undertaken. Parts of that unfinished book are included in Chapters I–IV of this one.

In answer to the repeated request that the methods of symmetrical and related components be presented very simply, the methods of solving unbalanced power system problems by means of components are analyzed and discussed in detail. The book has been divided into two volumes. Volume I deals largely with the determination of currents and voltages of fundamental frequency in power systems during unbalanced conditions by means of symmetrical and related components. Included in this volume are the electrical characteristics of overhead transmission circuits and information and data on transformers and synchronous machines which permit them to be represented by equivalent circuits in the solution of practical problems. Volume II will give additional characteristics of synchronous machines, equivalent circuits for types of transformers not included in Volume I, characteristics of insulated cables, induction machines, and other electrical equipment encountered in a-c power systems. Overvoltages from various causes and the effects of saturation in transformers and of amortisseur windings in synchronous machines will also be included in Volume II. In both volumes special attention is given to equivalent circuits and the solution of practical problems.

The author wishes to express appreciation to her associates who have assisted in the preparation of this book by helpful suggestions or critical reviews of completed chapters; especially to Mr. Charles Concordia for his willingness to act as consultant in regard to arrangement and presentation of material and to Miss Amelia De Lella

for her patience and good nature in typing and retyping the various editions and revisions of the notes, drawing preliminary figures, and proofreading.

EDITH CLARKE

SCHENECTADY, NEW YORK
June, 1943

CONTENTS

INTRODUCTION

The problems of the power transmission engineer at any given time may be divided roughly into three classes:

1. Problems which can be solved analytically by well-known methods in general use. The methods are satisfactory, because it is thought that all the factors influencing the problem are understood and can be evaluated, and the time required is not considered unduly long.
2. Problems which can be solved analytically and the various factors evaluated, but the time and labor required are excessive.
3. Problems for which there is no known analytic method of evaluating all the factors involved. This is not intended to imply that, for a given problem with all conditions specified, the engineer given sufficient time cannot provide a workable solution; but rather that, the effect of the various influences not being thoroughly understood, a different and independent problem is encountered with each change in given conditions.

The extensive use of mechanical and electrical calculating devices, factory tests on equipment, laboratory tests on miniature electrical systems, and field tests on actual systems have made it unnecessary to include a fourth class of problems — those for which there is no known solution.

The types of problems appearing under the three classifications change with time. Before the period of interconnection of power systems, many of the problems connected with the normal operation of a substantially balanced system were thought to be in class 1. Some, however, were placed in class 2, one example being the determination by calculation of load currents in balanced networks with multiple paths. The use of the a-c network analyzer[1] to determine current and voltage distribution in a balanced system moved this problem from class 2 into class 1. But when it was discovered that, under certain conditions of balanced system operation, a generating station at the distant end of a long transmission line could not be operated at its rated output and remain in synchronism with the rest of the system,

[1] For numbered references see the list at end of the introduction and at end of each chapter.

i.e., the system as designed was not stable under steady state, problems previously thought to be in class 1 dropped back into class 3. For a time the problems of system stability under both steady-state and transient conditions were in class 3; but after a few years they could be moved up into class 2. At the present time they probably still remain in class 2. Although the various factors involved are understood and the use of the a-c network analyzer materially reduces the time required for a given solution, the methods used are essentially step-by-step ones. They are the best available at present, but improvements are to be expected.

The application of the method of symmetrical components[2] to the solution of power system problems during unbalanced conditions, within a few years moved a long list of problems from classes 2 and 3 into class 1, the d-c calculating table[3] or the a-c network analyzer reducing the time required for solution.

There are certain problems which, although solvable by symmetrical components, can be solved more readily by a different set of components.[4,5] These components are simply related to symmetrical components and can be derived from them. By providing a different approach to the problem, they frequently allow a simpler solution.

Outstanding examples of problems in class 3 at the present time are those which involve non-linear relations. The effect upon system voltages and currents of unequal saturation in the phases of a transformer bank because of unsymmetrical system conditions is an example of a class 3 problem. Although solutions have been obtained for certain problems involving non-linear relations by making use of a differential[6] analyzer or a transient[7] analyzer, it will probably be some time before all problems of this type are removed from class 3.

The purpose of this book is to help the power transmission engineer solve some of his problems. Since it is expected that many of these problems will deal with systems during unbalanced conditions, where the use of symmetrical components and their related components will materially aid him, the greater part of the book is devoted to these components and their applications. But as he will also be expected to determine system conditions during normal operation, tables and charts are given to assist him in the solution of such problems.

Except for an occasional integral or differential equation, introduced for a better understanding of the fundamental principles involved, a knowledge of the elementary principles of alternating currents, algebra, plane geometry, trigonometry, and familiarity with electrical equipment are the only prerequisites for an understanding of this book.

In an endeavor to simplify the application of the method of symmetrical components to unbalance power system problems, those concepts necessary for its application, but not an integral part of the method itself, are discussed in Chapter I. Among these are vectors, complex quantities, operators, vector representation of sinusoidal currents and voltages, per unit quantities, one-line impedance diagrams, equivalent circuits, and the principle of superposition.

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CHAPTER I

DEFINITIONS AND FUNDAMENTAL CONCEPTS

A **vector** has magnitude and direction. A vector is represented by a straight line, of length proportional to the magnitude of the vector, extending in the direction of the vector and terminated by an arrowhead. The starting point of a vector is called the origin and the terminal point the terminus. In Fig. 1(*a*), O is the origin of the vector V and T is the terminus. Vectors having the same magnitude and direction are equal wherever their origins may be. In this book, unless specifically stated to the contrary, a vector, V, will be written with no distinguishing mark. When the magnitude alone of the vector V is specified, V will be enclosed in bars, thus, $|V|$.

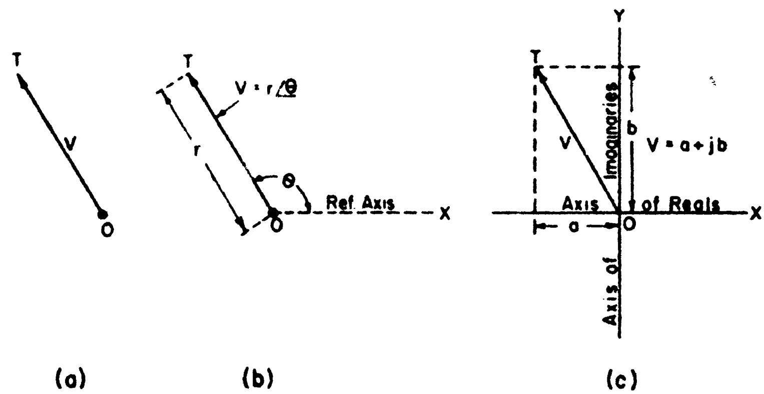


FIG. 1. (*a*) Vector. (*b*) In polar coordinates. (*c*) In rectangular coordinates.

A plane vector can be defined by two coordinates. If *polar coordinates* are used, the two coordinates are the magnitude of the vector and the angle it makes with the axis of reference, the angle being positive when measured in the counterclockwise direction. In Fig. 1(*b*), if OX is the reference axis, the vector V is specified if written $r\underline{/\theta}$, where r, a scalar quantity called the modulus, represents the magnitude of the vector, and θ, called the argument, is the angle measured in the counterclockwise direction from the reference axis to the positive direction of the vector. A positive angle θ is written $\underline{/\theta}$, a negative angle θ is written $\overline{\backslash\theta}$ or $\underline{/-\theta}$. In *Cartesian coordinates* the vector is defined by its projections upon two reference axes intersecting at a known angle. The simplest Cartesian coordinates are rectangular, with one axis horizontal and the other vertical. In Fig. 1(c) the horizontal and vertical axes are OX and OY, respectively. There will be a positive or negative projection on the horizontal axis and similarly on the vertical axis, projections to the right of the origin on the horizontal axis and above the origin on the vertical axis being positive. The axes are commonly referred to as the axes of *reals* and *imaginaries*, respectively. The letter j is used in electrical engineering to designate the projection on the vertical or imaginary axis. The vector expressed in this manner is written $a + jb$, where the signs of a and b may be either positive or negative. In Fig. 1(c) the real part of V is negative. A vector in the form $a + jb$ is said to be expressed in complex form. The magnitude of a vector in polar form is independent of the reference axis, but its argument is not. When expressed in complex form, both coordinates depend upon the choice of reference axes.

A **complex quantity**, as here used, is a complex number whose real and imaginary parts correspond to physical quantities. Complex quantities differ from vectors in that they have no direction of their own and consequently are independent of the reference axes. A complex quantity of magnitude different from unity attached to a vector rotates the vector through an angle and also changes its magnitude, producing a new vector of a different kind. The impedance of an electric circuit is a complex quantity. When this complex quantity is multiplied by a current vector the result is a voltage vector which, in the general case, differs in magnitude and phase from the current vector.

To change a vector or complex number expressed in the polar form $r\underline{/\theta}$ to the complex form $a + jb$, or vice versa, the reference axis for θ being the axis of reals, the following relations are employed (see Fig. 1):

$$a = r\cos\theta$$

$$b = r\sin\theta$$

$$a + jb = r(\cos\theta + j\sin\theta)$$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}\frac{b}{a}$$

$$r\underline{/\theta} = \sqrt{a^2 + b^2}\ \underline{/\tan^{-1}\frac{b}{a}}$$

where the signs of a and b and of the trigonometric functions should be observed.

Addition and subtraction of vectors or complex numbers can be performed either graphically or algebraically.

To find the *sum* of two or more vectors graphically, the origin of the second vector is placed at the terminus of the first vector, and so on with the others successively, the proper direction being given to each. The sum is the vector drawn from the origin of the first vector to the terminus of the last vector. (See Fig. 2(a).) The algebraic addition

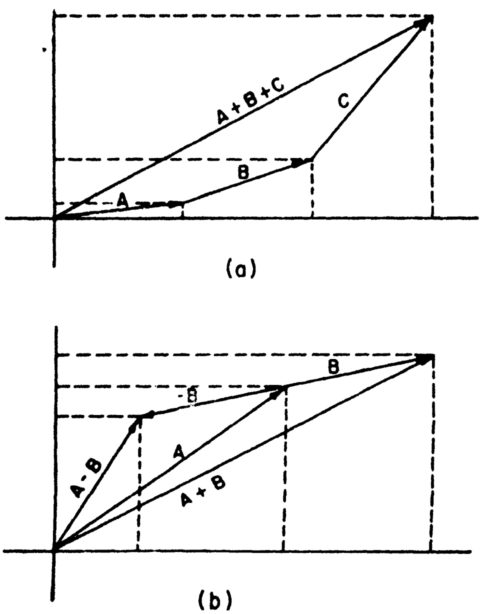


FIG. 2. (a) Sum of vectors A, B, and C. (b) Sum and difference of two vectors, A and B.

of vectors is readily performed if they are expressed in the complex form $a + jb$. The sum of all the real components gives the real part of the resultant, whereas the sum of all the j components gives the imaginary part of the resultant.

The *difference* of two vectors is defined as the result of adding to the first vector the second vector drawn 180° from its given direction. Figure 2(b) shows the graphical operations of adding the vectors A and B and subtracting B from A. The algebraic difference of two vectors expressed in complex form $a + jb$ is obtained by reversing the signs of both components of the second vector, then adding the real components and the imaginary components, just as in the addition of two given vectors.

When addition or subtraction of vectors is required, manipulations are more easily carried out with the vectors in complex form. When a vector is multiplied or divided by a complex number it may be preferable to express both in polar form.

Multiplication and Division of Vectors and Complex Numbers. By the use of Maclaurin's theorem it can be shown that

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$e^{-j\theta} = \cos\theta - j\sin\theta$$

where e is the base of Napierian logarithms

$$\left(e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.718 +\right)$$

The vector V can therefore be written

$$V = r\underline{/\theta} = r(\cos\theta + j\sin\theta) = re^{j\theta} \qquad [1]$$

In [1], θ is an angle expressed in either degrees or radians. If the vector is written $r\underline{/\theta}$, θ will, in general, be expressed in degrees; if written $re^{j\theta}$, θ will be in radians; if written $r(\cos\theta + j\sin\theta)$, θ may be in either degrees or radians but is usually in degrees. (π radians = 180°, where $\pi = 3.14159 +$.)

With vectors and complex numbers expressed in the form $re^{j\theta}$, multiplication and division may be performed by applying the rules for exponentials developed in algebra. Given a vector V_1 and a complex number Z_2, where

$$V_1 = r_1e^{j\theta_1} = r_1\underline{/\theta_1}$$
$$Z_2 = r_2e^{j\theta_2} = r_2\underline{/\theta_2}$$

Then, adding exponents of e when multiplying and subtracting when dividing,

$$V_1 \times Z_2 = r_1r_2e^{j(\theta_1+\theta_2)} = r_1r_2\underline{/\theta_1 + \theta_2} \qquad [2]$$

$$V_1 \div Z_2 = \frac{r_1e^{j\theta_1}}{r_2e^{j\theta_2}} = \frac{r_1}{r_2}e^{j(\theta_1-\theta_2)} = \frac{r_1}{r_2}\underline{/\theta_1 - \theta_2}(r_2 \neq 0) \qquad [3]$$

From [2] it follows that: ***To obtain the direct product of a vector and a complex number expressed in polar form, multiply their magnitudes (moduli) and add their angles (arguments)***, obtaining as a result a new vector. And, from [3]: ***To obtain the quotient of a vector and a complex number expressed in polar form, divide the modulus of the dividend by the modulus of the divisor and subtract the argument of the divisor from the argument of the dividend***, obtaining a new vector.

Take, for example,

$$V_1 = 2.000\underline{/60^\circ} = 1.000 + j1.732$$

$$Z_2 = 1.000\underline{/120^\circ} = -0.500 + j0.866$$

Then, applying the rules for multiplying and dividing,

$$V_1 \times Z_2 = 2.000\underline{/180^\circ} = -2.000 + j0$$

$$V_1 \div Z_2 = 2.000\overline{/60^\circ} = 1.000 - j1.732$$

A complex number may be raised to any power by multiplying it by itself a sufficient number of times. Thus,

$$(Z)^n = (r\underline{/\theta})^n = r^n\underline{/n\theta} \qquad [4]$$

Any desired root of a complex number may be obtained by extracting the given root of its modulus and dividing its argument by the given root. Thus,

$$\sqrt[n]{Z} = \sqrt[n]{r\underline{/\theta}} = \sqrt[n]{r}\,\underline{\Big/\frac{\theta + 2\pi k}{n}}, \text{ where } k = 0, 1, \cdots (n - 1) \qquad [5]$$

An **operator** is a complex number of unit magnitude. An operator is used to rotate a vector through an angle without changing its magnitude. A familiar operator is $-1 = 1\underline{/180^\circ}$. Attached to a vector, -1 rotates it through 180°. Another common operator is $j = 1\underline{/90^\circ}$ which rotates the vector to which it is attached through 90° in the positive direction, while $-j = 1\overline{/90^\circ}$ will rotate it 90° in the negative (clockwise) direction.

If the vector $V_1 = 300\underline{/120^\circ}$ is multiplied by the operator j, the resultant vector, V_2, is

$$V_2 = 300\underline{/120^\circ} \times 1\underline{/90^\circ} = 300\underline{/210^\circ} = -259.8 - j150$$

If V_1 is multiplied by $-j$, the resultant vector, V_2 is

$$V_2 = 300\underline{/120^\circ} \times 1\overline{/90^\circ} = 300\underline{/30^\circ} = 259.8 + j150$$

The Operator j. Powers of the operator j may be written in several ways, as indicated below:

$$j = 1\underline{/90^\circ} = 1\overline{/270^\circ} = 0 + j = j$$

$$j^2 = 1\underline{/180^\circ} = 1\overline{/180^\circ} = -1 + j0 = -1$$

$$j^3 = 1\underline{/270^\circ} = 1\overline{/90^\circ} = 0 - j = -j$$

$$j^4 = 1\underline{/360^\circ} = 1\underline{/0^\circ} = 1 + j0 = 1$$

$$j^5 = 1\underline{/450^\circ} = 1\underline{/90^\circ} = 0 + j = j$$

The Operator a. In dealing with balanced three-phase vectors, an operator which rotates vectors through 120° is very useful. It is commonly accepted practice to use a to represent this operator. Thus,

$$a = 1\,\underline{/120^\circ}$$

In the complex form, using the transformation relations given previously,

$$a = 1(\cos\theta + j\ \sin\theta) = \cos 120^\circ + j\ \sin 120^\circ = -0.500 + j0.866$$

The square of a is obtained by multiplying a by itself:

$$a^2 = (-0.500 + j0.866)^2 = -0.500 - j0.866$$

or

$$a^2 = a \times a = 1\,\underline{/120^\circ + 120^\circ} = 1\,\underline{/240^\circ} = 1\,/\overline{120^\circ}$$

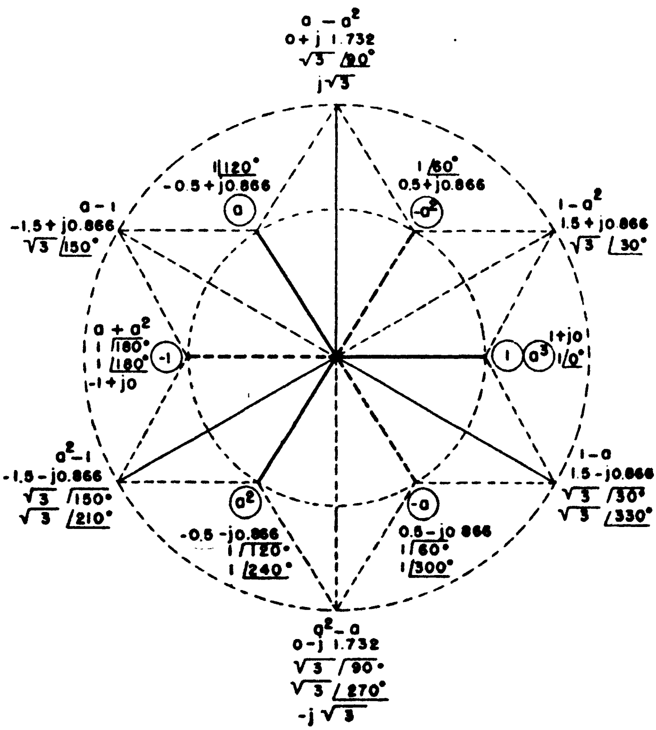


FIG. 3. Graphical representation of functions of the operator a. (F. A. Hamilton)

Functions of the operator a occur frequently in work with symmetrical components. Table I and Fig. 3 give some of these in convenient form. Where functions more complicated than those given in the table are encountered, they may always be reduced to a single complex number by replacing the powers of a by their equivalents in complex form from Table I or Fig. 3. If a function of a is a fraction with a in the denominator, either an algebraic operation such as factoring followed by division may be performed, or else the numerator and denominator may be multiplied by a quantity which will make the denominator a real number. The quantity which will accomplish this elimination of all but real numbers in the denominator is an operator capable of rotating the denominator to the axis of reals.

Examples.

(a) $$2a^2 + 3a + 5 = 2(-0.5 - j0.866) + 3(-0.5 + j0.866) + 5 = 2.5 + j0.866$$
$$= 2.648\,\underline{/19.1^\circ}$$

(b) $$\frac{1 - a^2}{a - a^2} = \frac{(1 - a)(1 + a)}{a(1 - a)} = \frac{1 + a}{a} = \frac{-a^2}{a} = -a$$

or

$$\frac{1 - a^2}{(a - a^2)} = \frac{(1 - a^2)}{(a - a^2)} \times \frac{(a^2 - a)}{(a^2 - a)} = \frac{a^2 - a^4 - a + a^3}{a^3 - a^4 - a^2 + a^3} = \frac{a^2 + 1 - 2a}{2 - (a^2 + a)}$$
$$= \frac{-3a}{3} = -a$$

(c) $$\frac{1 - a}{1 + a^2} = \frac{(1 - a)}{(-a)} \times \frac{(-a^2)}{(-a^2)} = \frac{1 - a^2}{1} = 1 - a^2$$

TABLE I

FUNCTIONS OF THE OPERATOR a

$$a = 1\,\underline{/120^\circ} = -0.5 + j0.866$$
$$a^2 = 1\,\underline{/240^\circ} = -0.5 - j0.866$$
$$a^3 = 1\,\underline{/360^\circ} = 1.0 + j0$$
$$a^4 = 1\,\underline{/120^\circ} = -0.5 + j0.866 = a$$
$$-a = 1\,\overline{/60^\circ} = 0.5 - j0.866$$
$$-a^2 = 1\,\underline{/60^\circ} = 0.5 + j0.866$$
$$1 + a + a^2 = 0$$
$$a + a^2 = -1$$
$$a - a^2 = 0 + j1.732 = \sqrt{3}\,\underline{/90^\circ}$$
$$a^2 - a = 0 - j1.732 = \sqrt{3}\,\overline{/90^\circ}$$
$$1 - a = 1.5 - j0.866 = \sqrt{3}\,\overline{/30^\circ}$$
$$1 - a^2 = 1.5 + j0.866 = \sqrt{3}\,\underline{/30^\circ}$$
$$a - 1 = -1.5 + j0.866 = \sqrt{3}\,\underline{/150^\circ}$$
$$a^2 - 1 = -1.5 - j0.866 = \sqrt{3}\,\overline{/150^\circ}$$

Sinusoidal Quantities. Sinusoidal currents and voltages have magnitudes which vary sinusoidally with time. The ordinate in Fig. 4 gives, at any time t measured from an initial time t_0, the instantaneous value of the current i, of oscillation frequency f. The equation for the current i may be written

$$i = I_m \sin (2\pi ft + \theta) = I_m \sin (\omega t + \theta) \qquad [6]$$

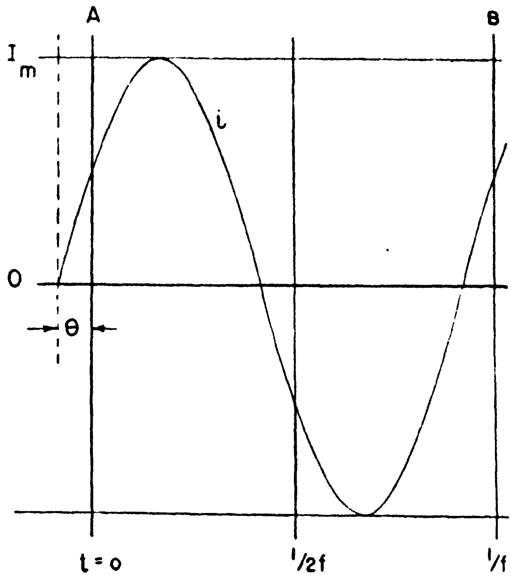


FIG. 4. Graph of instantaneous current, $i = I_m \sin (2\pi ft + \theta)$. Ordinates are the magnitude of i, and abscissas are time t in seconds.

In [6], I_m is the maximum or crest value of the current i, f the frequency in cycles per second, t the time in seconds, and θ the electrical angle between the point at which the current wave is zero and increasing in a positive direction and the point from which time is measured. θ is the *phase angle* and may be either positive or negative. At time $t = 0$ the magnitude of i is $I_m \sin \theta$.

The *period*, or the time required for the current to complete one cycle, is $1/f$ second and is indicated by T. A 60-cycle current, for example, requires $\frac{1}{60}$ second to complete one cycle. One cycle is shown in Fig. 4 between A and B. In [6], $(2\pi ft + \theta)$ is in radians. If θ is expressed in degrees and 2π is replaced by 360°, [6] becomes

$$i = I_m \sin \left(360° \frac{t}{T} + \theta \right) \qquad [7]$$

where t/T is number of periods in time t.

Vector Representation of Sinusoidal Currents and Voltages. The instantaneous values of the current i in Fig. 4 may be obtained by the projection on a fixed reference line of a plane vector of magnitude I_m revolving in the positive or counterclockwise direction at a constant angular velocity of $\omega = 2\pi f$ radians ($360f$ degrees) per second. In Fig. 5, the vector OA makes one revolution in time $T = 1/f$ second. Angle is measured from OX and represents the instantaneous phase; projection is on OY and represents the instantaneous magnitude. At time $t = 0$, the vector $OA = I_m$ makes the angle θ with OX, and at any time t the angle is $(\omega t + \theta)$. The position of OA shown in Fig. 5 corresponds to $t = 0$.

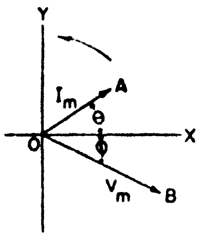


FIG. 5. Vector diagram at time $t = 0$ of sinusoidal quantities i and v.

Sinusoidal currents and voltages of the same frequency may be represented in the same vector diagram, each current and voltage being represented by a revolving vector whose length is proportional to the maximum magnitude of the quantity and whose displacement from the OX axis at time $t = 0$ is its phase angle. In Fig. 5 the projection on OY of the revolving vector OB of magnitude V_m, angular velocity $2\pi f$ radians per second, and phase angle ϕ (ϕ is negative in this case) represents the magnitude of the voltage v at any time t. The angle which OB makes with OX represents the phase of v at time t. The position of OB shown in Fig. 5 corresponds to $t = 0$.

At $t = 0$, i leads v by an angle, $\theta - \phi$, equal to the algebraic difference of the phase angles θ and ϕ; and, since OA and OB revolve at the same rate, i and v retain this relative phase displacement.

In those problems dealing with sinusoidal currents and voltages in which effective rather than crest values are considered, it is found more convenient to represent the voltages and currents by vectors whose magnitudes are proportional to effective values and to select a value of t such that one of the vectors lies along OX. The vector chosen to coincide with OX is called the *reference vector*, and the phase displacements of the other currents and voltages are then their relative phase displacements from the current or voltage selected as reference vector.

The *root mean square* value of an alternating current, written rms current, is the effective value of the current. The power loss, I^2R, in a resistance circuit with an alternating current of rms value I, is the same as the loss in the same circuit with a direct current I. The rms value of an alternating current during any given time is the square root of the average value of the square of the current during that time. This definition applies to non-sinusoidal as well as to sinusoidal currents.

If the current is sinusoidal, the rms current in terms of crest current is given by the following equation:

$$I_{\text{rms}} = \left[\int_0^{2\pi} \frac{(I_m \sin x)^2\,dx}{2\pi}\right]^{1/2} \tag{8}$$

Integration of [8], with the use of a table of integrals,* gives

$$I_{\text{rms}} = \left[\frac{I_m^2}{2\pi}\left[\tfrac{1}{2}x - \tfrac{1}{4}\sin 2x\right]_0^{2\pi}\right]^{1/2} = \frac{I_m}{\sqrt{2}} \tag{9}$$

With a sinusoidal voltage, the rms voltage is likewise the crest value of the voltage divided by $\sqrt{2}$:

$$V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \tag{10}$$

A-c ammeters and voltmeters read rms amperes and volts, respectively, whereas an oscillograph records instantaneous values versus time. Since the ratio of the rms to the crest value of sinusoidal currents and voltages is constant, one can readily be obtained from the other.

In the design of generators for polyphase systems, care is taken to secure sinusoidal (or approximately sinusoidal) generated voltages of equal rms values in all phases and of equal crest values, which occur successively in the phases in equal time intervals. For an n-phase generator, the time interval or *time phase displacement* in degrees is $360°/n$. For a three-phase generator it is 120°.

The *phase order* of the phase voltages in a polyphase circuit is the order in which they reach their crest values. The *positive-sequence phase order* for a polyphase system is the phase order of the generated voltages. Since the naming of the phases is arbitrary, it will be assumed that the phases are named so that the phase order of the generated voltages is $a, b, c, d, \cdots$. It should be pointed out, however, that in some three-phase power systems the phases are indicated by x, y, z(a, b, c, or 1, 2, 3) and the generated voltages have the phase order x, z, y(a, c, b or 1, 3, 2). In calculations of such systems, the phases can be renamed to conform to the chosen standard.

The phase voltages and line currents of a polyphase circuit are *balanced*, or *symmetrical*, if they are sinusoidal, equal in magnitude, and displaced from each other by equal phase angles. Balanced voltages and currents of positive-sequence phase order are called *positive-sequence voltages* and *currents*, respectively.

* Equation 261, *A Short Table of Integrals*, B. O. Peirce, Second Revised Edition, Ginn and Co.

A polyphase power system is balanced if the phase voltages and line currents at every point in the system are balanced and of the same frequency and phase order as the generated voltages, that is, if the voltages and currents are positive-sequence voltages and currents, respectively. For a polyphase system to be balanced, it is necessary that the generated voltages be balanced and the impedance to positive-sequence currents be the same in all phases. Figure 6(a) shows the vector diagram of the rms currents and voltages at a point of a balanced three-phase power system, with V_a as reference vector. In determining rms voltages and currents, and average power in a balanced polyphase system, only one phase of the system need be considered, because impedances to neutral (positive-sequence impedances) and rms currents and voltages are the same in all phases. Instantaneous currents and voltages in the phases will differ in time phase, but the difference in time phase between the current and voltage of any one phase will be the same as that in all other phases. A balanced polyphase power system can therefore be represented for purposes of calculation by a *one-line diagram* in which the applied voltages are voltages to neutral and the impedances are impedances to neutral. The one-line diagram of a balanced polyphase power system is a *positive-sequence diagram*, so called because all applied voltages are of positive-sequence phase order (i.e., of the phase order of the generated voltages), and the impedances are those met by balanced currents of positive-sequence phase order.

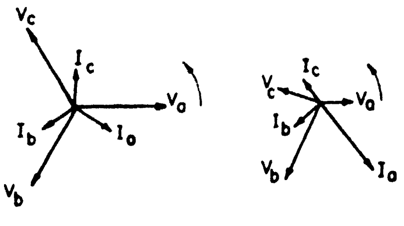


FIG. 6. Vector diagram of fundamental-frequency rms currents and voltages at a point in a three-phase system, with V_a as reference vector. (a) Balanced system. (b) Unbalanced system.

An *unbalanced polyphase power system* is one in which the voltages and currents are unbalanced. A system will be unbalanced if the generated voltages are unbalanced, or if the circuits which compose the system are unsymmetrical. When an unsymmetrical fault occurs on a system which was previously balanced, the currents and voltages become unbalanced because the phases have become unsymmetrical. The fundamental-frequency currents and voltages of an unbalanced system, being sinusoidal, can be represented by vectors. Fundamental frequency is impressed frequency. Figure 6(b) illustrates the vector representation of the fundamental-frequency currents and voltages at a point of an unbalanced three-phase power system with V_a, the voltage of phase a, as reference vector.

Voltage Rise, Voltage Drop, Direction of Current Flow, Average Power. In calculations in which sinusoidal voltages and currents are represented by vectors, it is important to distinguish between voltage rise and voltage drop and to adopt a convention for indicating positive direction of current flow. Since many of the conventions used in d-c calculations are also used in a-c calculations, these conventions and the laws governing d-c circuits will be discussed before those for a-c systems.

D-C Systems. According to established convention, the voltage generated in an open-circuited d-c dynamo is a rise or increase in voltage from negative to positive terminal; it may also be considered a drop or decrease in voltage from positive to negative terminal. If the d-c dynamo supplies an external resistance load connected across its terminals, positive direction of current flow is taken from the negative to the positive terminal through the machine and from the positive to the negative terminal through the load. By Ohm's law, a current flowing through a resistance causes a voltage drop (or a negative voltage rise) in the direction of current flow equal to the product of the current and the resistance. It also causes a voltage rise through the resistance in the opposite direction. Voltage rise and voltage drops have opposite signs when taken in the same direction through an element of the circuit, but the same signs when taken in opposite directions.

The voltage at a point in the system is the difference in potential between that point and some other point used as reference. Unless otherwise stated, the voltage to ground, or to some other reference point, is understood to mean the *rise* in voltage in going from the reference point to the given point.

In d-c circuits, where the direction of current flow is not known, an arrow is used to indicate the direction assumed as positive and the current is indicated by a symbol. (See Fig. 7(a).) Calculations are then based on the assumption of positive current flowing in the direction indicated by the arrow. If calculations give a negative current flowing in the indicated direction, it can be concluded that positive current flows in the opposite direction or the current flowing in the indicated direction is a negative current. Negative current flowing in a given direction is equal in magnitude to positive current flowing in the opposite direction.

When the voltage rise through a d-c machine from the negative to the positive terminal is positive and the current in the same direction is also positive, power *out* of the machine is positive and it is a generator; but, if the current in the same direction as the positive voltage rise is negative, power *out* of the machine is negative and it is a motor. Power into the motor, however, is positive. A generator sends out positive power or receives negative power; a motor sends out negative power or receives positive power.

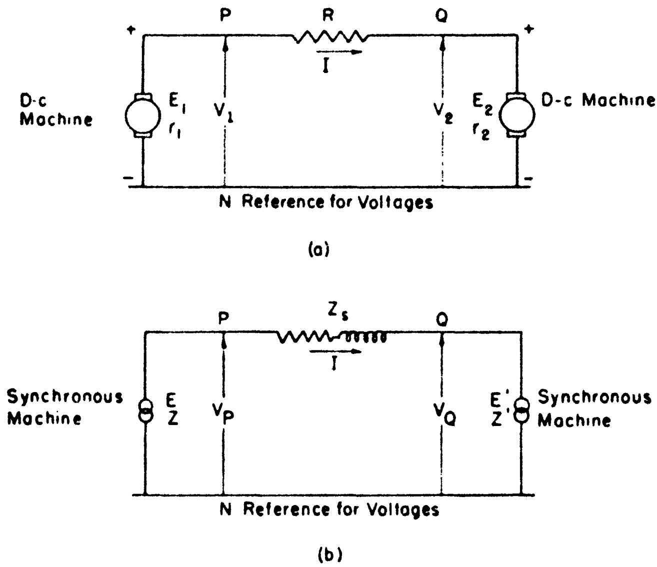


FIG. 7. (*a*) D-c system. (*b*) A-c system — single-phase or one-phase of balanced polyphase system.

A-C Systems. The conventions used in d-c systems will be applied to a-c systems in which fundamental-frequency currents and voltages are represented by vectors. The main difference is that in d-c systems currents and voltages can be only positive or negative, that is, can differ in phase only by exactly 0° or 180°, whereas in a-c systems, with a particular voltage or current selected as reference vector, the other currents and voltages may have any phase relation with the reference vector between 0° and 360°. If this distinction is borne in mind, and resistance in Ohm's law replaced by impedance, the conventions and principles used in d-c systems can be applied also to a-c systems.

In a-c systems, as in d-c systems, the voltage at a given point is the difference in potential between that point and some other point used as reference. Unless otherwise stated, it is the voltage rise in going from the reference point to the given point. This is a convention in general use. When the voltages at all points in a system are referred to the same point, and this point is of zero potential relative to the system, a single subscript will be used to indicate the voltage at a given point. Thus, the voltage at point a will be written V_a, where the magnitude of V_a is the rms voltage at a referred to a point of zero potential, and the phase angle of V_a depends upon the voltage or current vector selected as reference vector. If V_a is selected, its phase angle is zero and $V_a = |V_a|\underline{/0°}$.

The voltage at any point a, referred to a second point b, will be written

$$V_{ba} = V_a - V_b$$

where the magnitudes of V_a and V_b are the rms voltages at points a and b, respectively, referred to the zero-potential reference point for the system, and their phase angles are determined by the vector selected as reference vector. V_{ba} indicates the voltage rise in going from point b (indicated by the first subscript) to point a (indicated by the second subscript). This notation, which is arbitrary, is used in many books, but not in all. V_{ba} is an rms vector voltage of magnitude independent of the reference vector; its phase angle is the angle it makes with the reference vector.

Figure 7(b) shows two single-phase synchronous machines, or one phase of each of two polyphase machines. Let machines 1 and 2 have constant impedances Z and Z' and rms generated voltages E and E', respectively, as indicated. Terminal P of machine 1 is connected through an external impedance Z_s to terminal Q of machine 2; their other terminals are connected directly to N, a point of zero potential.

Following the convention that the positive direction of voltage rise through a machine is from the reference point to the terminal, E represents the rms voltage rise from N to P on open circuit in machine 1, and E' the rms voltage rise from N to Q in machine 2 on open circuit. If E' is equal in magnitude to E and also in phase, the generated voltages of the two machines, as connected in Fig. 7(b), oppose each other and no current will flow in the circuit. If, however, E' is different in magnitude or in phase from E there will be a current flowing in the loop circuit. To calculate this current I when the magnitudes and relative phases of E and E' and the impedances Z, Z', and Z_s are known, a positive direction for current flow will be assumed. The choice of the direction assumed as positive is arbitrary, but, having been assumed, the calculated current will be the current flowing in the assumed direction. Let the positive direction of current flow be from N to P through machine 1, as indicated by arrow in Fig. 7(b). As there is only one loop, the current I will flow from P to Q through the impedance Z_s, and from Q to N through the second machine. When there is more than one loop, it may be necessary to assign symbols to several currents and to indicate all assumed directions by arrows. The *voltage drop* around the loop in Fig. 7(*b*) in the assumed direction of current flow is given by the following equation, obtained by applying Ohm's law, which states that the *voltage drop in the direction of current flow is the product of the current and the impedance through which it flows*, and Kirchhoff's law, which states that *the sum of the voltage drops (or voltage rises) taken in the same direction around a closed loop is zero:*

$$-E + IZ + IZ_s + E' + IZ' = 0 \tag{11}$$

This is also the equation for voltage rise in the opposite direction around the loop.

The *voltage rise* around the closed loop in the direction of current flow is

$$E - IZ - IZ_s - E' - IZ' = 0 \tag{12}$$

This is also the equation for voltage drop in the opposite direction around the loop. Either equation may be written

$$E - IZ = E' + IZ_s + IZ' \tag{13}$$

In [13] the voltage rise through machine 1 to its terminal P is equated to the voltage rise through machine 2 and the impedance Z_s to the point P. The rise in voltage from the reference point to point P is the same no matter which path is taken.

$$V_P = E - IZ = E' + IZ_s + IZ' \tag{14}$$

where V_P is the voltage rise from the reference point N to point P.

From [14],

$$I = \frac{E - E'}{Z + Z' + Z_s} \tag{15}$$

The magnitude and phase, relative to the reference vector, of the current I in the direction assumed can be obtained from [15] when E and E' are referred to the same reference vector.

If the current had been assumed to flow in the opposite direction to that indicated in Fig. 7(*b*), its magnitude would be unchanged but its sign would be reversed, giving a current 180° out of phase with that given by [15]. Therefore, a known current flowing in a given direction can be replaced by a current of equal magnitude flowing in the opposite direction and 180° out of phase with it.

The *average power* P at any point in a single-phase system or in each phase of a balanced polyphase system is

$$P = |V| \times |I| \cos \theta \qquad [16]$$

where $|V|$ and $|I|$ are the scalar values of the effective voltage and current at the point and θ is the phase angle between V and I.

If the point is at the terminals of a machine, V represents the voltage rise through the machine. If I is the current flowing from the machine, P in [16] is the power out of the machine. If this power is positive, the machine is generating power; if negative, it is absorbing power. If I is the current flowing into the machine, P in [16] is the power into the machine. If this power is positive, the machine is receiving power; if negative, it is generating power.

PER CENT AND PER UNIT QUANTITIES

The numerical per unit value of any quantity is its ratio to the chosen base quantity of the same dimensions, expressed as a decimal. For example, if base voltage is taken as 110 kv, voltages of 99 kv, 110 kv, and 115 kv will be 0.90, 1.00, and 1.045, respectively, when expressed in per unit on the given base voltage. The chosen base voltage, 110 kv, is referred to as base voltage, 100% voltage, or unit voltage. The numerical value of a quantity in per unit is equal to its value in per cent divided by 100, when the base quantities are the same for the per unit as for the per cent system. Per unit values are more convenient to use in calculations than per cent values. When quantities expressed in per cent are multiplied, it is necessary to divide the product by some power of 100 to obtain the correct result in per cent. The confusion caused by the introduction of powers of 100 is avoided in per unit calculations.

If *base impedance in ohms* is defined as that impedance which will have a voltage drop across it of 100% of base voltage when 100% current flows through it, base impedance will be determined when base voltage and current have been specified. Defined in per unit terms, *base or unit impedance in ohms* is that value of impedance which will have unit voltage drop across it when unit current flows through it. Because of the definition of base impedance and the fundamental relation between current, voltage, and kva in electrical circuits, any two of the four quantities, current, voltage, kva, and impedance, may be selected as the independent base quantities and the other two will then be determined. In most calculations, it is found convenient to select voltage and kva as the independent base quantities and to define base power in kilowatts as numerically equal to base kva.

Single-Phase Systems. With voltage and kva as the two independent base quantities and base power numerically equal to base kva, the following relations exist in a single-phase circuit:

$$\text{Base current in amperes} = \frac{\text{base kva}}{\text{base voltage in kv}} = \frac{\text{base kva}}{\text{base kv}} \qquad [17]$$

$$\text{Base impedance in ohms} = \frac{\text{base voltage in volts}}{\text{base current in amperes}}$$

$$= \frac{(\text{base kv})^2 \times 10^3}{\text{base kva}} \qquad [18]$$

$$\text{Base power in kw} = \text{numerical value of base kva} \qquad [19]$$

An impedance Z, given in ohms, can be expressed in per cent or per unit of base impedance, without first calculating base impedance, when voltage and kva are the two independent base quantities:

$$Z \text{ (in \%)} = 100 \frac{Z \text{ (in ohms)}}{\text{base ohms}} \qquad [20]$$

$$Z \text{ (in per unit)} = \frac{Z \text{ (in ohms)}}{\text{base ohms}} \qquad [21]$$

Replacing base ohms in [20] and [21] by $\dfrac{(\text{base kv})^2 \times 10^3}{\text{base kva}}$ from [18],

$$Z \text{ (in \%)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 10} \qquad [22]$$

$$Z \text{ (in per unit)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 1000} \qquad [23]$$

Solving [22] and [23] for Z in ohms,

$$Z \text{ (in ohms)} = \frac{Z \text{ (in \%)} \times (\text{base kv})^2 \times 10}{\text{base kva}} \qquad [24]$$

$$Z \text{ (in ohms)} = \frac{Z \text{ (in per unit)} \times (\text{base kv})^2 \times 1000}{\text{base kva}} \qquad [25]$$

where

Base kva = the single-phase base kva

Base kv = the single-phase base voltage in kv

Example. If base kva and voltage are given as 5000 kva and 10,000 volts, respectively, from [17]–[19]:

$$\text{Base current} = \tfrac{5000}{10} = 500 \text{ amp}$$

$$\text{Base impedance} = \frac{10{,}000}{500} = 20 \text{ ohms}$$

$$\text{Base power} = 5000 \text{ kw}$$

A given impedance Z of 2 ohms is 10%, or 0.1 per unit of the base impedance 20 ohms; or, directly from [23] without first calculating base impedance,

$$Z \text{ (in per unit)} = \frac{2 \times 5000}{(10)^2 \times 1000} = 0.1$$

Polyphase Systems. Since a polyphase system under balanced conditions can be represented by a one-line diagram with impedances and voltages to neutral, [17]–[25], developed for a single-phase system, apply also to the one-line diagrams of balanced polyphase systems, base voltage and kva being line-to-neutral voltage and kva per phase, respectively.

Three-Phase Systems. For three-phase equipment, rated kva is customarily given for the three phases and rated voltage is the line-to-line voltage. In calculations of balanced three-phase circuits made on a line-to-neutral basis, base voltage will be a line-to-neutral voltage, and base kva the kva per phase. Base kva per phase is one-third the three-phase base kva and the base line-to-neutral voltage is the base line-to-line voltage divided by $\sqrt{3}$. With these values of base kva and voltage, [17]–[25], developed for a single-phase circuit, apply as well for a line-to-neutral circuit.

Equations [17]–[25] apply also to a three-phase delta-connected circuit, if base voltage is a line-to-line voltage and base kva one-third the three-phase base kva. In balanced three-phase power systems, line-to-line voltages at any point in the system are $\sqrt{3}$ times the line-to-neutral voltages at that point, and currents in delta-connected windings are $1/\sqrt{3}$ times the line currents flowing from the delta. Expressed in per unit of their respective base values, line-to-line and line-to-neutral voltages at any point in the system are the same in magnitude; likewise per unit line currents flowing from a delta and the per unit delta currents, when referred to their respective base currents, are the same in magnitude.

In a line-to-neutral circuit, an impedance to neutral Z, given in ohms, may be expressed in per cent or per unit of base impedance to neutral by equations involving three-phase base kva and line-to-line base voltage. It can be seen that the use of

$$\frac{\text{Three-phase base kva}}{3} \quad \text{and} \quad \frac{\text{line-to-line base voltage}}{\sqrt{3}}$$

in [22]–[25] will give a factor 3 in both numerator and denominator; so that, for three-phase circuits, [22]–[25] can be rewritten

$$Z \text{ (in \%)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 10} \tag{26}$$

$$Z \text{ (in per unit)} = \frac{Z \text{ (in ohms)} \times \text{base kva}}{(\text{base kv})^2 \times 1000} \tag{27}$$

$$Z \text{ (in ohms)} = \frac{Z \text{ (in \%)} \times (\text{base kv})^2 \times 10}{\text{base kva}} \tag{28}$$

$$Z \text{ (in ohms)} = \frac{Z \text{ (in per unit)} \times (\text{base kv})^2 \times 1000}{\text{base kva}} \tag{29}$$

where

Z = impedance to neutral
Base kva = the three-phase base kva
Base kv = the line-to-line base voltage in kv

Equations [26]–[29] are given for convenience in terms of three-phase base kva and line-to-line base voltage. In stating the base quantities selected for the solution of a three-phase system problem, three-phase kva and line-to line voltage are usually given as the base quantities. It should be remembered, however, that in a line-to-neutral diagram, base or unit voltage is *line-to-neutral voltage* and base or unit kva is *kva per phase*. Thus, if it is stated that base kva is 100,000 kva and base voltage 110 kv, it is understood that these base quantities are three-phase kva and line-to-line voltage; therefore, in the line-to-neutral impedance diagram base kva is one-third of 100,000 kva = 33,333 kva, base kv is $\frac{110}{\sqrt{3}} = 63.5$ kv.

Change in Base Quantities. In electric systems, some impedances are ordinarily given in ohms; transmission line impedances, for example. Some are given in per cent or per unit based on their kva and voltage ratings; machine reactances, for example. It is often necessary therefore to convert ohms to per unit, or vice versa, and to transform per unit impedances from their own kva and voltage bases to the kva and voltage bases selected as base values for the system.

From [22], [23], [26], and [27], it is apparent that impedances in per cent and per unit vary directly as their kva bases and inversely as the squares of their voltage bases. Impedances in per unit on any given kva base may be expressed on a new kva base by the equation,

$$\begin{matrix}\text{Per unit impedance} \\ \text{on new kva base}\end{matrix} = \begin{matrix}\text{per unit impedance} \\ \text{on given kva base}\end{matrix} \times \frac{\text{new kva base}}{\text{given kva base}} \qquad [30]$$

Impedances in per unit on a given voltage base may be expressed on a new voltage base by the equation,

$$\begin{matrix}\text{Per unit impedance} \\ \text{on new voltage base}\end{matrix} = \begin{matrix}\text{per unit impedance} \\ \text{on given voltage base}\end{matrix} \times \left(\frac{\text{given base voltage}}{\text{new base voltage}}\right)^2. \qquad [31]$$

Advantages of Per Unit or Per Cent Impedances. It is customary to express the reactances of electrical equipment in per cent or per unit based on their ratings. Ordinarily the rated kva and voltage constitute the given base quantities. The characteristics of electrical apparatus of various types and ratings can be readily compared when their reactances are expressed in this manner. Such comparisons show that the reactances of synchronous machines of different ratings, but of the same general type, fall within definite limits and that the reactances of transformers of the same rated voltages do not differ by a wide margin. It is frequently possible, therefore, when certain reactances are unknown, to assume values for them which will be satisfactory for use in calculations. This is especially true when the quantities to be determined are but slightly affected by variations in the unknown reactances.

Per unit quantities can be used to great advantage in determining currents, voltages, etc., throughout a three-phase system in which there are many circuits connected by transformers, operating at different voltages. If impedances in ohms are used, one circuit is selected as reference and all impedances are referred to that circuit. To refer an impedance in ohms on one side of a transformer to the other side, it is necessary to multiply this impedance by the square of the transformer turn ratio. This is a relatively simple procedure when there are but few circuits. As the number of circuits is increased, the advantages of per unit impedances are more pronounced. Impedances expressed in per unit on a chosen kva base, with the ratio of base voltages on the two sides of a transformer equal to the transformer turn ratio, are the same referred to either side of the transformer. For example, consider a single-phase transformer rated 5000 kva, 13,800 volts–138,000 volts, connecting circuits 1 and 2. The turn ratio, determined from the rating, is $n_2/n_1 = 138{,}000/13{,}800 = 10$. From [23], 4 ohms in the low voltage circuit in per unit on 5000 kva and 13,800 volts is

$$\frac{4 \times 5000}{(13.8)^2 \times 10^3} = 0.105 \text{ per unit, referred to circuit 1}$$

Referred to the high voltage circuit, 4 ohms in the low voltage circuit is

$$4 \times (10)^2 = 400 \text{ ohms}$$

From [23], 400 ohms referred to the high voltage circuit in per unit on 5000 kva and 138 kv is

$$\frac{400 \times 5000}{(138)^2 \times 10^3} = 0.105 \text{ per unit, referred to circuit 2}$$

The impedance in per unit is 0.105 referred to either circuit.

In the illustration above, base impedances in the two circuits were not calculated, since per unit impedances were determined directly from the given base quantities, kva and voltage. An alternate method is first to determine base impedances in the two circuits by [18] and then to express the impedances referred to circuits 1 and 2 in per unit of their respective base impedances. From [18],

$$\text{Base impedance in ohms in circuit 1} = \frac{(13.8)^2 \times 10^3}{5000} = 38 \text{ ohms}$$

$$\text{Base impedance in ohms in circuit 2} = \frac{(138)^2 \times 10^3}{5000} = 3800 \text{ ohms}$$

4 ohms is $\frac{4}{38} = 0.105$ per unit of (38 ohms) base ohms in circuit 1

400 ohms is $\frac{400}{3800} = 0.105$ per unit of (3800 ohms) base ohms in circuit 2

Transformer impedances are usually given in per cent based on the transformer rating. In a two-winding transformer, the kva ratings of the windings are the same and the ratio of rated voltages is usually the same as the turn ratio. In a two-winding transformer, the impedance of the transformer with exciting current neglected, expressed in per cent or per unit on the kva rating, is the same referred to either winding of the transformer, if base voltages in the two windings are in proportion to the turns. This follows directly from the illustration above.

In the one-line per unit impedance-to-neutral diagram (positive-sequence impedance diagram) of a balanced three-phase power system, all impedances are expressed in per unit of the base impedance of the circuit in which they are located; or, with kva and voltage as the independent base quantities, all per unit impedances are based on the same kva and on the base voltages of their respective circuits. In a system consisting of several circuits connected by transformers, if base voltage is arbitrarily chosen for some particular circuit, base voltages in the other circuits are not independent but are determined by the transformer turn ratios. With transformer exciting currents neglected, they are the voltages in these circuits at no load when the arbitrarily chosen base voltage is the voltage of the selected circuit. This is illustrated in the following problem.

Problem 1. Given a three-phase, 60-cycle transmission system, consisting of: a three-phase transmission line; a three-phase generator rated 15,000 kva, 4 kv, with a reactance of 25%; a transformer bank of three single-phase transformers connected Δ–Y, each transformer rated 10,000 kva, 4200–66,500/115,000 Y volts with low voltage taps. The operating tap is 3900–66,500 volts, with a leakage reactance of 10% based on rated kva and tap voltages. Base three-phase kva and base line-to-line voltage have been arbitrarily chosen as 100,000 kva and 110 kv, respectively, in the transmission line. What is base line-to-line voltage in the generator circuit? What are the per unit positive-sequence reactances of generator and transformer on the chosen base quantities?

Solution. Base line-to-line voltage in the generator circuit is determined by the arbitrarily chosen base line-to-line voltage in the transmission circuit and the transformer turn ratio of the operating tap. With 110-kv line-to-line voltage in the transmission circuit, base line-to-line voltage in the generator circuit is $3.9 \times \frac{110}{115} = 3.72$ kv. On a 30,000-three-phase-kva base and a base line-to-line voltage of 115 kv (10,000 kva per phase, $115/\sqrt{3} = 66.5$-kv line-to-neutral voltage), the transformer bank has a reactance of 10%. On a three-phase-kva base of 100,000 kva and a line-to-line voltage base of 110 kv (33,330 kva per phase, $110/\sqrt{3} = 63.5$-kv line-to-neutral voltage), the transformer leakage reactance in per unit from [30] and [31] is

$$0.10 \times \frac{100{,}000}{30{,}000} \times \left(\frac{115}{110}\right)^2 = 0.364$$

The per unit generator reactance on 100,000 kva and 3.72 kv, the base voltage of the generator circuit, is

$$0.25 \times \frac{100{,}000}{15{,}000} \times \left(\frac{4.0}{3.72}\right)^2 = 1.93$$

COMPONENTS OF CURRENT AND VOLTAGE AND SUPERPOSITION

The division of currents and voltages into components is not unfamiliar. Balanced currents and voltages are often divided into two components, one component in phase with a particular voltage or current used as a reference vector and the other component 90° out of phase with it. The voltage drop caused by a current divided into components is obtained by adding vectorially the drops due to its components.

Validity of the Use of Components. The calculation of currents and voltages in electric circuits by adding currents due to components of voltage, and voltages due to components of current, is justified by the *principle of superposition*. This principle states that the response to a force can be determined by adding the responses to the components of the force, provided the responses vary directly with the forces applied, i.e., if the equations involved are *linear*. Superposition can be rigorously applied to electric circuits only when the values of the circuit constants or parameters (resistance, leakance, inductance, capacitance) are independent of the current, voltage, or frequency which may be associated with them. Although the usual resistances, leakances, and capacitances in power circuits are not strictly constant, they may be approximately so within certain limts. This is also true of inductances where the flux paths are in non-magnetic mediums. Inductances associated with iron, where the flux density is not high over the current range of the circuit, may in many cases be considered to have essentially linear characteristics. When saturation must be taken into consideration, constant values of reactance *adjusted* to allow for saturation can frequently be used to advantage. Cases in which the effect of saturation upon reactance must be taken into account are pointed out in later chapters.

IMPEDANCE NETWORKS

The principle of superposition will be applied to impedance networks in which the self-impedances of the circuits and the mutual impedances between circuits at constant frequency are assumed constant.

Self- and Mutual Impedances. The *self-impedance* of a circuit is the ratio of the voltage drop in the circuit in the direction of current flow to the current when all other circuits are open. The *mutual impedance* between two circuits is the ratio of the voltage drop induced in one of the circuits to the current in the other circuit which induces it. Self-impedance will be indicated by Z with two subscripts, both referring to the circuit. The self-impedance of circuit A is Z_{aa}. The mutual impedance between two circuits will be indicated by Z with two subscripts which refer to the two circuits. Let I_a, I_b, I_c, $\cdots I_n$ and V_a, V_b, V_c, $\cdots V_n$ represent currents and voltage drops in circuits A, B, C, $\cdots N$, respectively, in the positive direction. With I_a flowing in circuit A and all other circuits open, $Z_{ba} = V_b/I_a$, $Z_{na} = V_n/I_a$, etc.

With I_b flowing in circuit B and all other circuits open, $Z_{ab} = V_a/I_b$, $Z_{nb} = V_n/I_b$, etc. With all circuits closed, the voltage drops are

$$\begin{aligned}
V_a &= I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} + \cdots + I_nZ_{an} \\
V_b &= I_aZ_{ba} + I_bZ_{bb} + I_cZ_{bc} + \cdots + I_nZ_{bn} \\
V_c &= I_aZ_{ca} + I_bZ_{cb} + I_cZ_{cc} + \cdots + I_nZ_{cn} \qquad [32] \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
V_n &= I_aZ_{na} + I_bZ_{nb} + I_cZ_{nc} + \cdots + I_nZ_{nn}
\end{aligned}$$

In [32] the first subscript of Z refers to the voltage and the second to the current associated with it. If the mutual impedances are the same viewed from either circuit, $Z_{ab} = Z_{ba}$, $Z_{cn} = Z_{nc}$, etc. As the voltage drop in any circuit is equal to the voltage applied to the circuit, $V_a, V_b, V_c, \cdots V_n$ in [32] can be equated to the applied voltages. If no voltage is applied to a closed circuit, the total voltage drop in it is zero. If any circuit is opened, its current becomes zero.

Driving-Point and Transfer Impedances. Let $E_1, E_2, \cdots E_n$ represent the voltages at terminal points 1, 2, $\cdots n$ of a network, where voltages are applied between the terminals and the zero-potential bus, and let $I_1, I_2, \cdots I_n$ be the corresponding currents. Positive direction of current flow is assumed to be *into the network* from the point at which voltage is applied. The resultant current which flows into the network from point 1 will be the difference between the component current sent into the network by E_1 acting alone and the sum of the component currents which the voltages $E_2, E_3, \cdots E_n$, each acting alone, send out the network at point 1. When a voltage is assumed to act alone, the other voltages of the network are reduced to zero and their points of application connected to the zero-potential bus through zero impedance. When the voltage E_1 is applied between point 1 and the zero-potential bus, with the other voltages of the system reduced to zero, there will be a current entering the network at point 1 and currents leaving the network at points 2, 3, $\cdots n$. The current entering the network at point 1 will depend upon the applied voltage and the impedance offered to it. This impedance is the *driving-point* impedance of point 1 and is defined as the ratio of the voltage at point 1 to current entering the network at 1, with no other voltages applied to the network. The current leaving the network at point 2 will likewise depend upon the voltage applied at point 1 and an impedance which takes into account the impedances of the other paths to the zero-potential bus as well as those in the direct path between points 1 and 2. This impedance is called the *transfer* impedance between points 1 and 2, and is defined as the ratio of the voltage at point 1 to the current at point 2 with the other voltages of the system reduced to zero. It can be shown[1] that in a static network composed of linear, bilateral elements without internal sources of energy, the transfer impedance between points 1 and 2 is the same as that between points 2 and 1, provided the applied voltages are of the same frequency. Such a network is called a *reciprocal* network. A bilateral element is a two-terminal circuit which has the same impedance viewed from either terminal. Any two-terminal rectifying device is a unilateral element.

The currents flowing into the network at the various points may be obtained by adding the currents produced by the various applied voltages, each acting alone, provided the network is a reciprocal one and superposition can be applied. Adding the currents due to each voltage acting alone,

$$
\begin{aligned}
I_1 &= \frac{E_1}{A_{11}} - \frac{E_2}{A_{21}} - \cdots - \frac{E_n}{A_{n1}} \\
I_2 &= -\frac{E_1}{A_{12}} + \frac{E_2}{A_{22}} - \cdots - \frac{E_n}{A_{n2}} \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
I_n &= -\frac{E_1}{A_{1n}} - \frac{E_2}{A_{2n}} \mp \cdots + \frac{E_n}{A_{nn}}
\end{aligned}
\qquad [33]
$$

where

$A_{11}, A_{22}, \cdots, A_{nn}$ = driving-point impedance at points 1, 2, and n, respectively

$A_{12}, \cdots, A_{1n}$ = transfer impedance between points 1 and 2, $\cdots$, between points 1 and n

Driving-point and transfer impedances are designated by double subscripts, the first subscript referring to the point at which the voltage is applied and the second to the point where current is measured. The letter A is used here rather than the usual Z to avoid confusing driving-point and transfer impedance with self- and mutual impedance, which are likewise designated by double subscripts. Equation [33] can also be written in terms of transfer and driving-point admittances, where admittances are reciprocals of the corresponding impedances.

ONE-LINE IMPEDANCE DIAGRAMS AND EQUIVALENT CIRCUITS

Two-Conductor Single-Phase Power System. When the problem is to determine the rms voltage between conductors at any point in the system or the line currents during normal operation or with faults between conductors, the two-conductor, single-phase system can be represented by a one-line impedance diagram in which the impedances are lumped impedances, equal to the sum of the equivalent impedances met by the outgoing and return current. The return path for current in the one-line diagram is a conductor, or bus, of zero impedance at zero potential. The current at any point in the one-line diagram represents the single-phase current at that point, and the voltage to the neutral bus represents the voltage between conductors at that point.

Balanced Three-Phase Power System. When the problem is to determine rms line currents and voltages to neutral, during normal operations or during three-phase balanced faults, the three-phase system can be represented by a one-line impedance diagram in which the impedances are impedances to neutral. The one-line diagram may be considered the diagram of one phase of the balanced system with a return path for the currents in a neutral conductor or bus of zero impedance at zero potential. The currents in the diagram are line currents and the voltage between any point and the neutral bus is the voltage to neutral at that point. The diagram of a balanced three-phase power system is called a positive-sequence diagram because the phase order of the balanced voltages at any point in the system is the same as the phase order of the generated voltage, and therefore positive.

In the one-line impedance diagram of a single-phase or a balanced three-phase power system, each piece of apparatus and each transmission circuit has its own *equivalent circuit*. These equivalent circuits will depend upon the purpose for which the one-line impedance diagram is to be used and the degree of precision required in the calculations to be made from it; therefore, under different conditions, the same piece of apparatus may have different equivalent circuits.

An oscillogram of the currents or voltages at a given point in a system during a disturbance may be shown, in addition to fundamental-frequency components, components of other frequencies. These components may include d-c components, one or more natural-frequency components determined by the system inductances and capacitances, even and odd harmonics, and sometimes subharmonics. Subharmonic frequencies are frequencies less than fundamental. For example, a third subharmonic in a 60-cycle system has a frequency of 20 cycles per second. The magnitude of a fundamental frequency component, in general, does not remain constant throughout the disturbance but decreases from its initial or subtransient value to its transient value and eventually reaches its steady-state or sustained value. With d-c components, natural-frequency components, harmonics, and subharmonics neglected, equivalent circuits for determining fundamental-frequency currents and voltages will depend upon whether subtransient, transient, or sustained currents and voltages are required. In transmission lines, transformers, and other static equipment the transition from subtransient to sustained impedance usually takes place within a fraction of a cycle of fundamental frequency. It is only in rotating equipment, that a distinction is made between subtransient, transient, and synchronous reactances when sinusoidal currents and voltages of fundamental frequency are to be determined.

An *equivalent circuit* for use in analytic calculations is a static network composed of self-impedance branches without mutual impedances between them which under the same operating conditions will produce, to the required degree of precision, the same conditions electrically at its terminals as the actual circuit (with or without mutual impedances) which it replaces. It should be noted that it is only at the *terminals* of the actual and equivalent circuits that the currents and voltages must be the same.

The number of branch impedances in an equivalent circuit, in its simplest form, will depend upon the number of terminals in the circuit it is to replace. A two-terminal equivalent circuit will have but one impedance. A three-terminal equivalent circuit will have three branch impedances, which may be connected either in Y or in Δ. A four-terminal equivalent circuit, in the general case, will have six branch impedances. The number of branch impedances is determined by the number of connections which can be made between terminals. The problem of determining the equivalent circuit for a given actual circuit (or combination of actual circuits) is that of determining the branch impedances of the equivalent circuit. One method of procedure is:

1. Draw the desired equivalent circuit with the required number of terminals, indicating the unknown branch impedances by symbols.
2. Test the given circuit and the assumed equivalent circuit by applying a voltage between the terminals taken two at a time, with the other terminals free, and determining the currents.
3. Equate the ratios of the applied voltages to the resultant currents in the given and equivalent circuits.

A second method of determining equivalent circuits is:

(1) Draw an equivalent circuit with the same number of terminals as the actual circuit, with each terminal connected to every other terminal through an impedance. (A three-terminal equivalent circuit will be a Δ.)

(2) With all terminals in the actual circuit connected to a common point by leads, insert a voltage in each lead in turn and measure the currents in the other leads.
(3) The voltage inserted in the lead to any terminal A divided by the current flowing in the lead from a second terminal B gives the impedance between terminals A and B to be used in the equivalent circuit. This is the transfer impedance between terminals A and B.

As the transfer impedance between any two terminals is the same measured from either terminal in a reciprocal static network, it is unnecessary to make all the measurements described in (2) above.

For networks which can be set up on a d-c calculating table or on an a-c network analyzer the second method for determining equivalent circuit is the more direct. Some of the branch impedances of an equivalent circuit determined on the a-c network analyzer may contain negative resistances; this makes the equivalent circuit unsuitable for subsequent use on the a-c network analyzer unless special negative resistance devices are used. Negative resistance offers no difficulty in an analytic solution.

In determining an equivalent circuit of a specified number of terminals, either of the methods may be used, or a combination of the two methods. Driving-point impedance is not required in the second method, but it may often be used to advantage in connection with the first method.

The following discussion of equivalent circuits assumes constant frequency and constant circuit parameters (the effect of saturation is not included).

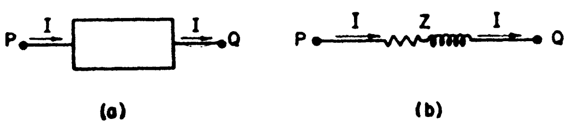


FIG. 8. (a) Two-terminal circuit. (b) Equivalent circuit to replace (a).

Two-Terminal Equivalent Circuit. A two-terminal circuit is shown in Fig. 8(a) and its assumed equivalent circuit in Fig. 8(b). The given circuit in Fig. 8(a) is represented as a box with two terminals, P and Q. The box may represent any impedance network between P and Q, with or without mutual impedances between the branches. Since there are only two terminals, the equivalent circuit will have but a single impedance branch, Z. For the circuit (b) to be the equivalent of (a) with the same voltage V applied between its terminals, the current I entering and leaving the equivalent circuit must be the same as that entering and leaving the given circuit. The impedance Z of the equivalent circuit may be determined by applying a voltage V between terminals P and Q in the given circuit and measuring the current I; then $Z = V/I$.

Two Circuits with Self-Impedances Z_{aa} and Z_{bb} Connected at Both Ends with Mutual Impedance Z_{ab} between Them. The given circuit for this case is shown in Fig. 9(a) and the desired equivalent circuit in

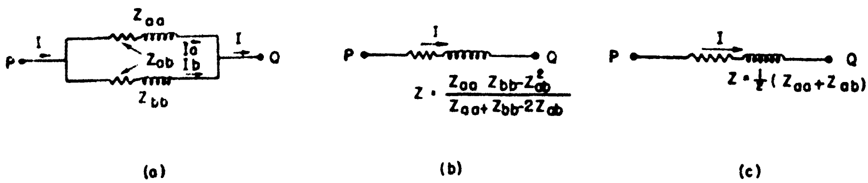


FIG. 9. (a) Two circuits connected at their terminals, with self-impedances Z_{aa} and Z_{bb}, and mutual impedance Z_{ab} between them. (b) Equivalent circuit for (a), $Z_{aa} \neq Z_{bb}$. (c) Equivalent circuit for (a), $Z_{aa} = Z_{bb}$.

Fig. 9(b), with impedance Z to be determined. The impedance Z of the equivalent circuit can be obtained as follows:

Applying a voltage V between terminals P and Q of the given circuit,

$$V = I_aZ_{aa} + I_bZ_{ab}$$

$$V = I_aZ_{ab} + I_bZ_{bb}$$

where

I_a and I_b = the currents in circuits a and b, respectively

and

$I = I_a + I_b$ = total current entering the circuit at terminal P and leaving at terminal Q

Solving for I_b in terms of I_a,

$$I_b = I_a \frac{Z_{aa} - Z_{ab}}{Z_{bb} - Z_{ab}} \qquad [34]$$

Substituting [34] in the original equations,

$$V = I_a \frac{Z_{aa}Z_{bb} - Z_{ab}^2}{Z_{bb} - Z_{ab}} \qquad [35]$$

From [34] and [35],

$$I_a = V \frac{Z_{bb} - Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

$$I_b = V \frac{Z_{aa} - Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

Adding I_a and I_b,

$$I = I_a + I_b = V\frac{Z_{aa} + Z_{bb} - 2Z_{ab}}{Z_{aa}Z_{bb} - Z_{ab}^2}$$

The impedance of the equivalent circuit is

$$Z = \frac{V}{I} = \frac{Z_{aa}Z_{bb} - Z_{ab}^2}{Z_{aa} + Z_{bb} - 2Z_{ab}} \tag{36}$$

For identical circuits, $Z_{aa} = Z_{bb}$ and [36] becomes

$$Z = \frac{Z_{aa}^2 - Z_{ab}^2}{2(Z_{aa} - Z_{ab})} = \frac{Z_{aa} + Z_{ab}}{2} \tag{37}$$

The equivalent circuit for two identical circuits with equal self-impedances Z_{aa}, connected at both ends, with mutual impedance Z_{ab} between them is shown in Fig. 9(*c*).

Three-Terminal Circuits. A three-terminal circuit can be represented by either a Y or a Δ. Figure 10(*a*) shows three terminals *a*, *b*, and *c*, connected by a Δ with self-impedance branches Z_{ab}, Z_{ac}, Z_{bc}.

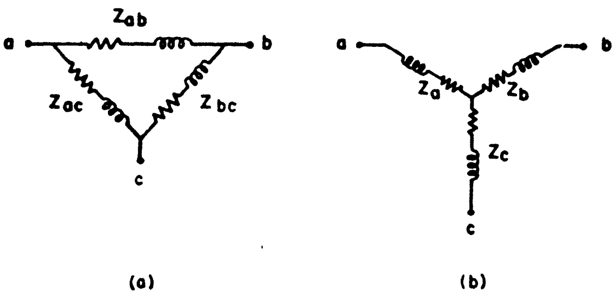


FIG. 10. (*a*) Δ-connected circuit between terminals *a*, *b*, and *c*. (*b*) Y-connected circuit between terminals *a*, *b*, and *c*.

Figure 10(*b*) shows the same terminals connected by a Y with self-impedance branches Z_a, Z_b, and Z_c. The Δ and Y must be exact equivalents of each other for conditions at their terminals. The relations[2] between the branch impedances of the Y and Δ circuits, determined by applying a voltage between the terminals taken two at a time with the third terminal open in both the Δ and the Y, and equating V/I for the two circuits, are

$$\begin{aligned} Z_a + Z_b &= \frac{Z_{ab}(Z_{ac} + Z_{bc})}{Z_{ab} + Z_{ac} + Z_{bc}} \\ Z_a + Z_c &= \frac{Z_{ac}(Z_{ab} + Z_{bc})}{Z_{ab} + Z_{ac} + Z_{bc}} \\ Z_b + Z_c &= \frac{Z_{bc}(Z_{ab} + Z_{ac})}{Z_{ab} + Z_{ac} + Z_{bc}} \end{aligned} \tag{38}$$

Subtracting each of the equations of [38] in turn from the sum of the other two, and dividing the resulting equation by 2,

$$
\begin{aligned}
Z_a &= \frac{Z_{ab}Z_{ac}}{Z_{ab} + Z_{ac} + Z_{bc}} \\
Z_b &= \frac{Z_{ab}Z_{bc}}{Z_{ab} + Z_{ac} + Z_{bc}} \\
Z_c &= \frac{Z_{ac}Z_{bc}}{Z_{ab} + Z_{ac} + Z_{bc}}
\end{aligned} \qquad [39]
$$

The branch impedances of the Y in terms of the Δ impedances are given by [39].

Equating the transfer impedances between terminals in the Δ to those in the Y,

$$
\begin{aligned}
Z_{ab} &= Z_a + Z_b + \frac{Z_aZ_b}{Z_c} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_c} \\
Z_{ac} &= Z_a + Z_c + \frac{Z_aZ_c}{Z_b} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_b} \\
Z_{bc} &= Z_b + Z_c + \frac{Z_bZ_c}{Z_a} = \frac{Z_aZ_b + Z_aZ_c + Z_bZ_c}{Z_a}
\end{aligned} \qquad [40]
$$

The branch impedances of the Δ in terms of the Y impedances are given by [40].

Equal Impedances in the Three Branches of the Δ or Y. If $Z_{ab} = Z_{ac} = Z_{bc}$, from [39],

$$Z_a = Z_b = Z_c = \frac{Z_{ab}}{3} \qquad [41]$$

If $Z_a = Z_b = Z_c$, from [40],

$$Z_{ab} = Z_{ac} = Z_{bc} = 3Z_a \qquad [42]$$

From [41] and [42] it follows that the branch impedances of an equivalent Y are one-third those of a given symmetrical Δ; and the branch impedances of the equivalent Δ are three times those of a given symmetrical Y. These relations hold when impedances are expressed in *ohms* or in *per unit on the same kva and voltage bases.*

If the impedances in [41] and [42], given in ohms, are expressed in per unit on the same base kva per phase, with Δ impedances based on line-to-line voltage and those of the Y on line-to-neutral voltage, from [23] they will be the same. The Δ impedances in ohms are three times the Y impedances, but the square of base line-to-line voltage is three times the square of base line-to-neutral voltage; therefore the impedances of the Y and Δ are equal when expressed in per unit on the same kva base and the base voltages of their respective circuits. This makes it possible to replace a symmetrical three-phase self-impedance circuit connected in Δ by an equivalent Y of the same per unit impedance per phase when conditions *outside* the circuit are considered, and per unit quantities are used.

A Circuit Consisting of Two Self-Impedances Z_{aa} and Z_{bb} Connected at One End, with Mutual Impedance Z_{ab} between Them. Figure 11 (*a*) shows the given circuit and Fig. 11 (*b*) the equivalent Y. The branches

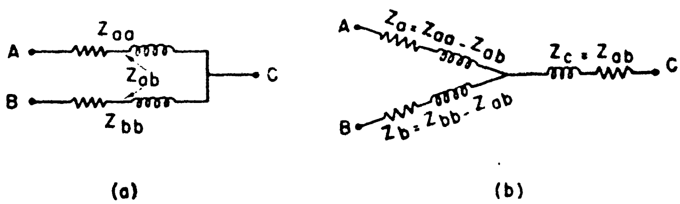


FIG. 11. (*a*) Two circuits connected at one set of terminals, with self-impedances Z_{aa} and Z_{bb} and mutual impedance Z_{ab} between them. (*b*) Equivalent circuit for (*a*).

Z_a, Z_b, and Z_c of the equivalent Y are determined by applying a voltage between terminals *A* and *C* with *B* isolated, between *B* and *C* with *A* isolated, and between *A* and *B* with *C* isolated, then equating V/I in each case.

The first two equations can be determined by inspection. The impedance between terminals *A* and *C* in the given circuit with terminal *B* isolated is Z_{aa}, and in the equivalent circuit $Z_a + Z_c$. Similarly, the impedance between terminals *B* and *C* with *A* isolated is Z_{bb} in the given circuit and $Z_b + Z_c$ in the equivalent circuit. With a voltage *V* applied between terminals *A* and *B* in the equivalent circuit, the impedance is $Z_a + Z_b$; in the given circuit, the current *I* enters at *A* and leaves at *B*, flowing in opposite directions through the self-impedances Z_{aa} and Z_{bb} in series. The voltage drop between terminals *A* and *B* is therefore

$$V = IZ_{aa} - IZ_{ab} + IZ_{bb} - IZ_{ab} = I(Z_{aa} + Z_{bb} - 2Z_{ab})$$

The three equations expressing Z_a, Z_b, and Z_c in terms of Z_{aa}, Z_{bb}, and Z_{ab} are

$$Z_a + Z_c = Z_{aa}$$

$$Z_b + Z_c = Z_{bb}$$

$$Z_a + Z_b = Z_{aa} + Z_{bb} - 2Z_{ab}$$

Solving these equations,

$$\begin{aligned} Z_a &= Z_{aa} - Z_{ab} \\ Z_b &= Z_{bb} - Z_{ab} \\ Z_c &= Z_{ab} \end{aligned} \qquad [43]$$

The values of Z_a, Z_b, and Z_c are indicated in Fig. 11(*b*). If terminals *A* and *B* of Fig. 11(*b*) are now connected, the circuit becomes a two-terminal circuit and checks Fig. 9(*b*) with $Z_{aa} \neq Z_{bb}$ and Fig. 9(*c*) if $Z_{aa} = Z_{bb}$.

Four-Terminal Circuits. A four-terminal circuit is shown in Fig. 12(*a*) as a box with four terminals. The general equivalent circuit

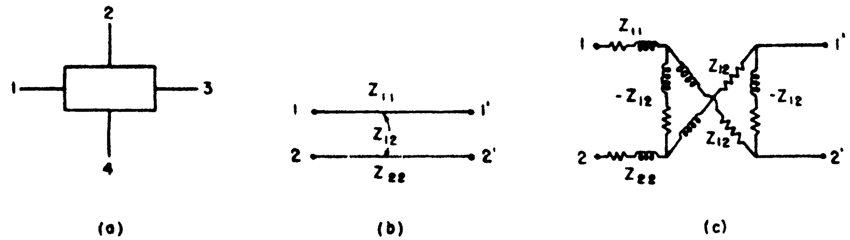


FIG. 12. (*a*) General four-terminal circuit. (*b*) Four-terminal circuit consisting of two circuits with self-impedances Z_{11} and Z_{22} and mutual impedance Z_{12} between them. (*c*) Equivalent circuit for (*b*).

will have a minimum of six branch impedances which can be evaluated when the nature of the given circuit is known. Two four-terminal circuits will be considered.

A Circuit Consisting of Two Self-Impedances Z_{11} and Z_{22}, Not Connected at Either End, with Mutual Impedance Z_{12} between Them. Figure 12(*b*) shows the given four-terminal circuit. The impedances directly connecting the four terminals of the equivalent circuit, determined from the transfer impedances in accordance with the second method, are[3,4]

$Z_{11} - \dfrac{Z_{12}^2}{Z_{22}}$ between terminals 1 and 1′

$Z_{22} - \dfrac{Z_{12}^2}{Z_{11}}$ between terminals 2 and 2′ [44]

$\dfrac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}}$ between terminals 1 and 2 and also between terminals 1′ and 2′

$-\dfrac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}}$ between terminals 1 and 2′ and also between terminals 2 and 1′

A simplification[5] of the equivalent circuit obtained by using the impedances given in [44] is shown in Fig. 12(*c*). In this equivalent circuit, the self-impedances Z_{11} and Z_{22} are placed in the terminal links outside the mesh. The impedances in the four-terminal mesh are then obtained from [44] for circuits with zero self-impedances: $Z_{11} = Z_{22} = 0$. For this condition, the impedances given by the first two equations in [44] are infinite; the other two equations give $-Z_{12}$ and Z_{12}, respectively. The self-impedance Z_{11} may be placed in a terminal link at either 1 or at 1′, and likewise Z_{22} may be placed at either 2 or 2′. If terminals 1′ and 2′ are connected, Fig. 12(*c*) becomes Fig. 11(*b*); if 1 and 2 are also connected, Fig. 12(*c*) becomes Fig. 9(*b*).

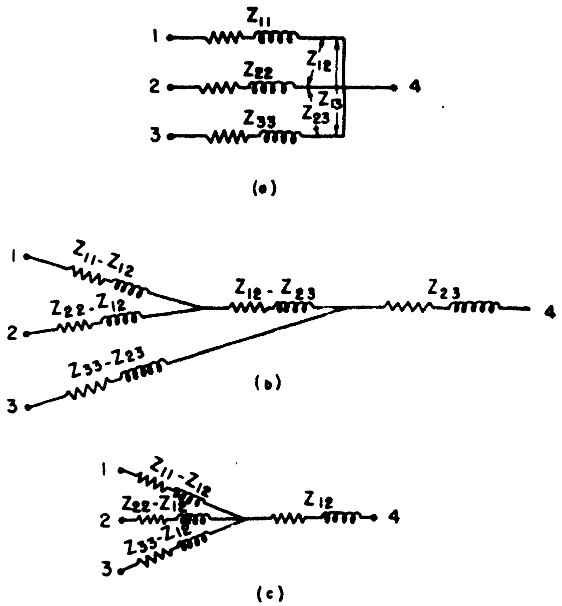


FIG. 13. (*a*) Four-terminal circuit consisting of three circuits connected at one set of terminals with self-impedances Z_{11}, Z_{22}, and Z_{33} and mutual impedances Z_{12}, Z_{13}, and Z_{23} between circuits. (*b*) Equivalent circuit for (*a*), if $Z_{13} = Z_{23} \neq Z_{12}$. (*c*) Equivalent circuit for (*a*), if $Z_{13} = Z_{23} = Z_{12}$.

A Circuit Consisting of Three Self-Impedances Z_{11}, Z_{22}, and Z_{33}, Connected at One End, with Mutual Impedances between Them — (a) Two Equal Mutual Impedances: $Z_{13} = Z_{23} \neq Z_{12}$. Figure 13(*a*) shows the given four-terminal circuit. Figure 13(*b*) is the equivalent circuit. That (*b*) is the equivalent of (*a*) can be shown by applying voltages between the four terminals in (*a*) and (*b*) taken two at a time with the other two terminals open and each time equating V/I in the two circuits.

(*b*) *Three Equal Mutual Impedances:* $Z_{13} = Z_{23} = Z_{12}$. With $Z_{23} = Z_{12}$ in Fig. 13(*b*), the equivalent circuit becomes that shown in Fig. 13(*c*).

Special Equivalent Circuits for Use in the One-Line Impedance Diagrams

Two-Winding Transformer. The two-winding transformer has four terminals. In most system calculations, however, it may be represented by a two- or a three-terminal equivalent circuit. An equivalent circuit has the same number of terminals as the equivalent of the circuit it replaces, but not necessarily the same number of terminals as the actual circuit itself. This is allowable, because the equivalent circuit is part of a one-line impedance diagram of the system to be used for certain specified calculations. Therefore certain connections can be made in the actual circuit which do not really exist, but, if they did exist, they would in no way affect the calculations to be made for the *given problem*.

Figure 14(*a*) shows a two-winding transformer with terminals A, A', B, and B'. A single-phase voltage E is applied to the primary winding AA'. The secondary winding BB' supplies a single-phase load through a two-conductor transmission line. The problem is to determine the equivalent circuit for the transformer to be used in a one-line impedance diagram of the single-phase system for calculating currents and voltages between conductors for various load conditions, or the currents during short circuits between conductors.

If the transformer in Fig. 14(*a*) with the four terminals A, A', B, B' is represented as in 14(*b*) with A' and B' connected, the given problem is in no way affected. The equivalent circuit, however, is required to replace a three-terminal equivalent of the actual circuit and not a four-terminal one. Since there are but three terminals, the equivalent circuit may be either an equivalent Y or Δ. An equivalent Y will be used, as shown in Fig. 14(*c*), with branch impedances Z_1, Z_2, and Z_3 to be determined.

Let the per unit self-impedance of winding AA' be Z_{11} and that of winding BB' be Z_{22} with mutual impedance Z_{12} between windings. There are three unknown impedances in the assumed equivalent circuit, Z_1, Z_2, and Z_3; therefore three tests will be required on the actual transformer to determine the desired equivalent circuit. Positive directions for currents are indicated by arrows in both circuits. Applying a voltage between A and A' with B isolated in Figs. 14(*b*) and (*c*), and then between B and B' with A isolated, reading the currents in both cases, and equating V/I in the transformer and in the equivalent circuit, two equations are obtained:

$$Z_{11} = Z_1 + Z_3 \qquad [45]$$

$$Z_{22} = Z_2 + Z_3 \qquad [46]$$

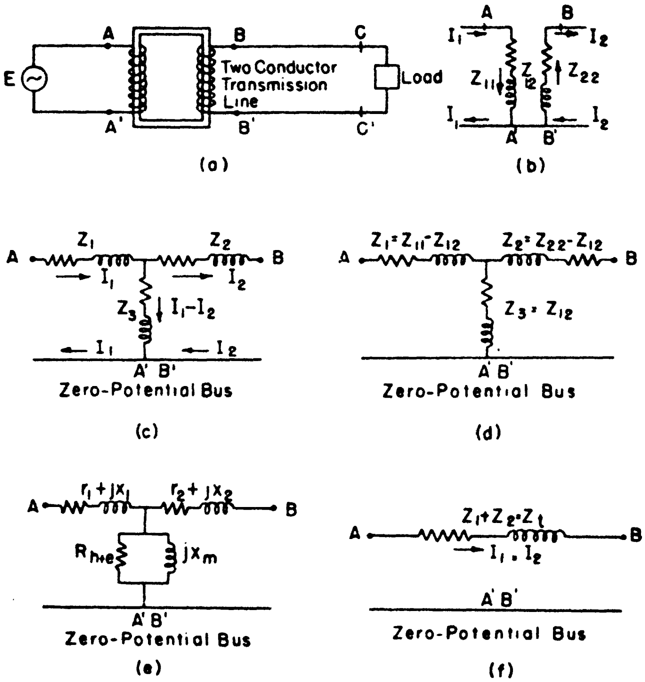


FIG. 14 (*a*) Two-line diagram of a single-phase two-conductor system with single-phase transformer. (*b*) Points A' and B' connected to give an equivalent of the transformer in (*a*) which satisfies a given problem. (*c*) Assumed equivalent circuit for (*b*) with Z_1, Z_2, and Z_3 to be determined. (*d*), (*e*) Three-terminal equivalent circuits for (*b*) with exciting current included. (*f*) Two-terminal equivalent circuit for (*b*) with exciting current neglected.

The third test can be made on the transformer by applying a voltage V to one winding AA' with the other winding BB' short-circuited, and in the equivalent circuit by applying a voltage between A and the zero-potential bus, with B connected to this bus. In the transformer,

$$V = I_1Z_{11} - I_2Z_{12}, \quad \text{in winding } AA'$$

$$0 = -I_1Z_{12} + I_2Z_{22}, \quad \text{in winding } BB'$$

Eliminating I_2 in these equations,

$$V = I_1\left(Z_{11} - \frac{Z_{12}^2}{Z_{22}}\right), \quad \text{and} \quad \frac{V}{I_1} = Z_{11} - \frac{Z_{12}^2}{Z_{22}}$$

In the equivalent circuit,

$$\frac{V}{I_1} = Z_1 + \frac{Z_2Z_3}{Z_2 + Z_3}$$

Therefore

$$Z_{11} - \frac{Z_{12}^2}{Z_{22}} = Z_1 + \frac{Z_2Z_3}{Z_2 + Z_3} \qquad [47]$$

Subtracting [47] from [45],

$$\frac{Z_{12}^2}{Z_{22}} = Z_3 - \frac{Z_2Z_3}{Z_2 + Z_3} = \frac{Z_3^2}{Z_2 + Z_3} \qquad [48]$$

Equations [45], [46], and [48] are satisfied if

$$\begin{aligned} Z_1 &= Z_{11} - Z_{12} \\ Z_2 &= Z_{22} - Z_{12} \\ Z_3 &= Z_{12} \end{aligned} \qquad [49]$$

The equivalent circuit for the transformer is shown in the one-line impedance diagram of Fig. 14(*d*), in which

$Z_1 = r_1 + jx_1 = Z_{11} - Z_{12} =$ leakage impedance of winding AA'

$Z_2 = r_2 + jx_2 = Z_{22} - Z_{12} =$ leakage impedance of winding BB'

$Z_3 = Z_{12} =$ mutual impedance between windings. Z_{12} is usually represented by x_m, the magnetizing reactance, paralleled with R_{h+e}, representing the hysteresis and eddy current losses, as in Fig. 14(*e*).

In transformers, the sum of the primary and secondary leakage reactances, $x_1 + x_2$, may be of the order of 10% and x_m at normal voltage about 3000%. For this reason, there is no measurable difference between the per unit self-impedance of the windings at normal voltage and the exciting or mutual impedance; therefore tests 1 and 2 described above would give but one per unit impedance. Usually but two tests are made, either 1 or 2, which gives the mutual impedance, and 3, which (with Z_3 large with respect to Z_1 and Z_2) gives $Z_1 + Z_2$, the sum of the resistances and leakage reactances of the two windings. In many problems the mutual impedance is considered infinite and the transformer replaced by a series impedance, $Z_t = Z_1 + Z_2$. It is then a two-terminal circuit without connection to the zero-potential bus, as shown in Fig. 14(*f*).

When Fig. 14(*d*) is compared with 11(*b*), it is seen that the equivalent circuit for the two-winding transformer for use in the one-line impedance diagram could have been obtained directly from the general three-terminal equivalent circuit representing two self-impedances connected at one end, with mutual impedance between them.

Two-Winding Transformer Banks in Balanced Three-Phase Systems. In three-phase power systems, the windings of the three transformers comprising the bank may be connected either in Δ or in Y. There are four possible arrangements. The primary windings may be connected either in Δ or in Y, and with either arrangement the secondary windings may likewise be connected either in Δ or in Y. With identical transformers in the bank, the circuits are symmetrical, and therefore the bank if connected Δ–Δ, Δ–Y, or Y–Δ can be replaced by an equivalent bank connected Y–Y, when conditions *outside* the bank are to be determined. Expressed in per unit on the rating of the transformer, the self-impedances of the windings and the mutual impedance between them are the same in the equivalent Y–Y bank as in the given bank. The *difference in phase* of the currents and voltages on opposite sides of a Δ–Y transformer bank is discussed in Chapter III.

With a Y–Y bank (or an equivalent Y–Y bank) in a balanced three-phase power system, the neutral points of the Y's may be connected without affecting the balanced system. Then the equivalent circuit for the bank, to be used in the one-line impedance-to-neutral diagram of the balanced system for determining currents and voltages outside the bank, will be a three-terminal circuit, one terminal to be connected to the primary circuit, one to the secondary circuit, and the third to the neutral bus. Since per unit impedances in a line-to-neutral diagram are based on line-to-neutral voltage and the kva per phase, the per unit equivalent circuit for use in the one-line diagram will be the same as that of any one of the three single-phase units. See Fig. 14(*e*) if exciting current is considered, and Fig. 14(*f*) if neglected. Figure 15 gives the equivalent circuit of a Δ–Y connected transformer bank for use in a one-line diagram of a balanced three-phase system.

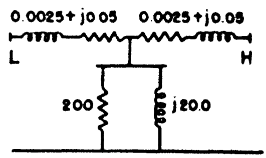


FIG. 15. Per unit equivalent circuit based on rating of a Δ–Y transformer bank on an equivalent Y–Y basis. Each unit rated 10,000 kva, 4000–66,500/115,000 Y volts. Leakage reactance 10%, resistance 0.5%, magnetizing current 5%, hysteresis and eddy current 0.5%. One-half the total leakage reactance and copper resistance for the two windings has been assumed for each winding.

Three-Winding Transformer Banks in Balanced Three-Phase Systems.[6] With three identical transformers in the bank and balanced currents and voltages in the system, any of the windings connected in Δ can be replaced by an equivalent Y of the same per unit impedances.

If exciting current is neglected, the equivalent circuit for the three-winding transformer bank for use in the single-phase one-line diagram for determining balanced currents and voltages outside the bank is a three-terminal circuit.

The three windings of each single-phase unit are called primary,

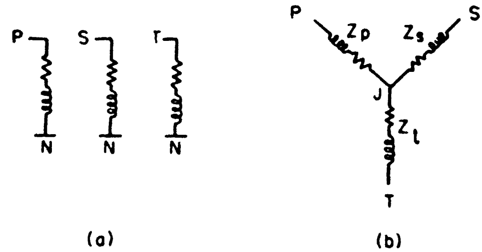


FIG. 16. (a) One phase of a three-winding transformer bank (if connected in Δ, replaced by its equivalent Y). (b) Equivalent circuit of (a) for use in the positive-sequence diagram. With exciting current neglected, there is no connection to the neutral bus N. Terminals P, S, and T are to be connected to the primary, secondary, and tertiary circuits, respectively.

secondary, and tertiary. These windings are shown in Fig. 16(a). Let

Z_{ps} = per unit leakage impedance in primary and secondary windings with the tertiary winding open

Z_{pt} = per unit leakage impedance in primary and tertiary windings with the secondary winding open

Z_{st} = per unit leakage impedance in secondary and tertiary windings with the primary winding open

Impedances are based on voltages which are proportional to the number of turns in the windings, and therefore the given impedance between two windings with the third open is the same referred to either of the two windings. Since the kva ratings of the three windings are not usually equal, Z_{ps}, Z_{pt}, and Z_{st} as defined above are expressed on the same kva base.

Figure 16(b) shows the assumed equivalent circuit with branch impedances Z_p, Z_s, and Z_t to be evaluated. Following the procedure given above,

$$Z_p + Z_s = Z_{ps}$$

$$Z_p + Z_t = Z_{pt}$$

$$Z_s + Z_t = Z_{st}$$

Solving the three simultaneous equations,

$$\begin{aligned} Z_p &= \tfrac{1}{2}(Z_{ps} + Z_{pt} - Z_{st}) \\ Z_s &= \tfrac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \\ Z_t &= \tfrac{1}{2}(Z_{pt} + Z_{st} - Z_{ps}) \end{aligned} \qquad [50]$$

Z_p, Z_s, and Z_t in Fig. 16(*b*) are replaced by their values given in [50]. Equivalent circuits for transformers are further discussed in this volume and in Vol. II.

Three-Phase Synchronous Machines. In the design of synchronous machines, care is taken to have the phases symmetrical and the generated voltages approximately sinusoidal and balanced. The windings of the machine may be connected in either Y or Δ. If they are connected in Δ, the Δ can be replaced by an equivalent Y for determining currents and voltages of fundamental frequency outside the machine. As a first approximation, the machine can be represented by an equivalent circuit consisting of a generated voltage between the neutral (or equivalent neutral) of the machine and a lumped impedance, representing the impedance to neutral of the machine under balanced loads. The generated voltage in the equivalent circuit represents the generated voltage of one phase. Figure 17 shows the equivalent circuit of a synchronous machine for use in the positive-sequence diagram. N is the neutral of the machine, Z the impedance to neutral, T the terminal of one phase, and E the generated voltage in that phase. The values to be assigned to Z and E depend upon the problem. The resistance component of the positive-sequence impedance is relatively small and can usually be neglected. If initial short-circuit current of fundamental frequency is to be determined, subtransient reactance will be used in Z, and E will be the voltage behind subtransient reactance. In transient stability studies, or in short-circuit studies after subtransient effects have disappeared, transient reactance and the voltage behind transient reactance may be used in Z and E. For steady-state operation, equivalent steady-state reactance and the voltage behind this reactance are used. These reactances and the corresponding voltages are discussed further in Vol. II.

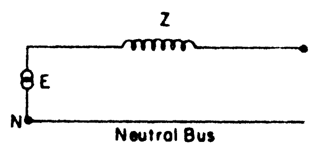


Fig. 17. Equivalent circuit for a synchronous machine with generated voltage, E, and impedance to neutral, Z, for use in the positive-sequence one-line diagram.

Transmission Circuits. If capacitance and leakance are neglected, a symmetrical three-phase transmission circuit (or one assumed symmetrical) has equal impedances in the three phases under balanced operation. It can therefore be represented in the positive-sequence diagram as a lumped self-impedance Z.

$$Z = \ell(r + jx) = \ell(r + j2\pi fL) \tag{51}$$

where ℓ is the length of line in miles, r the resistance in ohms per mile, L the inductance in henries per mile, and f the frequency in cycles per second. Equivalent circuits for transmission circuits with distributed constants are discussed in Chapter VI.

THREE-PHASE FAULTS

A three-phase fault, since it is symmetrical in the three phases, does not unbalance the system. The fundamental-frequency currents and voltages resulting from the fault are of the same phase order as the generated voltages, and can be calculated from the positive-sequence one-line impedance diagram and the operating conditions previous to the occurrence of the fault.

D-C Components. The d-c component of fault current in any phase will depend upon the point of the voltage wave at which the fault is applied. If the phase voltage is at its crest value, which corresponds to zero flux linkages in the phase, the current wave with resistance neglected will be a sinusoidal wave symmetrical about the current axis. If the phase voltage is at its zero value, which corresponds to maximum flux linkages in the phase, the current wave with resistance neglected will be a completely offset wave. Its initial crest value will be equal to twice that of the symmetrical current wave. The rms value of the symmetrical current wave is $(1/\sqrt{2})$ times its crest value. Let this rms value be I. Then the d-c component of the completely offset wave is $\sqrt{2}I$. The initial rms value of the completely offset wave with resistance neglected is

$$\sqrt{I^2 + (\sqrt{2}I)^2} = \sqrt{3}I$$

When the d-c component has decayed to zero, the wave becomes symmetrical about the current axis. In three-phase short-circuit calculations, initial symmetrical rms fault currents are calculated by using subtransient reactances and allowance is made for the d-c component if the maximum possible rms current is required. As the currents and voltages of the three phases are displaced from each other by 120°, the current of only one phase can have the approximate maximum rms value given above. Also, only one phase could have an initial symmetrical current wave.

Transient currents of fundamental frequency are currents which exist after d-c components and subtransient effect have disappeared. They are symmetrical about the current axis and their rms values are determined by using transient reactances. Sustained short-circuit currents are symmetrical about the current axis and are determined by using steady-state reactances.

Fundamental Frequency Components. Two methods are available for determining three-phase short-circuit currents of fundamental frequency.

1. Currents in the fault and throughout the system can be expressed in terms of the applied (generated) voltages and the transfer and driving-point impedances with the fault on the system by [33]. This method requires the determination of the generated voltages from given operating conditions previous to the occurrence of the fault.

2. Any fault point F in the network may be considered a terminal point and the voltage V_f at this point determined from the given operating conditions previous to the occurrence of the fault. V_f is applied through zero impedance between F and the zero-potential bus. The initial currents may then be expressed in terms of the driving-point and transfer impedances of the network and the voltages at all terminal points, F included. When initial currents are expressed with the point F at which the fault will occur as one of the terminal points, none of the transfer and driving-point impedances will be changed by the fault. The only change in [33] resulting from the fault will be in the voltage V_f at the fault, which becomes zero. Before the fault the voltage at F was V_f; after the fault it is zero. The change in voltage is therefore $-V_f$. Applying the principle of superposition, the currents in the system can be determined by adding to the load currents the currents resulting from the voltage $-V_f$ applied at F, with all other applied voltages equated to zero. Because of the fault, the changes in the currents flowing *into the network* at terminal points 1, 2, $\cdots n$, and F are

$$
\begin{aligned}
\Delta I_1 &= \frac{V_f}{A_{f1}} \\
\Delta I_2 &= \frac{V_f}{A_{f2}} \\
&\cdots\cdots\cdots\cdots \\
\Delta I_n &= \frac{V_f}{A_{fn}} \\
\Delta I_f &= -\frac{V_f}{A_{ff}}
\end{aligned}
\qquad [52]
$$

where A_{ff} is the driving-point impedance at point F, and $A_{f1}, A_{f2}, \cdots A_{fn}$ are the transfer impedances between F and $1, 2, \cdots n$, respectively.

ΔI_f represents the change in the current flowing from the fault into the network. Since there was no current flowing into the network from the fault before the fault occurred, ΔI_f is the fault current flowing *into the network*. If I_f is the current flowing *from the network* into the fault,

$$I_f = -\Delta I_f = \frac{V_f}{A_{ff}} = \frac{V_f}{Z_1} \qquad [53]$$

From [53], the current flowing into the fault can be determined from the prefault voltage and the driving-point impedance at the fault. In a positive-sequence network, the driving-point impedance at any point is the positive-sequence impedance viewed from that point and is designated by Z_1. To determine the three-phase fault current by method 2, it is necessary to know only V_f, the prefault voltage, and Z_1, the positive-sequence impedance viewed from the fault. The currents in the network due to the fault can be determined from the fault current and the positive-sequence impedance diagram of the system, or the changes in the currents entering the network at terminal points can be determined from [52]. The current at any point in the network due to the fault is added to the load current to obtain the resulting current at that point.

The fault and system currents calculated by the two methods given above are the initial **symmetrical rms values,** which exist for only a short time after the occurrence of the fault and are determined by the use of subtransient reactances. At the instant the fault occurs, the voltages in the positive-sequence system behind the subtransient reactances of the synchronous machines on the system will remain fixed in magnitude and retain their relative phase angles. The fluxes linking the rotor circuits cannot change instantly and the rotors cannot change their relative angular positions instantly; therefore the magnitudes and phases of the voltages behind the subtransient reactances, being proportional to the fluxes, will remain fixed at the first instant. If currents and voltages at subsequent intervals are to be determined, changes in relative angular positions of the rotors should be taken into account, if their effect is appreciable, as well as the change of machine reactances from subtransient to transient and finally to steady-state reactances.

The simplest way to determine three-phase short-circuit currents in a large three-phase system is by means of a d-c calculating table, if resistances and capacitances can be neglected. On the d-c table, a voltage V_f is applied between the neutrals of the machines, connected at a common point, and the zero-potential bus of the table. This corresponds to the condition of no load when all per unit generated voltages are equal and in phase. The procedure with the d-c table would be just the same if the system were assumed to be operating under load and the currents due to the fault were determined by applying a voltage $-V_f$ at the point of fault, equal in magnitude and opposite in sign to the voltage which existed there before the fault, following the second method given above. Load current can be added to short-circuit current to obtain resultant currents. Since load currents, in general, are not in phase with short-circuit currents, the error in neglecting them is small.

If an a-c network analyzer is used, the positive-sequence network is set up and the generated voltages adjusted in magnitude and phase until the operating conditions previous to the occurrence of the fault are obtained. Connection is then made between the point of the system where the fault occurs and the neutral bus. The currents in the system will include load currents as well as currents resulting from the fault. Currents are initial symmetrical rms values when subtransient reactances and the voltages behind these reactances are used. Solution on the a-c network analyzer corresponds to the first method given above.

To illustrate the procedure for determining the currents and voltages during a three-phase fault by the two methods given above, a simple problem will be solved.

Problem 2. A synchronous generator supplies power to a synchronous motor connected directly to its terminals. The positive-sequence subtransient reactances of the generator and motor on a certain kva base and the rated voltage of the machines are 40 and 200%, respectively. Resistance is neglected. The current supplied by the generator is 50% of base current. The power factor at the bus is unity and the voltage is 98% of rated voltage. Find the initial symmetrical rms current in the fault, the generator, and the motor when a three-phase fault occurs on the bus.

Solution by Second Method. Figure 18(*a*) shows a one-line diagram of the system, part (*b*) the positive-sequence diagram. With V_t, the voltage at the bus before the fault occurred as reference vector,

$$V_f = V_t = 0.98 + j0$$
$$I_g = 0.5 + j0$$
$$I_m = -0.5 + j0$$

where I_g and I_m denote currents in the generator and motor, respectively, positive direction of current flow being from the neutrals of the machines towards the fault.

The positive-sequence impedance viewed from the fault in Fig. 18(*b*) is

$$Z_1 = \frac{j0.40 \times j2.00}{j2.40} = j0.333$$

The current flowing into the fault, from [53], is

$$I_f = \frac{V_f}{Z_1} = \frac{0.98}{j0.333} = -j2.94$$

The distribution of the fault current in the system can be obtained from the impedance diagram. In the simple system of Fig. 18(b), I_f divides inversely as the parallel impedances in its paths. Thus,

$$I_f \text{ (from generator)} = -j2.94 \times \frac{2.00}{2.40} = -j2.45$$

$$I_f \text{ (from motor)} = -j2.94 \times \frac{0.40}{2.40} = -j0.49$$

Adding the currents due to the fault and the load currents in generator and motor,

$$I_g = 0.5 - j2.45$$
$$I_m = -0.5 - j0.49$$

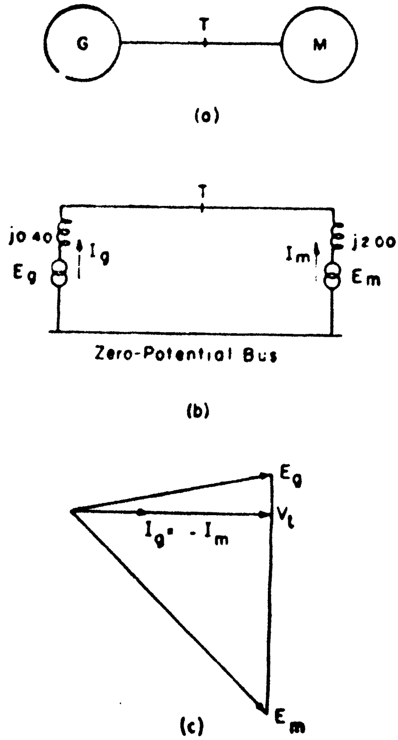


FIG. 18. (a) One-line diagram of system of Problem 2. (b) Positive-sequence diagram. (c) Voltage and current vector diagram before the fault.

Solution by First Method. The internal voltages of both machines must be calculated from given operating conditions before this method can be applied. With V_t, the voltage at the bus before the fault occurs, as reference vector, the voltages behind subtransient reactances are

$$E_g = V_t + I_g Z_g = 0.98 + (0.5 \times j0.40) = 0.98 + j0.20 = 1.00 \underline{/11.5^\circ}$$

$$E_m = V_t + I_m Z_m = 0.98 + (-0.5 \times j2.00) = 0.98 - j1.00 = 1.40 \overline{\backslash 45.6^\circ}$$

where the subscripts g and m refer to generator and motor, respectively. Figure 18(c) gives the voltage and current vector diagram for the given operating conditions. After the fault occurs,

$$I_g \text{ (in generator)} = \frac{E_g}{Z_g} = \frac{0.98 + j0.20}{j0.40} = 0.5 - j2.45$$

$$I_m \text{ (in motor)} = \frac{E_m}{Z_m} = \frac{0.98 - j1.00}{j2.00} = -0.5 - j0.49$$

$$I_f \text{ (in fault)} = \frac{E_g}{Z_g} + \frac{E_m}{Z_m} = -j2.94$$

Problem 3. A three-phase, 60-cycle generator, rated 5000 kva, 11 kv, with a transient reactance of 30% on its rating, supplies power through a three-phase current

3. "Cisoidal Oscillations," by George A. Campbell, *Proc. A.I.E.E.*, 1911, April, Vol. 30, pp. 789–824.
4. "Simultaneous Faults on Three-Phase Systems," by Edith Clarke, *A.I.E.E. Trans.*, Vol. 50, 1931, pp. 919–941.
5. "Equivalent Circuits — I," by Frank M. Starr, *Proc. A.I.E.E.*, Vol. 51, 1932, pp. 287–298.
6. "Theory of Three-Circuit Transformers," by A. Boyajian, *A.I.E.E. Trans.*, February, 1924, pp. 508–529.

CHAPTER II

SYMMETRICAL COMPONENTS — BASIC EQUATIONS FOR THREE-PHASE SYSTEMS

The first paper indicating the possibilities of resolving an unbalanced system of currents into positive- and negative-sequence components of current was published in 1912 by L. G. Stokvis.[1] A second paper[2] dealing with third-harmonic voltages in alternators was presented under the sponsorship of André Blondel at a meeting of the French Academy of Science in 1914. It is interesting to note that positive- and negative-sequence currents as they are now known were a by-product of Stokvis's main endeavor, which was to find a means of determining the magnitude of the third-harmonic voltage produced by unbalanced line-to-line loads. A more detailed treatment of the resolution into positive- and negative-sequence currents of the unbalanced currents in three-phase ungrounded systems was given[3] in 1915.

In 1918, Dr. C. L. Fortescue presented before the American Institute of Electrical Engineers a paper[4] which introduced the concept of zero-sequence currents and voltages and provided a general method for the solution of unbalanced polyphase systems. In this paper Dr. Fortescue proved that "*a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining* $(n - 1)$ *systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems* . . ."

Many unbalanced problems hitherto solved only with difficulty are now solved as routine problems by the method of symmetrical components. Chief among these is the determination of currents and voltages of fundamental frequency in systems during unsymmetrical short circuits. This important application of the method of symmetrical components was made available by two papers[5,6] in 1925 and another[7] in 1926. The application of the method to the ever present problem of unsymmetrical short circuits indicated to power engineers its value as a tool in the study of system performance, and it was but a short time before the method was widely used. The publication of a series[8] of articles and courses in engineering schools and commercial organizations further stimulated interest in symmetrical components. Many engineers have contributed in the application of the method to various problems encountered in power system operation.

The method of symmetrical components is a general one, applicable to any polyphase system. Because of the widespread use of three-phase systems and the greater familiarity which electrical engineers have with them, symmetrical component equations will be developed for them first.

THREE-PHASE SYSTEMS

In three-phase power systems, sinusoidal currents and voltages of fundamental frequency are represented for purposes of calculation by vectors revolving at an angular velocity, $\omega = 2\pi f$ radians per second. (See Chapter I.) The components which replace them must therefore be sinusoidal quantities of the same frequency, represented by vectors revolving at the same angular velocity. Since the angles between vectors revolving at the same rate are fixed, a set of three voltage or current vectors, V_a, V_b, and V_c, and the components which are to replace them, can be represented in the same vector diagram, with any current or voltage vector revolving at the same rate as reference vector.

Choice of Components. Any three co-planar vectors V_a, V_b, and V_c can be expressed in terms of three new vectors V_1, V_2, and V_3 by three simultaneous linear equations with constant coefficients. Thus,

$$V_a = c_{11}V_1 + c_{12}V_2 + c_{13}V_3 \qquad [1]$$

$$V_b = c_{21}V_1 + c_{22}V_2 + c_{23}V_3 \qquad [2]$$

$$V_c = c_{31}V_1 + c_{32}V_2 + c_{33}V_3 \qquad [3]$$

where the choice of coefficients is arbitrary, except for the restriction that the determinant* made up of the coefficients[9] must not be zero.

Each of the original vectors has now been replaced by three vectors, making a total of nine vectors. The nine vectors consist of three groups or systems of three vectors each. These systems are as follows:

1st system: $c_{11}V_1, \quad c_{21}V_1, \quad c_{31}V_1$

2nd system: $c_{12}V_2, \quad c_{22}V_2, \quad c_{32}V_2$

3rd system: $c_{13}V_3, \quad c_{23}V_3, \quad c_{33}V_3$

When values are assigned to the coefficients, the relations between the vectors of each system are fixed. It follows therefore that the three

* For a discussion of determinants, see Appendix A.

original vectors can be determined when the three new vectors are known. The purpose of expressing the three original vectors in terms of three new vectors is to simplify calculations, and thereby to gain a better understanding of a given problem and its related problems. With this thought in mind, two conditions should be satisfied in selecting systems of components to replace three-phase current and voltage vectors:

1. Calculations should be simplified by the use of the chosen systems of components. This is possible only if the impedances (or admittances) associated with the components of current (or voltage) can be obtained readily by calculation or test.
2. The systems of components chosen should have physical significance and be an aid in determining power system performance.

It will be seen that symmetrical components satisfy both these requirements.

Symmetrical Components. Although there are many ways of choosing the coefficients in [1]–[3] so that a system of three vectors can be replaced by three systems of vectors, consisting of three vectors each, there is only one way in which it can be replaced by three systems, each consisting of three *symmetrical* vectors. A system of three symmetrical vectors is one in which the three vectors are equal in magnitude and displaced from each other by equal angles. If V_a, V_b, and V_c are a set of voltage or current vectors, referring to phases a, b, and c, respectively, of a three-phase system, the three systems of three symmetrical vectors replacing V_a, V_b, V_c are the following:

1. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 120°, and the component of phase c lagging the component of phase b by 120°, as in Fig. 1 (a).
2. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 240°, and the component of phase c lagging the component of phase b by 240°, as in Fig. 1 (b).
3. A system of three vectors equal in magnitude displaced from each other by 0° or 360°, as in Fig. 1 (c).

In the first two systems of revolving vectors there is a sequence of phases; in the third system there is none, the three vectors being in phase. In the first system, Fig. 1 (a), taking counterclockwise direction as positive, the time order of arrival of the component vectors at a fixed axis of reference is abc. In the second system, Fig. 1 (b), the time order of arrival at the fixed axis of reference is *acb*. In both systems the vectors are displaced from each other by 120°, but the *phase order* of one system is the reverse of the other.

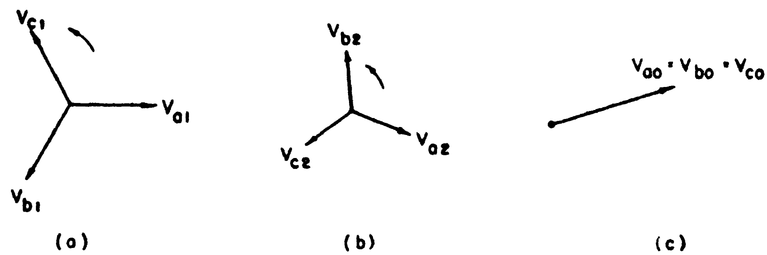


Fig. 1. (*a*) Positive-, (*b*) negative- and, (*c*) zero-sequence systems of vectors.

Meaning of Positive, Negative, and Zero Sequence. By a *positive-phase-sequence*, or *positive-sequence*, system of vectors is meant a system of three vectors equal in magnitude and 120° apart in phase, in which the time order of arrival of the phase vectors at a fixed axis of reference is the same as that of the generated voltages. The phases have been arbitrarily named so that the phase order of the generated voltages is *abc*. (See Chapter I.) In Fig. 1(*a*), the system of vectors is a positive-sequence system. The vectors in Fig. 1(*b*), which are also equal in magnitude and 120° apart in phase, arrive at a fixed axis in the phase order *acb* instead of *abc*. The system of vectors in Fig. 1(*b*) has consequently been called a *negative-sequence* system. If the vectors representing currents or voltages in the three phases are not separated in time phase, there will be no sequence of phases, and the currents or voltages, as the case may be, will vary simultaneously in each phase. A vector diagram for this system, which has been called *zero sequence*, is shown in Fig. 1(*c*). The vectors which form the zero-sequence system are equal in magnitude and in phase.

Notation. In Fig. 1 the double subscript notation has been employed, the first subscript indicating the phase, the second the sequence. Small letters *a*, *b*, and *c* are used to indicate the phases, while numerals 0, 1, and 2 are employed to designate respectively zero, positive, and negative sequence.

Symmetrical Component Equations. Three given vectors V_a, V_b, and V_c are expressed in terms of their symmetrical components by the equations

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad [4]$$

$$V_b = V_{b1} + V_{b2} + V_{b0} \qquad [5]$$

$$V_c = V_{c1} + V_{c2} + V_{c0} \qquad [6]$$

where known relations exist between the vectors of each of the three sets of vectors, V_{a1}, V_{b1}, V_{c1}; V_{a2}, V_{b2}, V_{c2}; V_{a0}, V_{b0}, V_{c0}.

Phase *a* as Reference Phase. The choice of the phase to be regarded as the reference phase is arbitrary. With phase a as reference phase, and making use of the operator a (see Chapter I), the following relations exist:

For positive-sequence vectors,

$$V_{b1} = a^2 V_{a1}; \quad V_{c1} = a V_{a1}$$

For negative-sequence vectors,

$$V_{b2} = a V_{a2}; \quad V_{c2} = a^2 V_{a2}$$

For zero-sequence vectors,

$$V_{b0} = V_{a0}; \quad V_{c0} = V_{a0}$$

Substituting these relations in [4]-[6], there results

$$V_a = V_{a1} + V_{a2} + V_{a0} \tag{7}$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \tag{8}$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \tag{9}$$

Comparing [7]-[9] with [1]-[3], the constant coefficients required to express a set of three vectors in terms of their symmetrical components are 1, a^2, a for the positive-sequence components, 1, a, a^2 for the negative-sequence components, and 1, 1, 1 for the zero-sequence components.

The three vectors V_a, V_b, and V_c are expressed in terms of their symmetrical components by [7]-[9]. When the symmetrical components are known, V_a, V_b, and V_c can be obtained either algebraically or graphically from [7]-[9]. A graphical combination of the sequence components is given in Fig. 2. The zero-, positive-, and negative-sequence components are shown in parts (a), (b), and (c), respectively. The combination of the components to produce the set of unbalanced vectors is shown in Fig. 2(d), and in Fig. 2(e) the three unbalanced vectors without their components.

Resolution of Unbalanced Vectors into Their Symmetrical Components. The resolution of a given unbalanced system of three vectors into symmetrical components can be carried out either graphically or analytically. As the most common procedure is the analytical one, this will be given first, and the graphical method, which follows readily from it, will be explained afterwards.

Analytical Method. The three symmetrical component vectors V_{a1}, V_{a2}, and V_{a0} can be expressed in terms of the original vectors V_a, V_b, and V_c by a solution of the simultaneous linear vector equations [7]–[9]. This will be done by two methods.

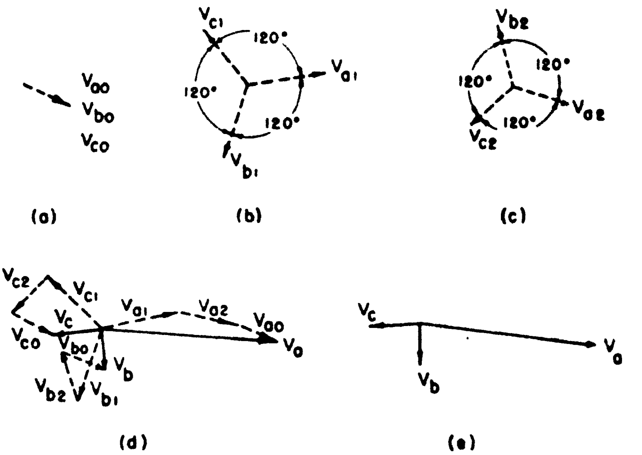


FIG. 2. Symmetrical component systems and their synthesis into three vectors.

Adding [7]–[9], remembering that $(1 + a + a^2) = 0$,

$$V_a + V_b + V_c = (1 + a + a^2)V_{a1} + (1 + a + a^2)V_{a2} + 3V_{a0}$$
$$= 3V_{a0}$$

Therefore

$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) \tag{10}$$

Multiplying [7], [8], and [9] by 1, a, and a^2, respectively, and adding,

$$V_a + aV_b + a^2V_c = (1 + a^3 + a^3)V_{a1} + (1 + a^2 + a^4)V_{a2}$$
$$+ (1 + a + a^2)V_{a0} = 3V_{a1}$$

Therefore

$$V_{a1} = \tfrac{1}{3}(V_a + aV_b + a^2V_c) \tag{11}$$

Multiplying [7], [8], and [9] by 1, a^2, and a, respectively, and adding,

$$V_a + a^2V_b + aV_c = 3V_{a2}$$

Therefore

$$V_{a2} = \tfrac{1}{3}(V_a + a^2V_b + aV_c) \tag{12}$$

An alternate method of solving [7]–[9] is by means of determinants. The solution of vector equations, except in special cases such as the present one, is usually most easily performed by determinants. Since determinants will be used extensively in this book, a few simple rules for their use are given in Appendix A. Using [A–5], Appendix A, the following equations may be written directly from [7]–[9]:

$$V_{a1} = \frac{1}{D}[(a - a^2)V_a - (1 - a^2)V_b + (1 - a)V_c]$$

$$V_{a2} = \frac{1}{D}[-(a^2 - a)V_a + (1 - a)V_b - (1 - a^2)V_c]$$

$$V_{a0} = \frac{1}{D}[(a - a^2)V_a - (a^2 - a)V_b + (a - a^2)V_c]$$

where $D = (a - a^2) - (a^2 - a) + (a - a^2) = 3(a - a^2)$. Simplifying the above equations, they reduce to [10]–[12].

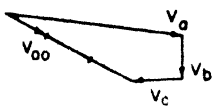


FIG. 3. Graphical method of securing zero-sequence component V_{a0}.

Graphical method. The graphical method of analyzing three unbalanced vectors follows directly from [10], [11], and [12]. Let the given vectors be those of Fig. 2(*e*). The zero-sequence component V_{a0} is found by adding the three given vectors and then dividing the vector closing the polygon by three. Figure 3 shows this process, a graphical method of solving [10].

The positive-sequence component V_{a1} is determined by solving [11] graphically. To V_a is added aV_b, i.e., V_b rotated through 120°. To the sum of V_a and aV_b is added a^2V_c, i.e., V_c rotated through 240°. The positive-sequence component is secured by dividing the vector which forms the closing side of the resulting polygon by three. Figure 4 illustrates this process.

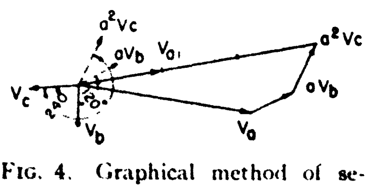


FIG. 4. Graphical method of securing positive-sequence component V_{a1}.

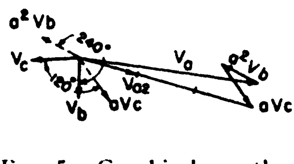


FIG. 5. Graphical method of securing negative-sequence component V_{a2}.

By solving [12] graphically, the negative-sequence component V_{a2} is obtained as pictured in Fig. 5.

Special Case — Zero-Sequence Components Absent. When the sum of the three vectors V_a, V_b, and V_c is zero, it follows from [10] that there are no zero-sequence components, and the three vectors will be expressed in terms of positive- and negative-sequence components only.

With $V_a + V_b + V_c = 0$, any one of the three vectors can be expressed in terms of the other two. Replacing V_c by $-V_a - V_b$ in [11] and [12],

$$V_{a1} = \tfrac{1}{3}[(1 - a^2)V_a + (a - a^2)V_b]$$

$$V_{a2} = \tfrac{1}{3}[(1 - a)V_a + (a^2 - a)V_b]$$

Replacing $(1 - a^2)$, $(a - a^2)$, $(1 - a)$, and $(a^2 - a)$ by their values in polar form given in Table I, Chapter I,

$$V_{a1} = \tfrac{1}{3}[\sqrt{3}V_a \underline{/30^\circ} + \sqrt{3}V_b \underline{/90^\circ}] = \frac{1}{\sqrt{3}}[V_a + V_b \underline{/60^\circ}]\underline{/30^\circ} \qquad [13]$$

$$V_{a2} = \tfrac{1}{3}[\sqrt{3}V_a \overline{/30^\circ} + \sqrt{3}V_b \overline{/90^\circ}] = \frac{1}{\sqrt{3}}[V_a + V_b \overline{/60^\circ}]\overline{/30^\circ} \qquad [14]$$

Using the complex form instead of the polar form for $(1 - a)$, etc.,

$$V_{a1} = \tfrac{1}{3}\left[V_a\left(\frac{3}{2} + j\frac{\sqrt{3}}{2}\right) + V_b(j\sqrt{3})\right]$$

$$= \frac{V_a}{2} + \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90^\circ} \qquad [15]$$

$$V_{a2} = \tfrac{1}{3}\left[V_a\left(\frac{3}{2} - j\frac{\sqrt{3}}{2}\right) + V_b(-j\sqrt{3})\right]$$

$$= \frac{V_a}{2} - \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90^\circ} \qquad [16]$$

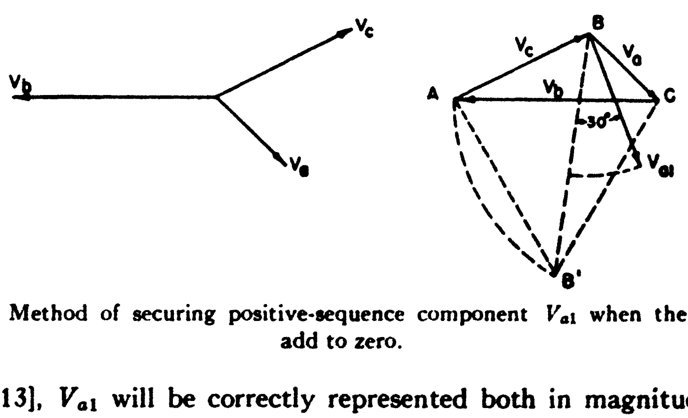


FIG. 6. Method of securing positive-sequence component V_{a1} when the vectors add to zero.

From [13], V_{a1} will be correctly represented both in magnitude and phase if (as in Fig. 6) V_b is rotated 60° in the positive direction and added to V_a, the resultant vector BB' being then divided by $\sqrt{3}$ and rotated positively 30° about B as a center.

Referring to Fig. 7 and equation [14], if V_b is rotated negatively 60° and added to V_a, the resultant vector BB' divided by $\sqrt{3}$, and the new vector thus obtained rotated negatively 30° about B as a center, V_{a2} is obtained.

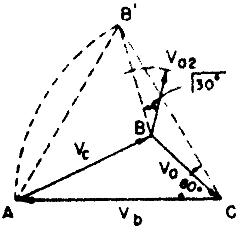


FIG. 7. Method of securing negative-sequence component V_{a2} when the vectors add to zero.

The positive- and negative-sequence components may also be obtained from the same diagram. In [15] and [16], V_{a1} and V_{a2} are given as the sum and difference of two vectors; one is $V_a/2$, the other $\frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°}$. To obtain a graphical construction, the vector $\left(\frac{V_a}{2} + V_b\right)$ must be turned through 90° and divided by $\sqrt{3}$ before it is added and subtracted from $V_a/2$ to give V_{a1} and V_{a2}, respectively.

A vector may be turned through 90° and its magnitude divided by $\sqrt{3}$ if a 30°, 60°, 90° triangle is constructed upon it with the 60° angle opposite it. In triangle ADE, Fig. 8, using scalar values, $DE = AD \tan 30° = AD/\sqrt{3}$; but vector $DE = 1/\sqrt{3}$ vector

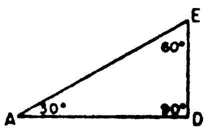


FIG. 8. Construction for dividing a vector AD by $\sqrt{3}$ and rotating it through 90°.

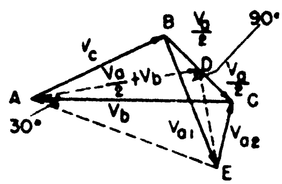


FIG. 9. Positive- and negative-sequence components obtained from the same diagram when the vectors add to zero.

$AD\underline{/90°}$. Equations [15] and [16] are solved graphically in Fig. 9. AD is drawn from A to D, the midpoint of BC. Then vector $DA = (V_a/2) + V_b$. DE is drawn at D, making a positive angle of 90° with DA; AE is drawn from A making an angle of 30° with AD and closing the triangle ADE. Then

$$\text{vector } DE = \frac{\text{vector } DA}{\sqrt{3}}\underline{/90°} = \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°}$$

$$\text{vector } BE = \text{vector } BD + \text{vector } DE$$

$$= \frac{V_a}{2} + \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\underline{/90°} = V_{a1} \qquad [17]$$

$$\text{vector } EC = \text{vector } DC - \text{vector } DE$$
$$= \frac{V_a}{2} - \frac{1}{\sqrt{3}}\left(\frac{V_a}{2} + V_b\right)\angle 90^\circ = V_{a2} \qquad [18]$$

With no zero-sequence components, V_{a1} and V_{a2} have been expressed in terms of V_a and V_b; they can also be expressed in terms of V_a and V_c, or V_b and V_c.

Voltage and Current Vectors. In the preceding developments V_a, V_b, and V_c have been used to represent a set of three vectors in a three-phase system rotating at the same rate. If voltage vectors are represented by V and current vectors by I, the basic symmetrical component equations for voltages and currents will be written:

Symmetrical Component Equations for Voltages

$$V_a = V_{a1} + V_{a2} + V_{a0} \qquad [7]$$
$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \qquad [8]$$
$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \qquad [9]$$
$$V_{a0} = \tfrac{1}{3}(V_a + V_b + V_c) \qquad [10]$$
$$V_{a1} = \tfrac{1}{3}(V_a + a V_b + a^2 V_c) \qquad [11]$$
$$V_{a2} = \tfrac{1}{3}(V_a + a^2 V_b + a V_c) \qquad [12]$$

Symmetrical Component Equations for Currents

$$I_a = I_{a1} + I_{a2} + I_{a0} \qquad [19]$$
$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0} \qquad [20]$$
$$I_c = a I_{a1} + a^2 I_{a2} + I_{a0} \qquad [21]$$
$$I_{a0} = \tfrac{1}{3}(I_a + I_b + I_c) \qquad [22]$$
$$I_{a1} = \tfrac{1}{3}(I_a + a I_b + a^2 I_c) \qquad [23]$$
$$I_{a2} = \tfrac{1}{3}(I_a + a^2 I_b + a I_c) \qquad [24]$$

In the above equations, the voltage vectors V_a, V_b, V_c may be the voltages to ground of phases a, b, c at a specified point in the three-phase system; or they may be voltages to neutral, line-to-line voltages, generated voltages, induced voltages, in fact, any set of three voltage vectors revolving at the same rate which may exist in a three-phase system. Likewise, the three current vectors may be the three line currents, the three currents in a Δ-connected circuit, the currents flowing into a fault from the three conductors, etc. In the work which follows, V and I with appropriate subscripts will be used to indicate various voltage and current vectors, depending upon the type of problem to be solved.

Alternative Equations for Numerical Calculations. If a and a^2 in [8], [9], [11], [12], [20], [21], [23], and [24] are replaced by $\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)$, respectively, the following equations more suitable for numerical calculations are obtained:

$$V_b = V_{a0} - \tfrac{1}{2}(V_{a1} + V_{a2}) - j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) \qquad [25]$$

$$V_c = V_{a0} - \tfrac{1}{2}(V_{a1} + V_{a2}) + j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) \qquad [26]$$

$$V_{a1} = \tfrac{1}{3}\left[V_a - \tfrac{1}{2}(V_b + V_c) + j\frac{\sqrt{3}}{2}(V_b - V_c)\right] \qquad [27]$$

$$V_{a2} = \tfrac{1}{3}\left[V_a - \tfrac{1}{2}(V_b + V_c) - j\frac{\sqrt{3}}{2}(V_b - V_c)\right] \qquad [28]$$

$$I_b = I_{a0} - \tfrac{1}{2}(I_{a1} + I_{a2}) - j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) \qquad [29]$$

$$I_c = I_{a0} - \tfrac{1}{2}(I_{a1} + I_{a2}) + j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) \qquad [30]$$

$$I_{a1} = \tfrac{1}{3}\left[I_a - \tfrac{1}{2}(I_b + I_c) + j\frac{\sqrt{3}}{2}(I_b - I_c)\right] \qquad [31]$$

$$I_{a2} = \tfrac{1}{3}\left[I_a - \tfrac{1}{2}(I_b + I_c) - j\frac{\sqrt{3}}{2}(I_b - I_c)\right] \qquad [32]$$

Instantaneous Power. The instantaneous power at any point in a three-phase system in the direction of current flow obtained by adding the instantaneous power in the three phases is

$$p = p_a + p_b + p_c = i_a v_a + i_b v_b + i_c v_c \qquad [33]$$

where p, v, and i indicate instantaneous power, voltage, and current, and subscripts a, b, and c refer to phases a, b, and c, respectively.

With sinusoidal currents and voltages of the same frequency in the three phases, instantaneous phase currents and voltages can be replaced by their instantaneous symmetrical components. Instantaneous power then becomes

$$p = (v_{a1} + v_{a2} + v_{a0})(i_{a1} + i_{a2} + i_{a0}) + (v_{b1} + v_{b2} + v_{b0}) \times (i_{b1} + i_{b2} + i_{b0}) + (v_{c1} + v_{c2} + v_{c0})(i_{c1} + i_{c2} + i_{c0}) \qquad [34]$$

The instantaneous zero-sequence components of voltages and currents are equal in the three phases and the sum of the instantaneous positive- or negative-sequence currents or voltages at any instant is zero. Making use of these relations, [34], when expanded, is

$$p = (v_{a1}i_{a1} + v_{b1}i_{b1} + v_{c1}i_{c1}) + (v_{a2}i_{a2} + v_{b2}i_{b2} + v_{c2}i_{c2}) + 3v_{a0}i_{a0} + (v_{a1}i_{a2} + v_{b1}i_{b2} + v_{c1}i_{c2}) + (v_{a2}i_{a1} + v_{b2}i_{b1} + v_{c2}i_{c1}) \quad [35]$$

Instantaneous symmetrical components of voltage and current are expressed in terms of their rms vector values (see Chapter I) by the following equations:

$$\begin{aligned}
v_{a1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha) \\
v_{b1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha - 120°) \\
v_{c1} &= \sqrt{2}|V_{a1}| \sin (\omega t + \alpha + 120°) \\
i_{a1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1) \\
i_{b1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1 - 120°) \\
i_{c1} &= \sqrt{2}|I_{a1}| \sin (\omega t + \alpha + \theta_1 + 120°) \\
v_{a2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta) \\
v_{b2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta + 120°) \\
v_{c2} &= \sqrt{2}|V_{a2}| \sin (\omega t + \beta - 120°) \\
i_{a2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2) \\
i_{b2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2 + 120°) \\
i_{c2} &= \sqrt{2}|I_{a2}| \sin (\omega t + \beta + \theta_2 - 120°) \\
v_{a0} &= v_{b0} = v_{c0} = \sqrt{2}|V_{a0}| \sin (\omega t + \gamma) \\
i_{a0} &= i_{b0} = i_{c0} = \sqrt{2}|I_{a0}| \sin (\omega t + \gamma + \theta_0)
\end{aligned} \quad [36]$$

where $|V_{a1}|$, $|V_{a2}|$, $|V_{a0}|$ and $|I_{a1}|$, $|I_{a2}|$, $|I_{a0}|$ are scalar values of the rms positive-, negative-, and zero-sequence vector voltages and currents, respectively; α, β, and γ are the phase angles by which the voltage vectors V_{a1}, V_{a2}, and V_{a0}, respectively, lead the reference vector (see Fig. 5, Chapter I). θ_1, θ_2, and θ_0 are the phase angles by which the positive-, negative-, and zero-sequence current vectors lead the corresponding voltage vectors, with θ positive for leading currents and negative for lagging currents.

Replacing instantaneous symmetrical components in [35] by their values in terms of rms vector quantities from [36], and making use of the trigonometric equations

$$\sin A \sin B = \tfrac{1}{2}[\cos (A - B) - \cos (A + B)]$$

and

$$\cos A + \cos (A + 120°) + \cos (A - 120°) = 0$$

the instantaneous three-phase power is

$$\begin{aligned} p = {} & 3|V_{a1}||I_{a1}| \cos \theta_1 + 3|V_{a2}||I_{a2}| \cos \theta_2 + 3|V_{a0}||I_{a0}| \cos \theta_0 \\ & - 3|V_{a0}||I_{a0}| \cos (2\omega t + 2\gamma + \theta_0) \\ & - 3|V_{a1}||I_{a2}| \cos (2\omega t + \alpha + \beta + \theta_2) \\ & - 3|V_{a2}||I_{a1}| \cos (2\omega t + \alpha + \beta + \theta_1) \end{aligned} \quad [37]$$

The instantaneous three-phase power consists of three constant terms and three variable terms which pulsate sinusoidally at double impressed frequency. If currents and voltages are expressed in amperes and volts, respectively, power will be in watts; if currents and voltages are expressed in per unit of base phase current and phase voltage, respectively, power will be in per unit of *base power per phase*, base kilowatts being equal numerically to base kva. If instantaneous power is expressed in per unit of three-phase base kilowatts, numerically equal to three-phase base kva, all terms on the right-hand side of the equality sign in [37] are divided by three.

Average Three-Phase Power in Terms of Symmetrical Components. Since the average power of the double frequency terms in [37] is zero, the average three-phase power P is

$$P = 3|V_{a1}||I_{a1}| \cos \theta_1 + 3|V_{a2}||I_{a2}| \cos \theta_2 + 3|V_{a0}||I_{a0}| \cos \theta_0 \quad [38]$$

where θ_1, θ_2, and θ_0 are the positive-, negative-, and zero-sequence power factor angles which are positive for leading current and negative for lagging currents. The average power given by [38] will be in watts if currents and voltages are in amperes and volts, respectively. It will be in per unit of *base power per phase* if currents and voltages are in per unit of base phase current and base phase voltage, respectively.

The average three-phase power in per unit of *three-phase base kilowatts* (numerically equal to three-phase base kva) is

$$P = |V_{a1}||I_{a1}| \cos \theta_1 + |V_{a2}||I_{a2}| \cos \theta_2 + |V_{a0}||I_{a0}| \cos \theta_0 \quad [39]$$

Average Three-Phase Power in Terms of Phase Voltages and Currents.

$$P = |V_a||I_a| \cos \theta_a + |V_b||I_b| \cos \theta_b + |V_c||I_c| \cos \theta_c \quad [40]$$

where θ_a, θ_b, and θ_c are the angles by which phase currents lead their respective phase voltages. Power will be in watts if currents and volt-

CHAPTER III

SHORT CIRCUITS ON SYSTEMS WITH ONE POWER SOURCE

In Chapter II, currents, voltages, and power at any point in a three-phase system are expressed in terms of the symmetrical components of current and voltage at that point. In this chapter, the method of symmetrical components is used to determine fundamental-frequency currents and voltages during a short circuit at the terminals of a symmetrical three-phase unloaded synchronous generator or on an unloaded circuit in series with a generator and transformer bank. In Chapter IV, the application is extended to any symmetrical three-phase power system, normally balanced but rendered unbalanced by an unsymmetrical fault.

Sequence Impedances of Symmetrical Three-Phase Circuits. In dealing with sinusoidal currents and voltages of fundamental frequency, the impedances offered to positive-sequence currents in the three phases of a circuit will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents, with only positive-sequence currents flowing in the circuit. Likewise, the impedances offered to negative-sequence currents in the three phases will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents with only negative-sequence currents flowing in the circuit. Zero-sequence currents by definition are the same in magnitude and phase in the three phases; therefore their sum is not zero, and there must be a return path in which the sum $3I_{a0}$ can flow or the zero-sequence impedance of the circuit will be infinite. The impedance per phase met by zero-sequence currents in a symmetrical three-phase circuit with only zero-sequence currents flowing is the impedance (or equivalent impedance) offered to any one of the three equal currents flowing in the phases and their sum returning through the earth or some other conductor to which the neutral is connected. Figure 1 shows the path of zero-sequence currents and indi-

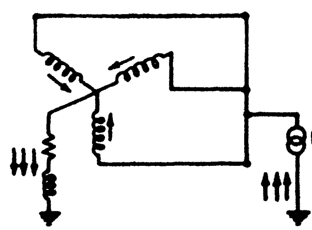


FIG. 1. Path of zero-sequence currents.

cates how zero-sequence impedance can be obtained by test. The zero-sequence impedance per phase of this circuit, which is Y-connected with neutral grounded through impedance, can be obtained by connecting the three terminals and applying a single-phase voltage to ground at the terminals. In a symmetrical circuit, the zero-sequence impedance per phase is three times the ratio of the applied voltage to the total current $3I_{a0}$.

Methods of calculating and measuring the sequence impedances of transformers, transmission lines, etc., are discussed in later chapters. It is shown in Chapter VIII that in a *symmetrical* static circuit without internal voltages the impedances to the currents of any sequence are the same in the three phases. It is further shown that currents of a given sequence produce voltage drops of like sequence only, and voltages of a given sequence produce currents of the same sequence only; consequently, there is no mutual coupling between the sequence systems. Since the impedance of a symmetrical static network to balanced three-phase currents is independent of the phase order, the positive- and negative-sequence impedances are equal; the zero-sequence impedance which includes the impedance of the return path of $3I_{a0}$, in the general case, is different from the positive- and negative-sequence impedances. In symmetrical rotating mchines, the impedances met by armature currents of a given sequence are equal in the three phases. As the impedance to currents of a given sequence depends upon their phase order relative to the direction of rotation of the rotor, positive-, negative-, and zero-sequence impedances are unequal in the general case.

For the present, let it be assumed that the sequence impedances of the symmetrical three-phase circuits discussed in this and the following chapter are known. Z_1, Z_2, and Z_0 will be used to indicate positive-, negative-, and zero-sequence impedances, respectively, of a symmetrical three-phase circuit or any portion of a symmetrical three-phase system. Let it also be assumed that there is no mutual coupling between the three sequence systems. The three sequence systems can then be considered separately, and phase currents and voltages determined by superposing their symmetrical components of current and voltage, respectively.

The division of currents and voltages into symmetrical components, with currents of each sequence meeting their own particular sequence impedances, is based on the principle of superposition (see Chapter I). Symmetrical components can be rigorously applied to electrical circuits only when the circuit impedances and admittances at the impressed frequency are constant. They can, however, be satisfactorily applied to many problems where the circuit parameters are not strictly constant, provided the resultant calculated phase voltages and currents are not of such magnitudes as to change materially the impedances and admittances *assumed* in making the calculations.

Generated or Internal Voltages. By generated or internal voltage is meant the voltage which would exist at the terminals of a machine on open circuit. It is the voltage behind the positive-sequence impedance, Z_1, of the generator (see Chapter I, Fig. 17), where Z_1 may be subtransient, transient, synchronous or equivalent steady-state impedance, depending upon the nature of the problem. The letter E will be used to designate generated voltages of machines, in order to distinguish them from voltages at the terminals or other points in the system, designated by the letter V. If the generated voltages in the three phases are E_a, E_b, and E_c, they can be resolved into their positive-, negative-, and zero-sequence components of generated voltages by [10]–[12], Chapter II, giving

$$\begin{aligned} E_{a0} &= \tfrac{1}{3}(E_a + E_b + E_c) \\ E_{a1} &= \tfrac{1}{3}(E_a + aE_b + a^2E_c) \\ E_{a2} &= \tfrac{1}{3}(E_a + a^2E_b + aE_c) \end{aligned} \qquad [1]$$

Since alternators are designed to generate balanced voltages, E_{a0} and E_{a2} in general will be zero. With balanced generated voltages,

$$\begin{aligned} E_a &= E_a \\ E_b &= a^2E_a = E_a\overline{\big/120°} = \left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right)E_a \\ E_c &= aE_a = E_a\underline{/120°} = \left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right)E_a \end{aligned} \qquad [2]$$

Voltage of the Neutral. In ungrounded circuits or circuits grounded through impedances, the neutrals of the circuits may or may not be at ground potential. Figure 2 shows a circuit with neutral N grounded through an impedance Z_n. If current flows in Z_n, the neutral N will not be at ground potential. That the voltage of the neutral referred to the ground is a zero-sequence voltage may be shown by applying equations [10]–[12], Chapter II. Since N is common to all three phases, the three voltages to ground

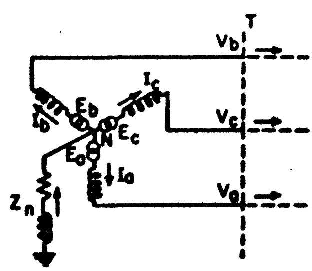


FIG. 2. Synchronous machine with neutral grounded through impedance.

as this point is approached become equal in the limit, thus: $V_a = V_b = V_c = V_n$. Hence

$$V_{a0} = \tfrac{1}{3}(V_n + V_n + V_n) = \frac{3V_n}{3} = V_n \qquad [3]$$

$$V_{a1} = \tfrac{1}{3}(V_n + aV_n + a^2V_n) = (1 + a + a^2)V_n = 0 \qquad [4]$$

$$V_{a2} = \tfrac{1}{3}(V_n + a^2V_n + aV_n) = (1 + a^2 + a)V_n = 0 \qquad [5]$$

From [4] and [5], it is evident that both the positive- and negative-sequence voltages from the neutral to ground are zero; and from [3], that the voltage of the neutral to ground is a zero-sequence voltage. For the positive- and negative-sequence systems, therefore, the expressions *voltage to neutral* and *voltage to ground* may be used interchangeably, but for the zero-sequence system it is important to distinguish between the two terms.

Reference for Voltages. The phase voltages at any point in a grounded system and their zero-sequence components of voltage will be referred to the ground at that point. The positive- and negative-sequence components of voltage are referred to neutral.

Convention for Direction of Current. For any given problem, the direction assumed as positive for current flow will be stated or indicated by arrows. (See Chapter I for a discussion of direction of current flow.)

Neutral or Ground Current. In Fig. 2, let I_n represent the neutral or ground current and I_a, I_b, I_c the three line currents, positive direction for neutral current being from the ground towards the neutral and for the line currents from the neutral towards the terminals. Applying Kirchhoff's law, that the sum of the currents flowing into a point is zero,

$$I_n = I_a + I_b + I_c$$

Replacing I_a, I_b, and I_c by their symmetrical components given by [19]–[21], Chapter II, and $(1 + a + a^2)$ by zero,

$$\begin{aligned} I_n &= (I_{a0} + I_{a1} + I_{a2}) + (I_{a0} + a^2I_{a1} + aI_{a2}) + (I_{a0} + aI_{a1} + a^2I_{a2}) \\ &= 3I_{a0} + I_{a1}(1 + a + a^2) + I_{a2}(1 + a^2 + a) = 3I_{a0} \qquad [6] \end{aligned}$$

From [6], the current from the ground flowing into the neutral of a circuit has no positive- or negative-sequence components, but is equal to three times the zero-sequence line current in the circuit. It follows therefore that, if Z_n = neutral grounding impedance,

$$V_n = -I_nZ_n = -3I_{a0}Z_n = -I_{a0}(3Z_n) \qquad [7]$$

Equation [7] states that *the zero sequence current flowing through three times the neutral grounding impedance produces the same voltage at the neutral as the neutral current flowing through the neutral grounding impedance.* This is a convenient relation which makes it possible to obtain the equivalent impedance offered to zero-sequence currents per phase by adding three times the impedance of the return path to the zero-sequence impedance of the symmetrical part of the system.

Symmetrical Components of Phase Voltages to Ground at the Terminals of a Symmetrical Machine in Terms of the Sequence Impedances and Symmetrical Components of Current. Figure 2 represents a symmetrical three-phase synchronous machine, with neutral grounded through an impedance Z_n. Positive direction of phase currents and their symmetrical components is taken from the neutral towards the terminals; that of the neutral current, from ground towards the neutral.

The positive-sequence component of the voltage of phase a at terminal T is equal to the generated positive-sequence component of voltage of phase a, E_{a1}, minus the voltage drop due to the positive-sequence current of phase a flowing through the positive-sequence impedance between the generator neutral and T. The negative-sequence component of the voltage of phase a at T is the generated negative-sequence component of voltage of phase a, E_{a2}, minus the voltage drop due to the negative-sequence current of phase a flowing through the negative-sequence impedance between N and T. The zero-sequence component of voltage at T is V_n (the voltage of the neutral) plus E_{a0} (the generated zero-sequence component of voltage of phase a) minus the voltage drop caused by zero-sequence current flowing through the zero-sequence impedance between N and T, where V_n is given by [7]. With balanced generated voltages, $E_{a2} = E_{a0} = 0$ and $E_{a1} = E_a$.

The equations for the symmetrical components of line-to-ground voltage of phase a at the terminals of a symmetrical three-phase synchronous machine with balanced generated voltages are

$$V_{a1} = E_a - I_{a1}Z_1 \qquad [8]$$

$$V_{a2} = -I_{a2}Z_2 \qquad [9]$$

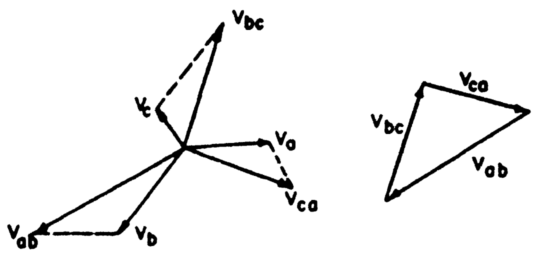
$$V_{a0} = V_n - I_{a0}Z'_0 = -I_{a0}(3Z_n + Z'_0) = -I_{a0}Z_0 \qquad [10]$$

where Z_1, Z_2, and Z'_0 are the positive-, negative-, and zero-sequence impedances, respectively, between machine neutral and terminals. $Z_0 = 3Z_n + Z'_0$ is the zero-sequence impedance between ground and T. Z'_0 in a Y-connected machine is a finite impedance. When the generator neutral is grounded, Z_n will also be finite; but when ungrounded, Z_n will be infinite.

When the symmetrical components of voltages are known, they can be substituted in [7]–[9] of Chapter II, and the line-to-ground voltages of the three phases obtained.

Note. V_{a0}, V_{a1}, and V_{a2} in [3]–[5] designate the symmetrical components of the neutral point N, while the same symbols in [8]–[10] refer to the terminal point T. Each point of a system will have its own symmetrical components of current and voltage, which, in general, will differ from those at other points.

Line-to-Line Voltages. The line-to-line voltages at any point T in a three-phase system will be the respective differences of the line-to-ground voltages at T, as shown in Fig. 3(a). When the differences



(a) (b)

FIG. 3. (a) Line-to-ground and line-to line voltages. (b) Sum of line-to-line voltages.

of the equations [7], [8], and [9] of Chapter II for the line-to-ground voltages in terms of their symmetrical components are taken, it is found that the zero-sequence components disappear. The line-to-line voltages are

$$
\begin{aligned}
V_{ab} &= V_b - V_a = (a^2 - 1)V_{a1} + (a - 1)V_{a2} \\
&= \sqrt{3}(V_{a1}\overline{/150^\circ} + V_{a2}\underline{/150^\circ}) \\
V_{bc} &= V_c - V_b = (a - a^2)V_{a1} + (a^2 - a)V_{a2} \\
&= \sqrt{3}(V_{a1}\underline{/90^\circ} + V_{a2}\overline{/90^\circ}) \\
V_{ca} &= V_a - V_c = (1 - a)V_{a1} + (1 - a^2)V_{a2} \\
&= \sqrt{3}(V_{a1}\overline{/30^\circ} + V_{a2}\underline{/30^\circ})
\end{aligned}
\qquad [11]
$$

Equations [11] express the line-to-line voltages in terms of the positive and negative sequence components of line-to-ground voltages. If V_{a1} and V_{a2} are expressed in volts, V_{ab}, V_{bc}, and V_{ca} will be in volts. If V_{a1} and V_{a2} are in per unit of base line-to-neutral voltage, V_{ab}, V_{bc}, and V_{ca} in [11] will also be in per unit of base line-to-neutral voltage.

The sum of the three line-to-line voltages shown in Fig. 3(b) is zero.

SHORT CIRCUITS

The simplest short-circuit problem, that presented by a fault at the terminals of a symmetrical Y-connected synchronous generator operated at no load with balanced generated voltages, will be treated first. Formulas will be developed for determining the three line currents and the line-to-ground and line-to-line voltages at the terminals of the generator for the following types of short circuits:

1. Three-phase fault.
2. Line-to-line fault.
3. Line-to-ground fault.
4. Double line-to-ground fault.

In applying the method of symmetrical components to the solution of problems involving a symmetrical three-phase system with but one point of dissymmetry, the procedure is to replace the phase currents and voltages at the point of dissymmetry by their symmetrical components of current and voltage. The phase currents and voltages may be any three currents and any three voltages associated with the three phases. These three currents and three voltages are the six unknowns to be determined. If the problem is to determine the line currents and the voltages to ground at the terminals of an unloaded generator with a fault at its terminals, the unknown currents and voltages are I_a, I_b, I_c, V_a, V_b, V_c, where I and V represent line currents and phase voltages to ground, respectively, and the subscripts refer to the phases. In any given problem, certain conditions are known about the unknown phase currents and voltages which can be expressed in equations. For example, if conductor a is faulted to ground at point P, the voltage to ground of phase a at P is zero, and the equation expressing this condition is $V_a = 0$. In a three-phase system, three equations can be written in terms of the three unknown phase currents and voltages at the point of dissymmetry. Three more equations are needed for a solution of the six unknowns. Equations [7]-[9] and [19]-[21] of Chapter II express the unknown phase voltages and currents, respectively, in terms of their symmetrical components, but they merely replace the six unknowns by six other unknowns: V_{a1}, V_{a2}, V_{a0}, I_{a1}, I_{a2}, I_{a0}. The advantage in using the six unknown components instead of the six unknown phase quantities is that the impedances met by the sequence currents can be determined either by calculation or test. This is not usually the case with phase impedances. However, if the phase impedances can also be readily obtained, there may be no advantage in introducing components; in fact, the use of phase quantities may give a simpler solution. If the generated voltages and the sequence impedances associated with each of the symmetrical components of current are known, three equations can be written expressing the components of voltage of each sequence at the fault in terms of the corresponding sequence current and the impedance associated with it. In the case of an unloaded symmetrical generator with a fault at its terminals, these three equations are given by [8]-[10]. Simultaneous solution of these three equations, together with the three equations determined by the boundary conditions, in which phase currents and voltages have been replaced by their symmetrical components of current and voltage, will give the six unknown symmetrical components of current and voltage. If more convenient, equations [10]–[12] and [22]–[24] of Chapter II can be used instead of [7]-[9] and [19]-[21]. The two sets of equations are not independent. One set expresses the phase quantities in terms of their symmetrical components; the other set, the symmetrical components in terms of the phase quantities. Either set may be used, or some equations from one set and some from the other, depending upon the problem. When the symmetrical components have been determined, the unknown phase currents and voltages are then calculated, using equations [7]-[9] and [19]-[21] of Chapter II.

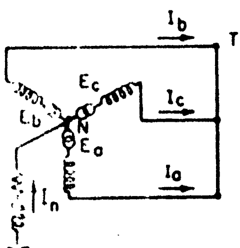


FIG. 4. Y-connected generator with three-phase short circuit at its terminals.

Three-Phase Fault. Since a symmetrical three-phase short circuit on a balanced system does not unbalance the system, it is evident that there will be only positive-sequence currents and voltages in the system. However, this case will be solved by the method of symmetrical components to illustrate the procedure with a balanced system.

(*a*) *Neutral Grounded.* Figure 4 shows a Y-connected generator with grounded neutral, and the three terminals short-circuited at T. The three equations expressing conditions at the fault are

$$V_b = V_a; \quad V_c = V_a; \quad I_a + I_b + I_c = 0$$

Equations [8]-[10] provide the other three simultaneous equations needed. Substituting V_a for V_b and V_c in [10]–[12] of Chapter II, and remembering that $1 + a + a^2 = 0$,

$$V_{a0} = V_a \qquad V_{a1} = 0 \qquad V_{a2} = 0$$

Substituting 0 for $I_a + I_b + I_c$ in [22] of Chapter II,

$$I_{a0} = 0$$

Substituting $V_{a2} = 0$ in [9],

$$I_{a2} = 0$$

Substituting $V_{a1} = 0$ in [8],

$$V_{a1} = 0 = E_a - I_{a1}Z_1$$

Therefore

$$I_{a1} = \frac{E_a}{Z_1} \tag{12}$$

Substituting $I_{a0} = 0$, $I_{a2} = 0$, and I_{a1} from [12] in [19]–[21] of Chapter II, the line currents with a three-phase fault are

$$I_a = I_{a1} = \frac{E_a}{Z_1} \tag{13}$$

$$I_b = a^2 I_{a1} = \frac{a^2 E_a}{Z_1} \tag{14}$$

$$I_c = a I_{a1} = \frac{a E_a}{Z_1} \tag{15}$$

Substituting $I_{a0} = 0$ in [10],

$$V_{a0} = -0 \cdot Z_0 \tag{16}$$

When Z_0 is finite, $V_{a0} = 0$. When Z_0 is infinite, V_{a0} in [16] is indeterminate. It may be evaluated, however, from the following considerations:

The point of fault T is common to the three phases; therefore the voltage from the fault to ground V_T, just like the voltage of the neutral to ground V_n, can have neither positive- nor negative-sequence components of voltage. (See [4] and [5].) The zero-sequence impedance between N and T is finite; therefore, with no zero-sequence current, there will be no zero-sequence voltage* drop, and T and N will be at the same potential above ground. In Fig. 4, T will be at ground potential, since the neutral point N is grounded.

(*b*) *Isolated Neutral.* With the neutral isolated but the fault point grounded, V_{a0} at T will be zero, and the voltage to ground at N, being the same as that at T, will be zero.

With the neutral and fault both isolated from ground, V_{a0} in [16] is mathematically indeterminate. However, in an ungrounded system in which capacitance and leakance are neglected, the term *voltage-to-ground* has no significance unless there should be an accidental ground

* It will be shown in Chapter VIII that in an *unsymmetrical* circuit there may be zero-sequence voltages present with no zero-sequence currents flowing.

on the system. In actual systems, leakance and capacitance to ground are always present, so that zero-sequence impedances to ground, although large, are not infinite. Since equal finite zero-sequence impedances to ground exist in the three phases of a symmetrical circuit with constant impedances, if there is no zero-sequence current, $V_{a0} = 0$ and $V_T = V_n = 0$.

The d-c voltage due to a static charge on an isolated system is to be distinguished from zero-sequence voltage, which is a single-phase voltage of fundamental frequency.

Problem 1. A generator whose subtransient, transient, and synchronous reactances are 12, 25, and 110%, respectively, and its resistance 0.6%, based on the rating of the generator, is operated at rated terminal voltage on open circuit. If a three-phase fault occurs, what are the initial symmetrical, transient, and sustained rms line currents expressed in per unit of rated current?

Solution. In a three-phase fault the rms values of I_b and I_c are equal to I_a in magnitude; therefore I_a only will be determined. Since $I_a = I_{a1}$, the first step will be to solve for I_{a1} by [13]. In this problem the resistance component of Z_1 will be neglected, since it is small relative to the reactance and its inclusion would not appreciably affect the magnitudes of the short-circuit currents.

The internal voltage of phase a, E_a, will be used as reference vector. Expressed in per unit, E_a will be 1.0 for all cases, since the generator is operated at rated terminal voltage on open circuit.

$$I_a = I_{a1} \text{ initial symmetrical rms} = \frac{1.0 + j0}{0 + j0.12} = -j8.33 \text{ per unit of rated current}$$

$$I_a = I_{a1} \text{ rms transient} = \frac{1 + j0}{0 + j0.25} = -j4.0 \text{ per unit of rated current}$$

$$I_a = I_{a1} \text{ rms sustained} = \frac{1 + j0}{0 + j1.10} = -j0.91 \text{ per unit of rated current}$$

Line-to-Line Fault. Figure 5 shows an unloaded Y-connected generator with grounded neutral and a line-to-line fault through zero fault impedance between terminals b and c. From the conditions of the problem it is evident that there can be no current in phase a, and that the currents in phases b and c are equal in magnitude and opposite in phase. The line-to-ground voltages at b and c must be the same, since there is no impedance between them.

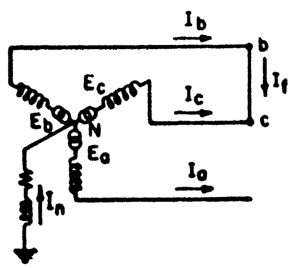


FIG. 5. Unloaded Y-connected generator with line-to-line short circuit at its terminals.

Expressing the conditions at the fault in equations,

$$I_a = 0 \qquad I_c = -I_b \qquad V_c = V_b$$

Substituting $I_a = 0$ and $I_c = -I_b$ in [22]–[24] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(0 + I_b - I_b) = 0$$

$$I_{a1} = \tfrac{1}{3}(0 + aI_b - a^2 I_b) = \frac{a - a^2}{3} I_b = j\frac{I_b}{\sqrt{3}} \qquad [17]$$

$$I_{a2} = \tfrac{1}{3}(0 + a^2 I_b - aI_b) = \frac{a^2 - a}{3} I_b = -j\frac{I_b}{\sqrt{3}}$$

Therefore

$$I_{a2} = -I_{a1} \qquad [18]$$

Subtracting [9] from [8] in Chapter II,

$$V_b - V_c = (a^2 - a)V_{a1} - (a^2 - a)V_{a2}$$

Since $V_b = V_c$,

$$V_{a1} = V_{a2} \qquad [19]$$

Replacing V_{a1} and V_{a2} in [19] by their values from [8] and [9], and substituting $-I_{a1}$ for I_{a2},

$$V_{a1} = E_a - I_{a1}Z_1 = V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \qquad [20]$$

Therefore

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}; \quad \text{and} \quad I_{a2} = -I_{a1} = -\frac{E_a}{Z_1 + Z_2} \qquad [21]$$

The current in the fault is

$$I_f = I_b = -I_c = a^2 I_{a1} + aI_{a2} + I_{a0} = (a^2 - a)I_{a1} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2} \qquad [22]$$

Substituting [21] in [20],

$$V_{a1} = V_{a2} = I_{a1}Z_2 = E_a\frac{Z_2}{Z_1 + Z_2} \qquad [23]$$

If the neutral is solidly grounded or grounded through an impedance, Z_0 is finite. Substituting $I_{a0} = 0$ in [10],

$$V_{a0} = -0 \cdot Z_0 = 0 \qquad [24]$$

When the generator neutral is ungrounded, Z_0 is infinite, and V_{a0} is indeterminate. As previously stated, in a system without an intentional or accidental ground when capacitance and leakance are neglected, V_{a0} is indeterminate; but, since capacitance and leakance to ground are always present, V_{a0} will be zero if the system is symmetrical and $I_{a0} = 0$.

Equations [17] and [24] give $I_{a0} = 0$, $V_{a0} = 0$. I_{a1}, I_{a2}, V_{a1}, and V_{a2} in terms of E_a and Z_1 and Z_2 are given by [21] and [23]. When numerical values of the symmetrical components have been calculated, the line currents and the line-to-ground voltages may be obtained by substituting them in [7]–[9] and [19]–[21] of Chapter II.

Problem 2. Find the line currents and the line-to-ground and line-to-line voltages in per unit of rated current and voltage, respectively, when a line-to-line fault through zero fault impedance occurs at the terminals of an unloaded generator with solidly grounded neutral. The field current is such as to produce rated voltage on open circuit. The generator is rated 15,000 kva, 13,800 volts, and the positive-, negative-, and zero-sequence impedances in per unit are $0.007 + j0.35$, $0.05 + j0.45$, and $0.007 + j0.06$, respectively. Draw the vector diagram giving the symmetrical components of current and voltage, line currents, and line-to-ground voltages.

Solution. Assume the fault to occur between terminals b and c. E_a, the generated voltage of phase a, is $13{,}800/\sqrt{3}$ in volts and 1.0 in per unit of rated line-to-neutral voltage. E_a will be taken as reference vector, and calculations will be made in per unit based on the generator rating.

From [17] and [24],

$$I_{a0} = 0; \quad V_{a0} = 0$$

Substituting $E_a = 1$ and the given per unit values of Z_1 and Z_2 in [21] and [23],

$$I_{a1} = -I_{a2} = \frac{E_a}{Z_1 + Z_2} = \frac{1.0}{0.057 + j0.80} = 1.247\overline{/85.9°} = 0.089 - j1.244$$

$$V_{a1} = V_{a2} = I_{a1}Z_2 = (0.089 - j1.244)(0.05 + j0.45) = 0.564 - j0.022$$
$$= 0.565\overline{/2.3°}$$

Substituting the symmetrical components in [19]–[21] and [7]–[9], of Chapter II,

$$I_a = 0 + I_{a1} - I_{a1} = 0$$

$$I_b = -I_c = 0 + a^2 I_{a1} - aI_{a1} = (a^2 - a)I_{a1} = -j\sqrt{3}I_{a1} = -2.155 - j0.154$$
$$= 2.160\overline{/175.9°}$$

$$V_a = 0 + V_{a1} + V_{a1} = 2V_{a1} = 1.128 - j0.044 = 1.129\overline{/2.3°}$$

$$V_b = V_c = 0 + a^2 V_{a1} + aV_{a1} = -V_{a1} = -0.564 + j0.022 = 0.565\underline{/177.7°}$$

From [11],

$$V_{ab} = V_b - V_a = -3V_{a1} = -1.692 + j0.066$$

$$V_{bc} = V_c - V_b = 0$$

$$V_{ca} = V_a - V_c = 3V_{a1} = 1.692 - j0.066$$

The symmetrical components of current and voltage, the line currents, and the line-to-ground voltages for this problem are given in the vector diagram of Fig. 6 in per unit of rated line current and line-to-neutral voltage with E_a as reference vector. If the vector length taken to represent unit voltage be assigned a scale value of $13{,}800/\sqrt{3} = 7970$ volts, and the length corresponding to unit current be considered as $15{,}000/(\sqrt{3}\ 13.8) = 628$ amp, the vector diagrams can be read in actual volts and amperes.

Note. It is suggested that the student close his book at this point and determine for himself the line currents and phase voltages to ground for the other types of short circuits. The following development, in that case, will serve as a check for his work.

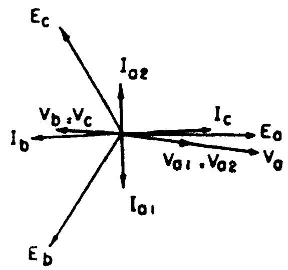


FIG. 6. Per unit vector diagram for Problem 2 with E_a the generated voltage of phase a as reference vector. Unit current scale one-third unit voltage scale.

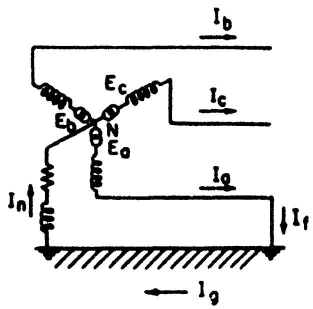


FIG. 7. Unloaded Y-connected generator with grounded neutral and line-to-ground fault on phase a.

Line-to-Ground Fault. (a) *Neutral of Generator Grounded.* Figure 7 shows a generator with grounded neutral, and a line-to-ground fault through zero fault impedance on phase a. The conditions at the fault are given by the following equations:

$$I_b = 0 \qquad I_c = 0 \qquad V_a = 0$$

Substituting $I_b = I_c = 0$ in [22]–[24] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

$$I_{a1} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

$$I_{a2} = \tfrac{1}{3}(I_a + 0 + 0) = \frac{I_a}{3}$$

Therefore

$$I_{a0} = I_{a1} = I_{a2} \qquad [25]$$

Substituting $V_a = 0$ in [7] of Chapter II,

$$V_{a1} = -(V_{a0} + V_{a2}) \qquad [26]$$

Substituting I_{a1} for I_{a0} and I_{a2} in [9] and [10],

$$V_{a2} = -I_{a2}Z_2 = -I_{a1}Z_2 \qquad [27]$$

$$V_{a0} = -I_{a0}Z_0 = -I_{a1}Z_0 \qquad [28]$$

Substituting [27] and [28] in [26],

$$V_{a1} = I_{a1}(Z_0 + Z_2) \qquad [29]$$

Substituting [29] in [8] and solving for I_{a1},

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0} \qquad [30]$$

The current in the fault I_f flows through ground and returns to the generator through the grounded neutral. From [6], neutral or ground current is a zero-sequence current equal to three times the zero-sequence line current flowing from the neutral. Therefore

$$I_f = I_g = I_n = 3I_{a0}$$

Also from Fig. 7, and equation [30],

$$I_f = I_g = I_n = I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a0} = \frac{3E_a}{Z_1 + Z_2 + Z_0} \qquad [31]$$

The fault current and the symmetrical components of current and voltage can be determined when I_{a1} has been evaluated. The line currents and voltages to ground at the generator terminals may then be obtained by substituting the values of the components in [7]–[9] and [19]–[21] of Chapter II.

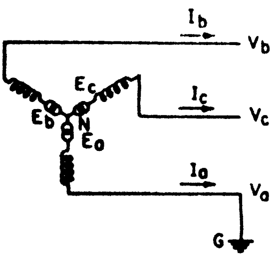


FIG. 8. Unloaded Y-connected generator with isolated neutral and line-to-ground fault on phase a.

(*b*) *Neutral of Generator Isolated.* Figure 8 shows a Y-connected generator with isolated neutral and phase a grounded through zero impedance. Since there is only one ground, neglecting capacitance and leakance, Z_0 will be infinite.

Substituting $Z_0 = \infty$ in [30],

$$I_{a1} = I_{a2} = I_{a0} = \frac{E}{Z_1 + Z_2 + \infty} = 0 \qquad [32]$$

Substituting 0 for I_{a1} and I_{a2} in [8] and [9],

$$V_{a1} = E_a \qquad V_{a2} = 0$$

Substituting $V_{a1} = E_a$ and $V_{a2} = 0$ in [26],

$$V_{a0} = -E_a$$

The zero-sequence voltage at the fault is $-E_a$. Since there is no zero-sequence current, there can be no zero-sequence voltage drop and the zero-sequence voltage of the neutral will be the same as that at the fault. Therefore

$$V_n = -E_a$$

The line-to-ground voltages are

$$V_a = V_{a1} + V_{a2} + V_{a0} = E_a + 0 - E_a = 0$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} = a^2 E_a + 0 - E_a = (a^2 - 1)E_a = E_{ab}$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} = a E_a + 0 - E_a = (a - 1)E_a = E_{ac}$$

The line-to-line voltages are

$$V_{ab} = V_b - V_a = (a^2 - 1)E_a = E_{ab}$$

$$V_{bc} = V_c - V_b = (a - a^2)E_a = E_{bc}$$

$$V_{ca} = V_a - V_c = (1 - a)E_a = E_{ca}$$

The line-to-line and line-to-neutral voltages will be unaffected by the fault, but the line-to-ground voltages will be unbalanced, one being zero and the other two equal to line-to-line voltage in magnitude. The voltage vector diagram is given in Fig. 9. E_a, E_b, and E_c are the generated voltages and also the terminal voltages to neutral or to ground before the fault, the neutral being at ground potential before the fault. V_a, V_b, and V_c are the phase voltages to ground and V_n the voltage of the neutral to ground after the fault. The fault causes the neutral to *shift* from its position of zero voltage before the fault to $-E_a$ after the fault.

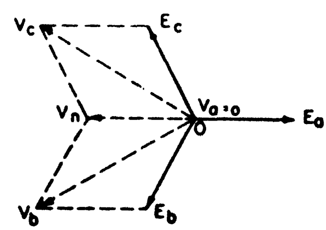


FIG. 9. Line-to-ground voltage vector diagram of isolated neutral generator with phase a grounded.

Problem 3. Find the per unit values of the symmetrical components of current and voltage, the line currents and the line-to-ground and line-to-line voltages for a fault of zero fault impedance on phase a of the generator described in Problem 2. Draw the vector diagram.

Solution. The given conditions are

$$E_a = 1.0; \quad Z_1 = 0.007 + j0.35; \quad Z_2 = 0.05 + j0.45; \quad Z_0 = 0.007 + j0.06$$

Substituting these values in [30], $I_{a1} = I_{a2} = I_{a0}$ is obtained. From [27]–[29], the three symmetrical components of voltage are obtained. Thus,

$$I_{a0} = I_{a2} = I_{a1} = \frac{1.0}{0.064 + j0.86} = 1.160\overline{/85.7^\circ} = 0.086 - j1.156$$

$$V_{a0} = -I_{a0}Z_0 = -(0.086 - j1.156)(0.007 + j0.06) = -0.070 + j0.003$$

$$V_{a2} = -I_{a2}Z_2 = -(0.086 - j1.156)(0.05 + j0.45) = -0.524 + j0.019$$

$$V_{a1} = I_{a1}(Z_0 + Z_2) = -(V_{a0} + V_{a2}) = 0.594 - j0.022$$

The line currents, the fault current, and the line-to-ground voltages obtained by substituting their symmetrical components in [19]–[21] and [7]–[9] of Chapter II are

$$I_a = I_f = I_g = I_n = 3I_{a1} = 0.258 - j3.468$$

$$I_b = I_c = 0$$

$$V_a = 0$$

$$\begin{aligned} V_b &= (-0.070 + j0.003) + a^2(0.594 - j0.022) + a(-0.524 + j0.019) \\ &= (-0.070 + j0.003) - \tfrac{1}{2}(0.070 - j0.003) - j\frac{\sqrt{3}}{2}(1.118 - j0.041) \\ &= -0.140 - j0.964 \end{aligned}$$

$$\begin{aligned} V_c &= (-0.070 + j0.003) + a(0.594 - j0.022) + a^2(-0.524 + j0.019) \\ &= (-0.070 + j0.003) - \tfrac{1}{2}(0.070 - j0.003) + j\frac{\sqrt{3}}{2}(1.118 - j0.041) \\ &= -0.070 + j0.973 \end{aligned}$$

The line-to-line voltages, obtained from the line-to-ground voltages, are

$$V_{ab} = V_b - V_a = V_b = -0.140 - j0.964$$

$$V_{bc} = V_c - V_b = 0.070 + j1.937$$

$$V_{ca} = V_a - V_c = -V_c = 0.070 - j0.973$$

In the vector diagram of Fig. 10, the symmetrical components of current and voltage, the line currents, and the line-to-ground voltages are given in per unit of rated line current and line-to-neutral voltage, with $E_a = 1$ as reference vector.

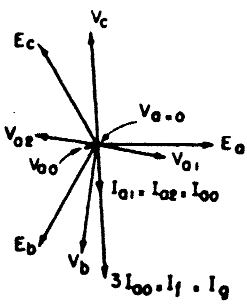


FIG. 10. Per unit vector diagram for Problem 3. Unit current scale one-third unit voltage scale.

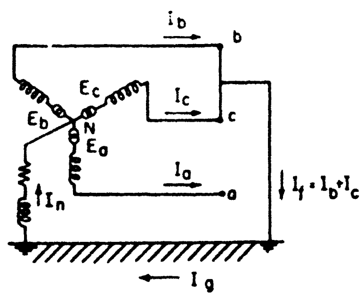


FIG. 11. Unloaded generator with grounded neutral and double line-to-ground fault at its terminals.

Double Line-to-Ground Fault. (*a*) *Neutral of Generator Grounded.* Figure 11 shows an unloaded generator with grounded neutral and terminals b and c grounded through zero impedances. The conditions at the fault are given by the following equations:

$$V_b = 0 \qquad V_c = 0 \qquad I_a = 0$$

Replacing V_b and V_c by zero in [10]–[12], Chapter II,

$$V_{a1} = V_{a2} = V_{a0} = \frac{V_a}{3} \qquad [33]$$

Replacing I_a by zero in [19] of Chapter II and solving for I_{a1},

$$I_{a1} = -(I_{a0} + I_{a2}) \qquad [34]$$

Substituting for V_{a2} and V_{a0} in [33] their values from [9] and [10],

$$V_{a1} = -I_{a2}Z_2 = -I_{a0}Z_0 \qquad [35]$$

Therefore

$$I_{a2} = -\frac{V_{a1}}{Z_2} \quad \text{and} \quad I_{a0} = -\frac{V_{a1}}{Z_0} \qquad [36]–[37]$$

Substituting [36] and [37] in [34],

$$I_{a1} = -(I_{a2} + I_{a0}) = V_{a1}\left(\frac{1}{Z_2} + \frac{1}{Z_0}\right) = V_{a1}\frac{Z_2 + Z_0}{Z_2Z_0} \qquad [38]$$

Therefore

$$V_{a1} = V_{a2} = V_{a0} = I_{a1}\frac{Z_2Z_0}{Z_2 + Z_0} \qquad [39]$$

From [36], [37], and [39], I_{a2} and I_{a0} can be expressed in terms of I_{a1}. Thus

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} \qquad [40]$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} \qquad [41]$$

Replacing V_{a1} in [8] by its value from [39] in terms of I_{a1} and solving for I_{a1},

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_0Z_2}{Z_0 + Z_2}} = \frac{E_a(Z_0 + Z_2)}{Z_0Z_1 + Z_0Z_2 + Z_1Z_2} \qquad [42]$$

At this point it is interesting to note that the value of I_{a1} was determined in the line-to-line fault by $Z_1 + Z_2$, in the single line-to-ground fault by $Z_1 + Z_0 + Z_2$, but in the present case by the sum of Z_1 and the parallel value of Z_0 and Z_2.

After the numerical value of I_{a1} has been calculated from [42] the other symmetrical components can be obtained, and from the symmetrical components the line currents and the line-to-ground and line-to-line voltages determined.

(*b*) *Neutral of Generator Isolated.* Figure 12 shows an unloaded generator with isolated neutral and phases *b* and *c* grounded through zero impedance. From Fig. 12 the conditions of the problem are

$$I_a = 0 \qquad I_b + I_c = 0 \qquad V_b = 0 \qquad V_c = 0$$

Substituting 0 for I_a and $-I_b$ for I_c in [22]–[24] of Chapter II and 0 for V_b and V_c in [10]–[12] of Chapter II,

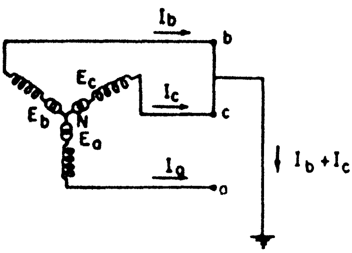


FIG. 12. Unloaded generator with isolated neutral and double line-to-ground fault at its terminals.

$$I_{a0} = 0 \tag{43}$$

$$I_{a2} = -I_{a1} \tag{44}$$

$$V_{a0} = V_{a1} = V_{a2} = \frac{V_a}{3} \tag{45}$$

Replacing I_{a2} by $-I_{a1}$ in [9],

$$V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \tag{46}$$

Substituting [46] in [45],

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}Z_2 \tag{47}$$

Substituting $V_{a1} = I_{a1}Z_2$ from [47] in [8] and solving for I_{a1},

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \tag{48}$$

Substituting [48] in [47],

$$V_{a1} = V_{a2} = V_{a0} = E_a \frac{Z_2}{Z_1 + Z_2} \tag{49}$$

Comparing the symmetrical components of currents for the double line-to-ground fault with isolated neutral with those for a line-to-line fault, it can be concluded that when a double line-to-ground fault occurs at the terminals of a generator with isolated neutral the line currents are the same whether the point of fault is isolated or grounded. Also, if the line-to-line voltages for the two cases are compared, they will be found to be identical. For the line-to-line fault with isolated neutral, the zero-sequence voltage is mathematically indeterminate, but practically it is zero. For the double line-to-ground fault with isolated neutral, it has been shown that

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}Z_2 = \frac{E_aZ_2}{Z_1 + Z_2}$$

The grounding of the fault point causes the neutral to shift from its initial position to $\dfrac{E_aZ_2}{Z_1 + Z_2}$.

Problem 4. Using per unit quantities, find the line currents, the fault current, and the line-to-ground and line-to-line voltages for a double line-to-ground fault of zero fault impedance on terminals b and c of a solidly grounded generator rated 10,000 kva, 13.8 kv, with positive-, negative-, and zero-sequence reactances of 30, 40, and 5%, respectively. The generator was operated at normal terminal voltage on open circuit before the fault.

Solution. From the given conditions

$$E_a = 1.0; \quad Z_0 = 0 + j0.05; \quad Z_1 = 0 + j0.30; \quad Z_2 = 0 + j0.40$$

Selecting E_a as reference vector, the value of I_{a1} calculated from [42] is

$$I_{a1} = \frac{1 + j0}{0 + j0.30 + \dfrac{(0 + j0.05)(0 + j0.40)}{0 + j0.45}} = \frac{1}{j0.30 + \dfrac{-0.02}{j0.45}}$$

$$= \frac{1}{j0.30 + j0.044} = \frac{1}{j0.344} = -j2.90$$

From [39]–[41] the other symmetrical components are

$$V_{a0} = V_{a1} = V_{a2} = (-j2.90)\frac{(0 + j0.05)(0 + j0.40)}{0 + j0.45} = (-j2.90)(j0.044) = 0.13$$

$$I_{a2} = -(-j2.90)\frac{0 + j0.05}{0 + j0.45} = j0.32$$

$$I_{a0} = -(-j2.90)\frac{0 + j0.40}{0 + j0.45} = j2.58$$

Substituting the symmetrical components in [19]–[21] and [7]–[9] of Chapter II (to shorten numerical work use [29], [30], [25], and [26] of Chapter II),

$$I_a = j2.58 - j2.90 + j0.32 = 0$$

$$I_b = j2.58 - 0.5(-j2.58) - j0.866(-j3.22) = -2.79 + j3.87$$

$$I_c = j2.58 - 0.5(-j2.58) + j0.866(-j3.22) = 2.79 + j3.87$$

$$I_f = I_b + I_c = I_g = I_n = 3I_{a0} = 3 \times j2.58 = j7.74$$

$$V_a = 3(0.13) = 0.39$$

$$V_b = (1 + a^2 + a)V_{a1} = 0$$

$$V_c = (1 + a + a^2)V_{a1} = 0$$

$$V_{ab} = V_b - V_a = -0.39$$

$$V_{bc} = V_c - V_b = 0$$

$$V_{ca} = V_a - V_c = 0.39$$

Windings of the Generator Connected in Δ

If the windings of the generator are connected in Δ, there is no path for zero-sequence currents and the zero-sequence impedance of the generator is infinite. For calculating conditions at its terminals, the Δ-connected generator can be replaced by an equivalent Y-connected generator with isolated neutral. The positive- and negative-sequence impedances of the equivalent Y-connected generator, if expressed in ohms, will be one-third those of the given Δ-connected generator; if expressed in per unit, they will be the same as those of the given Δ-connected generator. (See Chapter I.) For faults at the terminals of a Δ-connected generator, the line currents and the line-to-ground and line-to-line voltages at the fault will be the same as for a Y-connected ungrounded generator having the same per unit positive- and negative-sequence impedances based on the rating of the generator.

Voltages of Δ-Connected Windings. Voltages across windings connected in Δ are line-to-line voltages. Line-to-line voltages are expressed in terms of the symmetrical components of the line-to-ground voltages of phase a by [11]. When the line-to-ground voltages have only positive-sequence components of voltage, V_{a2} in [11] is zero and the Δ voltages have only positive-sequence components of voltage; these equations then become

$$\begin{aligned} V_{ab1} &= V_{b1} - V_{a1} = (a^2 - 1)V_{a1} = \sqrt{3}V_{a1}\overline{/150^\circ} \\ V_{bc1} &= V_{c1} - V_{b1} = (a - a^2)V_{a1} = \sqrt{3}V_{a1}/90^\circ = j\sqrt{3}V_{a1} \\ V_{ca1} &= V_{a1} - V_{c1} = (1 - a)V_{a1} = \sqrt{3}V_{a1}\overline{/30^\circ} \end{aligned} \qquad [50]$$

When the line-to-ground voltages have only negative-sequence components of voltages, V_{a1} in [11] is zero and the Δ voltages have only negative-sequence components of voltage; these equations then become

$$\begin{aligned} V_{ab2} &= V_{b2} - V_{a2} = (a - 1)V_{a2} = \sqrt{3}V_{a2}\underline{/150^\circ} \\ V_{bc2} &= V_{c2} - V_{b2} = (a^2 - a)V_{a2} = \sqrt{3}V_{a2}\overline{/90^\circ} = -j\sqrt{3}V_{a2} \\ V_{ca2} &= V_{a2} - V_{c2} = (1 - a^2)V_{a2} = \sqrt{3}V_{a2}\underline{/30^\circ} \end{aligned} \qquad [51]$$

Equations [50] and [51] express positive- and negative-sequence components of line-to-line voltages in terms of the positive- and negative-sequence components, respectively, of V_a. Figures 13(a) and (b) show these relations graphically.

As the line-to-line voltages themselves constitute a set of three-phase vectors, they can be resolved into positive- and negative-sequence components; but they will have no zero-sequence components since their sum is zero. (See Fig. 3(b).) The choice of the line-to-line voltage to be selected as reference is arbitrary. The simplest relations between the symmetrical components of I_a and the symmetrical components of the line-to-line voltages given by [50] and [51] are those for the components of V_{bc} which involve phase differences of ±90°. Those for V_{ab} or V_{ca} involve ±150° or ±30°, respectively. The components of

V_{bc} ($= V_c - V_b$) will therefore be selected as reference components, where the symmetrical components of V_{bc} in terms of the symmetrical components of V_a are

$$\begin{aligned} V_{bc0} &= 0 \\ V_{bc1} &= (a - a^2) V_{a1} = \sqrt{3} V_{a1} \underline{/90^\circ} = j\sqrt{3} V_{a1} \\ V_{bc2} &= (a^2 - a) V_{a2} = \sqrt{3} V_{a2} \overline{\diagup 90^\circ} = -j\sqrt{3} V_{a2} \end{aligned} \qquad [52]$$

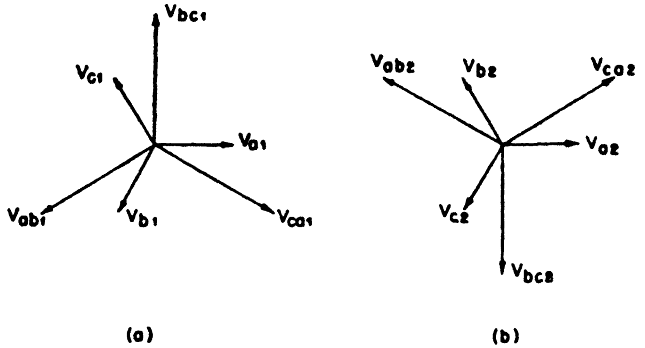


FIG. 13. Voltage vector diagrams of components of line-to-line and line-to-neutral voltages. (*a*) Positive sequence. (*b*) Negative sequence.

The symmetrical components of V_a in terms of the symmetrical components of V_{bc} from [52] are

$$\begin{aligned} &V_{a0} \text{ is indeterminate} \\ V_{a1} &= \frac{V_{bc1}}{\sqrt{3}} \overline{\diagup 90^\circ} = -j\frac{V_{bc1}}{\sqrt{3}} \\ V_{a2} &= \frac{V_{bc2}}{\sqrt{3}} \underline{/90^\circ} = j\frac{V_{bc2}}{\sqrt{3}} \end{aligned} \qquad [53]$$

The line-to-line voltages will be expressed in terms of the symmetrical components of V_{bc} by equations similar to those used to express V_a, V_b, and V_c in terms of the symmetrical components of V_a, except that there are no zero-sequence components in line-to-line voltages. See equations [7]–[9] of Chapter II. From Fig. 13(*a*), $V_{ca1} = a^2 V_{bc1}$, $V_{ab1} = aV_{bc1}$; from Fig. 13(*b*), $V_{ca2} = aV_{bc2}$, $V_{ab2} = a^2 V_{bc2}$. Therefore

$$\begin{aligned} V_{bc} &= V_{bc1} + V_{bc2} \\ V_{ca} &= V_{ca1} + V_{ca2} = a^2 V_{bc1} + a V_{bc2} \\ V_{ab} &= V_{ab1} + V_{ab2} = a V_{bc1} + a^2 V_{bc2} \end{aligned} \qquad [54]$$

Equations [52] and [53] express the relations between the symmetrical components of the line-to-line voltage V_{bc} and the line-to-ground voltage V_a when both are expressed in volts, or in per unit on a *common voltage base.*

If the components of line-to-line voltages are expressed on *base line-to-line voltage,* and the components of line-to-ground voltages on *base line-to-neutral voltage,* the $\sqrt{3}$ disappears, and [52] and [53] become

$$\begin{aligned} V_{bc0} &= 0 \\ V_{bc1} &= V_{a1}\angle 90^\circ = jV_{a1} \\ V_{bc2} &= V_{a2}\overline{/90^\circ} = -jV_{a2} \end{aligned} \qquad [55]$$

and

$$\begin{aligned} &V_{a0} \text{ is indeterminate} \\ V_{a1} &= V_{bc1}\overline{/90^\circ} = -jV_{bc1} \\ V_{a2} &= V_{bc2}\angle 90^\circ = jV_{bc2} \end{aligned} \qquad [56]$$

Currents in Δ-Connected Windings. With $V_{bc} = V_c - V_b$ selected as reference phase for line-to-line voltages, where V_{bc} represents the rise in voltage in going from b to c, currents in Δ-connected windings will be expressed in terms of the symmetrical components of the current in the winding bc. The choice, however, of positive direction for current flow in the winding bc is arbitrary. (See Chapter I for a discussion of voltage rise, voltage drop, and positive direction of current flow.) Consider, for example, a Y-connected generator supplying a pure resistance Y-connected load at its terminals as in Fig. 14(a). Arrows are used to indicate positive direction of current flow in phase a. The current I_a flowing out of the generator and into the load flows in the direction of voltage rise through the generator but in the direction of voltage drop through the load. By analogy, positive direction for current flow in the Δ windings will be taken in the direction of voltage rise (i.e., from b to c) when positive direction for line currents is away from the Δ terminals; and in the direction of voltage drop (i.e., from c to b) when positive direction for line currents is towards the Δ terminals. This convention is indicated in Figs. 14(b) and (c), where I with two subscripts represents current flowing from the point indicated by the first subscript towards the point indicated by the second.

It should be noted that arrows in Figs. 14(a), (b), and (c) are used to indicate the direction of current flow and should not be confused with the phase of the current. For example, in Fig. 14(a), with a unity power factor load delivered by the generator, I_a in the generator flowing from N to T is in phase with the terminal voltage indicated by the vector V_a. For this case, the directional arrow accompanying I_a in the generator might also represent its phase, but the directional arrow accompanying I_a flowing into the load from T to N' could not represent its phase (which is the same as that of I_a flowing from the generator) on the same current vector diagram. Likewise in the Δ

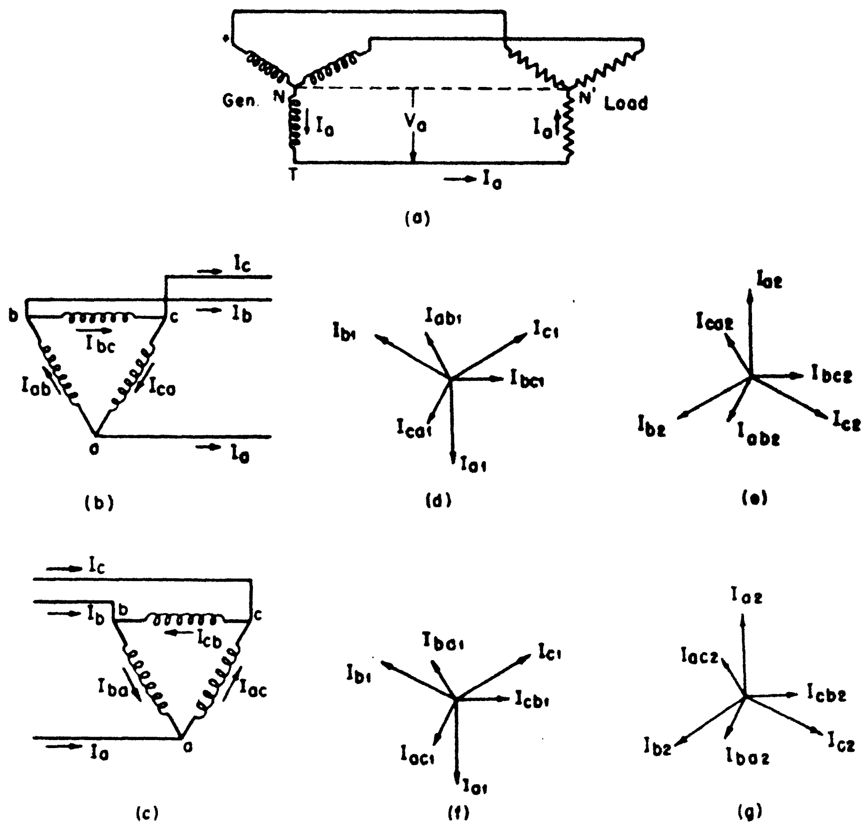


Fig. 14. (*a*) Y-connected generator supplying Y-connected resistance load. (*b*) and (*c*) Currents in Δ-connected windings and line currents flowing *from* and *toward* the Δ, respectively. (*d*) and (*e*) Positive- and negative-sequence vector diagrams, respectively, of currents indicated by arrows in (*b*) with I_{bc1} and I_{bc2} as reference vectors. (*f*) and (*g*) Positive- and negative-sequence vector diagrams, respectively, of currents indicated by arrows in (*c*) with I_{cb1} and I_{cb2} as reference vectors.

circuits of Figs. 14(*b*) and (*c*), for a unity power factor load delivered by the Δ of Fig. 14(*b*) and received by the Δ of Fig. 14(*c*), I_{bc} in Fig. 14(*b*) is in phase with V_{bc} and I_{cb} in Fig. 14(*c*) is also in phase with V_{bc}. The arrow accompanying I_{bc} and representing its direction might also represent its phase, but the arrow accompanying I_{cb} could not represent its phase (which is the same as that of I_{bc}) on the same vector diagram.

In Fig. 14(b), applying Kirchhoff's law,

$$\begin{aligned} I_a &= I_{ca} - I_{ab} \\ I_b &= I_{ab} - I_{bc} \\ I_c &= I_{bc} - I_{ca} \end{aligned} \qquad [57]$$

The currents I_{bc}, I_{ca}, and I_{ab} are expressed in terms of the symmetrical components of I_{bc} by the following equations, analogous to those of [54] with the addition of zero-sequence currents which may be present in the Δ,

$$\begin{aligned} I_{bc} &= I_{bc1} + I_{bc2} + I_{bc0} \\ I_{ca} &= a^2 I_{bc1} + a I_{bc2} + I_{bc0} \\ I_{ab} &= a I_{bc1} + a^2 I_{bc2} + I_{bc0} \end{aligned} \qquad [58]$$

Replacing I_a, I_b, and I_c in [57] by their values in terms of the symmetrical components of I_a given by [19]–[21] of Chapter II, and I_{bc}, I_{ca}, and I_{ab} by their values in terms of the symmetrical components of I_{bc} given by [58], the following relations are obtained:

$$\begin{aligned} I_{a0} &= 0 \\ I_{a1} &= -j\sqrt{3} I_{bc1} = \sqrt{3} I_{bc1} \overline{/90^\circ} \\ I_{a2} &= j\sqrt{3} I_{bc2} = \sqrt{3} I_{bc2} \underline{/90^\circ} \end{aligned} \qquad [59a]$$

and

I_{bc0} is indeterminate

$$\begin{aligned} I_{bc1} &= \frac{j}{\sqrt{3}} I_{a1} = \frac{I_{a1}}{\sqrt{3}} \underline{/90^\circ} \\ I_{bc2} &= \frac{-j}{\sqrt{3}} I_{a2} = \frac{I_{a2}}{\sqrt{3}} \overline{/90^\circ} \end{aligned} \qquad [59b]$$

Equations [59] give the relations between the symmetrical components of Δ current and line current flowing from the Δ when both are expressed in amperes or in per unit on a common current base.

The corresponding relations between the symmetrical components of the Δ current I_{cb} and the line current I_a flowing towards the Δ, obtained in a similar manner, are

$$\begin{aligned} I_{a0} &= 0 \\ I_{a1} &= -j\sqrt{3} I_{cb1} = \sqrt{3} I_{cb1} \overline{/90^\circ} \\ I_{a2} &= j\sqrt{3} I_{cb2} = \sqrt{3} I_{cb2} \underline{/90^\circ} \end{aligned} \qquad [60a]$$

and

$$I_{cb0} \text{ is indeterminate}$$

$$I_{cb1} = j\frac{I_{a1}}{\sqrt{3}} = \frac{I_{a1}}{\sqrt{3}}\angle 90^\circ \qquad [60b]$$

$$I_{cb2} = -j\frac{I_{a2}}{\sqrt{3}} = \frac{I_{a2}}{\sqrt{3}}\overline{\angle 90^\circ}$$

The positive- and negative-sequence current vector diagrams for Δ currents I_{bc}, I_{ca}, and I_{ab} and line currents flowing from the Δ are shown in Figs. 14(d) and (e); and for the Δ currents I_{cb}, I_{ba}, and I_{ac} and the line currents flowing towards the Δ in Figs. 14(f) and (g). In these diagrams, currents are in amperes or in per unit on a common current base.

When line and Δ currents are expressed in per unit, each on its own current base, the $\sqrt{3}$ in [59]-[60] disappears.

Summary of Relations between Components of Current and Voltage in the Δ and Components of Line Current and Voltage to Ground at the Δ Terminals. When currents flow from a Δ into the line, or from the line into a Δ, there can be no zero-sequence components of current in the line. Zero-sequence current may appear in the delta as a circulating current, but its magnitude cannot be determined from the line currents. With line and Δ currents expressed in per unit on their respective base currents, and line-to-ground voltages and Δ voltage in per unit of base line-to-neutral voltage and base line-to-line voltage, respectively, the following relations obtain:

$$\begin{aligned} &V_{bc1} \text{ leads } V_{a1} \text{ by } 90^\circ\text{:} \quad V_{bc1} = jV_{a1} \\ &V_{bc2} \text{ lags } V_{a2} \text{ by } 90^\circ\text{:} \quad V_{bc2} = -jV_{a2} \\ &V_{bc0} = 0\text{;} \quad V_{a0} \text{ is indeterminate} \end{aligned} \qquad [61]$$

When positive direction of current flow is away from the Δ,

$$\begin{aligned} &I_{bc1} \text{ leads } I_{a1} \text{ by } 90^\circ\text{:} \quad I_{bc1} = jI_{a1} \\ &I_{bc2} \text{ lags } I_{a2} \text{ by } 90^\circ\text{:} \quad I_{bc2} = -jI_{a2} \end{aligned} \qquad [62a]$$

When positive direction of current flow is towards the Δ,

$$\begin{aligned} &I_{cb1} \text{ leads } I_{a1} \text{ by } 90^\circ\text{:} \quad I_{cb1} = jI_{a1} \\ &I_{cb2} \text{ lags } I_{a2} \text{ by } 90^\circ\text{:} \quad I_{cb2} = -jI_{a2} \end{aligned} \qquad [62b]$$

It should be noted that $V_{cb1} = -V_{bc1}$; $V_{cb2} = -V_{bc2}$; $I_{cb1} = -I_{bc1}$; $I_{cb2} = -I_{bc2}$.

Problem 5. In Problem 2, assume the generator Δ-connected and determine the currents in the generator windings and the voltages across them from the positive- and negative-sequence components of line currents and line-to-ground voltages at the generator terminals.

Solution. From the solution of Problem 2 with E_a as reference vector,

$$I_{a1} = -I_{a2} = 0.089 - j1.244 \text{ per unit of rated line current}$$

$$V_{a1} = V_{a2} = 0.564 - j0.022 \text{ per unit of rated line-to-neutral voltage}$$

With $I_{a1} = -I_{a2}$, from [62a],

$$I_{bc1} = I_{bc2} = jI_{a1} = 1.244 + j0.089 \text{ per unit of rated } \Delta \text{ current}$$

With $V_{a1} = V_{a2}$, from [61],

$$V_{bc1} = -V_{bc2} = jV_{a1} = 0.022 + j0.564 = \text{per unit of rated line-to-line voltage}$$

Base voltage in the Δ windings is 13,800 volts. Base current is 15,000/(3 × 13.8) = 362 amp.

Substituting I_{bc1} and I_{bc2} in [58] with $I_{bc0} = 0$,

$$I_{bc} = 2I_{bc1} = 2.488 + j0.178 \text{ per unit of rated } \Delta \text{ current} = 900 + j65 \text{ amp.}$$

$$I_{ca} = -I_{bc1} = -1.244 - j0.089 \text{ per unit of rated } \Delta \text{ current} = -450 - j32 \text{ amp.}$$

$$I_{ab} = -I_{bc1} = -1.244 - j0.089 \text{ per unit of rated } \Delta \text{ current} = -450 - j32 \text{ amp.}$$

Substituting V_{bc1} and V_{bc2} in [54],

$$V_{bc} = 0$$

$$V_{ca} = (a^2 - a)V_{bc1} = -j\sqrt{3}V_{bc1} = 0.977 - j0.038 \text{ per unit of rated line-to-line voltage} = 13{,}500 - j525 \text{ volts}$$

$$V_{ab} = (a - a^2)V_{bc1} = +j\sqrt{3}V_{bc1} = -0.974 + j0.038 \text{ per unit of rated line-to-line voltage} = -13{,}500 + j525 \text{ volts}$$

Faults on Circuits in Series with an Unloaded Generator

Equations for the symmetrical components of currents and voltages at the fault, with a short circuit at the terminals of an unloaded generator, can be extended by analogy to unloaded circuits in series with the generator. When there is no intervening transformer bank, the positive-, negative-, and zero-sequence impedances of the series circuits are added directly to the positive-, negative-, and zero-sequence impedances, respectively, of the generator, giving total impedances Z_1, Z_2, and Z_0 viewed from the fault. These impedances replace the sequence impedances of the generator in the formulas developed for faults at the generator terminals.

When there is a transformer bank between the generator and the fault, and currents and voltages are to be determined on both sides of the bank, one-line impedance diagrams for each of the sequence systems are useful.

Sequence Networks. A symmetrical three-phase system rendered unbalanced by a fault can be resolved into positive-, negative-, and zero-sequence systems. In each of these systems the currents and voltages are symmetrical. Their three-phase impedance networks have equal impedances in the three phases and can therefore be replaced for purposes of calculation by an equivalent single-phase network and represented by a one-line impedance diagram. In the one-line sequence impedance diagrams of a three-phase network each piece of apparatus and each transmission circuit is replaced by its equivalent circuits. An equivalent circuit at its terminals should represent the apparatus or circuit which it replaces to the degree of precision required in the problem. See Chapter I for a general discussion of equivalent circuits and the development of equivalent circuits for use in positive-sequence one-line impedance diagrams.

In a symmetrical system, the one-line impedance diagram is the same, regardless of which phase is used as reference phase, but with the reference phase specified the currents and voltages in the single-phase network are those of the reference phase. The terms *positive-*, *negative-*, and *zero-sequence networks* will be used to indicate single-phase networks in which currents and voltages of positive-, negative-, and zero-sequence, respectively, are those of the reference phase. In each sequence network, the sequence voltages are referred to the zero-potential bus for the network.

Positive-Sequence Network. The positive-sequence one-line impedance diagram of a symmetrical three-phase system is discussed in Chapter I, and equivalent circuits are developed for use in this impedance diagram. All neutral points are at zero potential in the positive-sequence system and are therefore connected to a common point which is zero potential for the network. Generated voltages of the reference phase are represented in the positive-sequence network as applied between the zero-potential bus and the impedance of the generator. See Chapter I, Fig. 17.

Negative-Sequence Network. The negative-sequence, as well as the positive-sequence, voltage of all neutral points is zero. (See [4] and [5].) In the one-line impedance diagram of the negative-sequence system, all neutrals are therefore connected to a common point which is zero potential for the network. The negative-sequence network is similar to the positive, except that there are no generated negative-sequence voltages in a system balanced before the fault occurred. In a symmetrical three-phase static circuit, the positive- and negative-sequence impedances are equal; the equivalent circuits developed in Chapter I for use in the positive-sequence one-line impedance diagram can be used also in the negative-sequence network if the given circuit is a symmetrical three-phase static circuit, with or without mutual impedances between phases. As previously stated, the negative-sequence impedances of rotating machines are, in general, different from positive-sequence impedances.

Zero-Sequence Network. The zero-sequence system is not a three-phase system, since the phase currents and voltages are equal in magnitude and in phase. It is a single-phase system, with equal currents and equal voltages in the three phases at all points of the given three-phase system. The currents and voltages in the zero-sequence network are the same, regardless of which phase is selected as reference phase. The reference for zero-sequence voltages at any point in a grounded system is the ground at that particular point.

The reference for zero-sequence voltages is of a different character from that for positive- and negative-sequence voltages. In positive- or negative-sequence systems all neutral points are at the *same potential.* Neutral points in either system can therefore be connected to a common point. On the other hand, the ground is not necessarily at the same potential at all points. Therefore, in the one-line diagram of the zero-sequence system, the individual equivalent circuits for the various circuits and apparatus of the system must be so constructed that the zero-sequence voltages to ground at the terminals of these circuits are correctly given when referred to the zero-potential bus of the network. The zero-potential bus for the zero-sequence network does not represent the potential of the ground at any particular point, but is the reference ground for zero-sequence voltages at all points of the system. In Chapter XI a further discussion of the reference for zero-sequence voltages is given.

Equivalent circuits for the zero-sequence network depend upon the impedance met by the zero-sequence currents flowing in the three phases and their sum, $3I_{a0}$, flowing through neutral impedance and returning through the ground or a neutral conductor. If there is no complete path for zero-sequence currents in a circuit, the zero-sequence impedance is infinite. In drawing the zero-sequence impedance diagram, an infinite impedance is represented as an open circuit. Thus a Y-connected circuit with ungrounded neutral has infinite impedance to zero-sequence currents. As the equivalent circuit is used to determine voltages as well as currents, the opening is placed at the point where the impedance becomes infinite. For a Y-connected circuit of three equal self-impedances Z, with neutral N ungrounded and terminals at T, as in Fig. 15(a), a finite impedance Z is indicated in the equivalent circuit of Fig. 15(b) between N and T, and an open circuit between N and the zero-potential bus. Point T is to be connected into the zero-sequence diagram of the system. With finite impedance between T and N, no current will flow in this impedance; and, if there is no induced voltage, T and N are at the same zero-sequence potential.

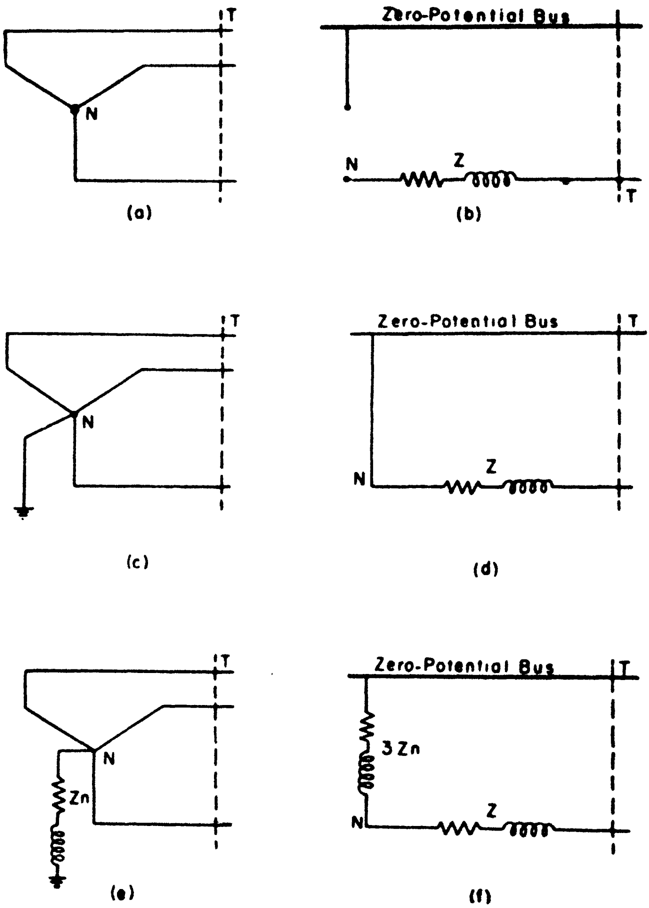


FIG. 15. Symmetrical Y-connected circuits and their zero-sequence equivalent circuits.

With the neutral solidly grounded, as in Fig. 15(c), N in the equivalent circuit of Fig. 15(d) is connected directly to the zero-potential bus. With the neutral grounded through an impedance Z_n, as in Fig. 15(e), N in the equivalent circuit shown in Fig. 15(f) is connected through $3Z_n$ to the zero-potential bus.

A Δ-connected circuit provides no path for zero-sequence currents flowing in the line. Viewed from its terminals, its zero-sequence impedance is infinite. The Δ voltages are line-to-line voltages and, since their sum is zero, they can have no zero-sequence components. The equivalent zero-sequence circuit for the symmetrical Δ-connected circuit of Fig. 16(*a*) with terminals at *T* is shown in Fig. 16(*b*). In this equivalent circuit there is an open circuit at *T* on the Δ side, indicating infinite impedance viewed from *T* towards the Δ; but beyond

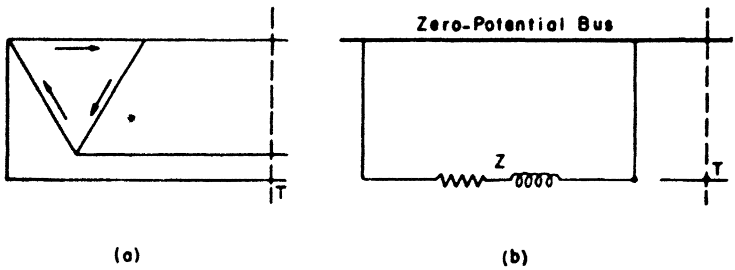


FIG. 16. Symmetrical Δ-connected circuit and its zero-sequence equivalent circuit.

the opening there is a connection to the zero-potential bus, indicating that (1) there may be zero-sequence voltages at *T* but there can be none across the phases of the Δ and (2) there may be zero-sequence currents in the Δ but there can be none in the line at *T*.

Equivalent Circuits for Transformer Bank of Three Identical Single-Phase Units. Equivalent circuits for two-winding transformer banks to be used in the positive-sequence one-line impedance diagram are shown in Chapter I, Figs. 14(*e*) and (*f*): in Fig. 14(*e*) the magnetizing current is included; in Fig. 14(*f*) it is neglected. Figure 16(*b*) of Chapter I gives the equivalent circuit for a three-winding transformer bank with magnetizing current neglected. These equivalent circuits, developed for the positive-sequence one-line impedance diagram, are independent of the phase order of the balanced currents, and therefore are also the equivalent circuits to be used in the negative-sequence one-line impedance diagram.

The line-to-neutral positive- or negative-sequence leakage impedance of the two-winding bank (or equivalent line-to-neutral impedance when one or both sets of windings are Δ-connected), if expressed in per cent or per unit based on its rating with exciting currents neglected, is the same referred to either side of the bank (see Chapter I). If expressed in ohms, the impedances of the bank referred to the two sides will be proportional to the squares of the equivalent line-to-neutral turns. In problems involving the determination of currents and voltages on both sides of a transformer bank, solutions are simplified if currents, voltages, and impedances are expressed in per cent or per unit on a common kva base with base line-to-neutral voltages on the two sides of the bank directly proportional to the equivalent line-to-neutral turns.

Transformer banks made up of three identical single-phase units offer the same impedances to zero-sequence currents as to positive-sequence currents, provided there is a path for zero-sequence currents. Figure 17(a) shows the path of per unit zero-sequence currents in a Y–Δ transformer bank between P and Q, with the neutral of the Y solidly grounded and magnetizing current neglected. Figure 17(b)

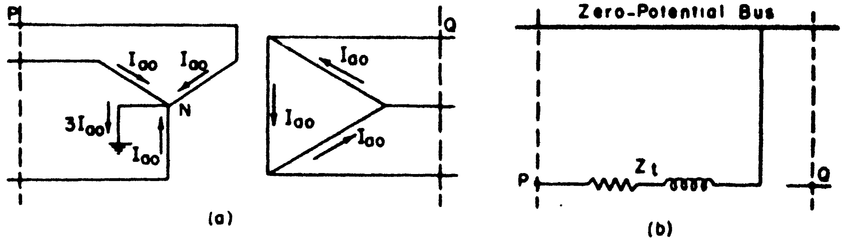


FIG. 17. (a) Path of zero-sequence currents in Y–Δ transformer bank with grounded neutral and negligible exciting current. (b) Zero-sequence equivalent circuit for (a) between P and Q, where Z_t is the per unit leakage impedance of one unit. If the neutral of the Y in (a) is grounded through Z_n, Z_t in (b) is replaced by $Z_t + 3Z_n$.

gives the zero-sequence equivalent circuit. Viewed from Q the impedance is infinite; viewed from P it is Z_t, the transformer leakage impedance. If the neutral of the Y is grounded through Z_n, the impedance Z_t in Fig. 17(b) is replaced by $(Z_t + 3Z_n)$.

Figure 18 gives additional equivalent circuits for transformer banks made up of three identical single-phase units, with exciting currents neglected. The two-winding banks are connected Y–Y, Y–Δ, and Δ–Δ, with the neutrals of the Y's grounded or ungrounded. The three-winding banks are connected Y–Y–Y, Y–Y–Δ, Y–Δ–Δ, and Δ–Δ–Δ, with the Y's grounded or ungrounded. The equivalent series impedance of the two-winding transformer between primary and secondary terminals, indicated by P and S, respectively, is Z_{ps}. The impedances of the three-winding transformer between primary, secondary, and tertiary terminals, indicated by P, S, and T, respectively, taken two at a time with the other winding open, are Z_{ps}, Z_{pt}, and Z_{st}, the subscripts indicating the terminals between which the impedances are measured. (See Chapter I.) It is suggested that the student draw these equivalent circuits for himself before looking at Fig. 18.

Shift in Phase of Positive- and Negative-Sequence Line-to-Neutral Voltages and Line Currents in Passing through a Y-Δ or Δ-Y Transformer Bank of Three Identical Single-Phase Units. The two possible ways of connecting transformers Y–Δ are shown in Figs. 19(a)

and (b). Capital letters refer to the Δ side of the bank and small letters to the Y side; phase A on the Δ side is the phase connected to the transformer phases corresponding to phases b and c on the Y side,

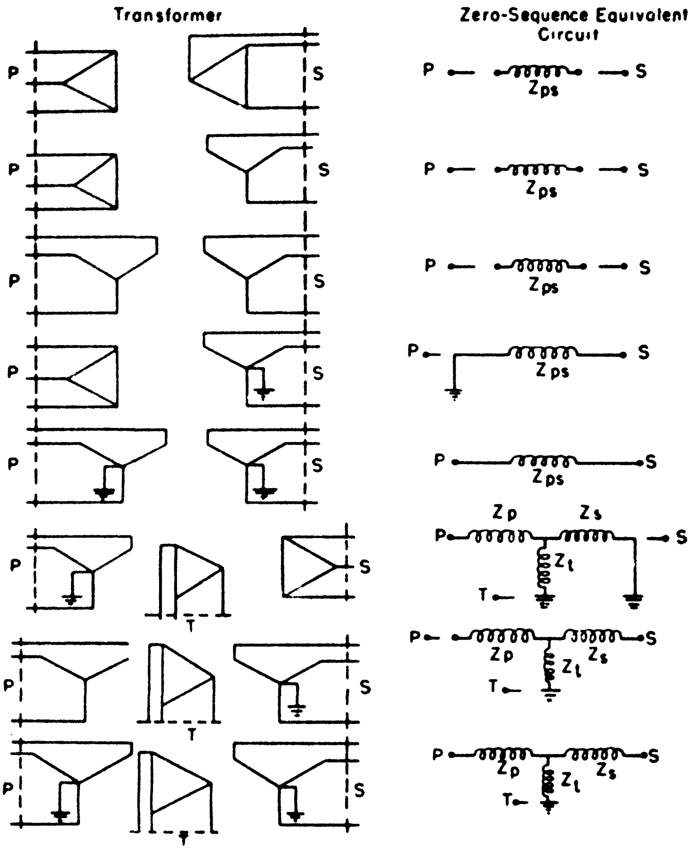


FIG. 18. Zero-sequence equivalent circuits for transformer banks of three identical single-phase units — exciting currents neglected.

$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$; $Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt})$; $Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$

as shown. Figures 19(c) and (d) give the positive-sequence voltage vector diagrams of connection diagrams (a) and (b), respectively, neglecting the voltage drop through the bank. Expressed in per unit, each on its own voltage base, V_{CB1} and V_{a1} in Fig. 19(c) are equal, as they are generated by the same per unit flux; and therefore

$$V_{A1} = jV_{CB1} = jV_{a1} \tag{63}$$

In the positive-sequence per unit voltage vector diagram of Fig. 19(d)

for the connection diagram of Fig. 19(b), $V_{BC1} = V_{a1}$; and

$$V_{A1} = -jV_{BC1} = -jV_{a1} \tag{64}$$

For the connection diagram of Fig. 19(a), the line-to-neutral voltage V_{A1} on the Δ side of the bank leads by 90° the positive-sequence line-to-neutral voltage V_{a1} on the Y side of the bank; but for the connection

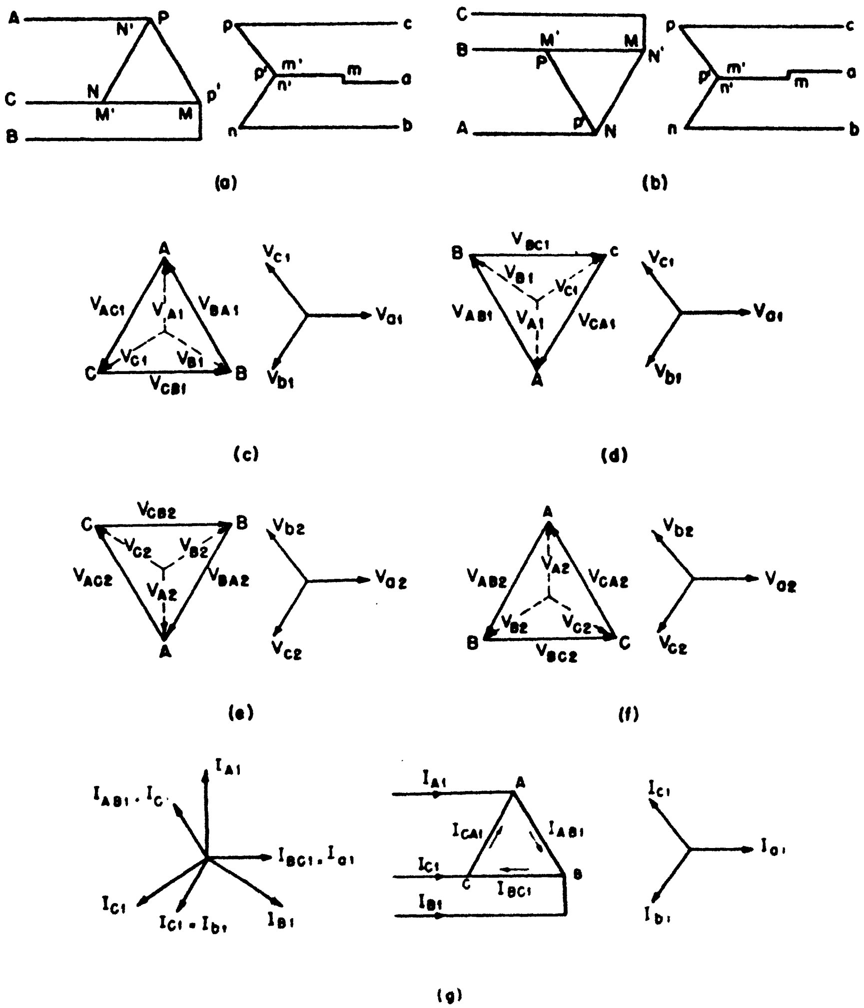


FIG. 19. (a) and (b) Possible connections of Δ-Y transformer banks. (c) and (d) Positive-sequence voltage vector diagrams for connection diagrams (a) and (b), respectively. (e) and (f) Negative-sequence voltage vector diagrams for connection diagrams (a) and (b), respectively. (g) Vector diagrams for positive-sequence currents in connection diagram (a).

of Fig. 19(*b*), V_{A1} lags V_{a1} by 90°. In Fig. 19, comparing (*a*) with (*c*) and (*b*) with (*d*), it may be seen that the positive-sequence voltage vector diagram in either case is the same as the transformer connection diagram; therefore, when the connection diagram is given, the phase relation between V_{A1} and V_{a1} can be determined by inspection.

Figure 19, parts (*e*) and (*f*), give the per unit negative-sequence voltage vector diagrams of connection diagrams (*a*) and (*b*), respectively, neglecting the voltage drop through the transformer bank. Making use of the relation $V_{A2} = jV_{BC2} = -jV_{CB2}$ of [56], in Fig. 19(*e*),

$$V_{A2} = -jV_{CB2} = -jV_{a2} \qquad [65]$$

In Fig. 19(*f*),

$$V_{A2} = jV_{BC2} = jV_{a2} \qquad [66]$$

Comparing [63] and [65], and [66] and [64], the positive-sequence line-to-neutral voltage is shifted 90° either forward or backward, depending upon the connection diagram, while the negative-sequence line-to-neutral voltage is shifted 90° in the direction opposite to the shift in phase of the positive-sequence voltage for the same connection diagram.

The shift in phase of positive- and negative-sequence line currents in passing through a Y–Δ or Δ–Y transformer bank, with transformer exciting currents neglected, must correspond exactly to the shift in phase of line-to-neutral voltages, with the voltage drop through the impedance of the bank neglected. If this were not the case, the kva and power on the two sides of the bank would not be equal with exciting current, resistance, and voltage drop through the bank neglected.

The shift in phase of positive-sequence line currents in passing through the Δ–Y connection given by Fig. 19(*a*) is shown in Fig. 19(*g*). With I_{a1} on the Y side as reference vector, and positive direction of current flow away from the neutral of the Y on the Y side and towards the Δ on the Δ side, the current in the Δ winding marked $M'M$ will flow from B to C, as indicated by the arrow. With exciting current neglected, I_{BC1} is in phase with I_{a1}. Arrows on the Δ side are used in the connection diagram to indicate direction of current flow but *do not indicate phase* referred to I_{a1}. The positive-sequence line current I_{A1} flowing towards the Δ in per unit from [62*b*] is

$$I_{A1} = -jI_{CB1} = jI_{BC1} = jI_{a1}$$

The current vector diagram is given to the left of the connection diagram in Fig. 19(*g*).

For the same connection diagram, Fig. 19(*a*), it can be shown that

$$I_{A2} = -jI_{a2}$$

For the connection diagram of Fig. 19(*b*),

$$I_{A1} = -jI_{a1}$$
$$I_{A2} = jI_{a2}$$

It can be concluded that positive-sequence line-to-neutral voltages and positive-sequence line currents are shifted 90° in phase in the same direction in passing through a Δ-Y or Y-Δ transformer bank; the negative-sequence line-to-neutral voltages and line currents are shifted 90° in the direction opposite to the positive-sequence shift in passing through the same bank. Whether the shift for positive-sequence currents and voltages is 90° forward or 90° backward will depend upon the manner of connecting the windings, the positive-sequence voltage vector diagram being the same as the transformer connection diagram. The direction of the 90° shift is unimportant in determining currents and voltages during faults on systems already in operation, unless the relative phases of currents and voltages on the two sides of the bank are to be compared; it is very important to operating engineers when two circuits, each with a Δ-Y bank, are connected in parallel.

In solving short-circuit problems in which the connection diagram of the Δ-Y bank is not given, it will be assumed that positive-sequence line currents and voltages to neutral are shifted 90°, either forward or backward, and that negative-sequence line currents and voltages to neutral are shifted 90° in the direction opposite to the positive-sequence shift. The only difference in rotating positive-sequence components forward 90° and negative-sequence components backward 90°, instead of rotations in the opposite directions for each, will be a difference in sign for all phase currents and voltages whose components have been rotated. This is illustrated in Problem 6.

Figure 20(*a*) shows a one-line diagram of a system consisting of generator, transformer bank, and transmission line in series, with the distant end of the line open. The connection diagram is given by Fig. 20(*b*). The transformer bank is connected Δ-Y with the Y on the line side grounded through impedance Z_n. The generator is Δ-connected. The positive- and negative-sequence diagrams are given by Figs. 20(*c*) and (*d*), respectively. In these diagrams, transformer exciting current is neglected and the transformer bank is replaced by its equivalent Y-Y bank. Line capacitance is neglected, and the line replaced by its series sequence impedances. Points *P*, *Q*, and *F* in all diagrams correspond to points *P*, *Q*, and *F* in the one-line system diagram. The generator, transformer, and line impedances are connected in series in the positive- and negative-sequence diagrams. In the zero-sequence impedance diagram, Fig. 20(*e*), the point *P* between two Δ-connected windings is represented as a point of open circuit, since its zero-sequence impedance is infinite in both directions. Should zero-sequence voltages be present in the generator, they would cause

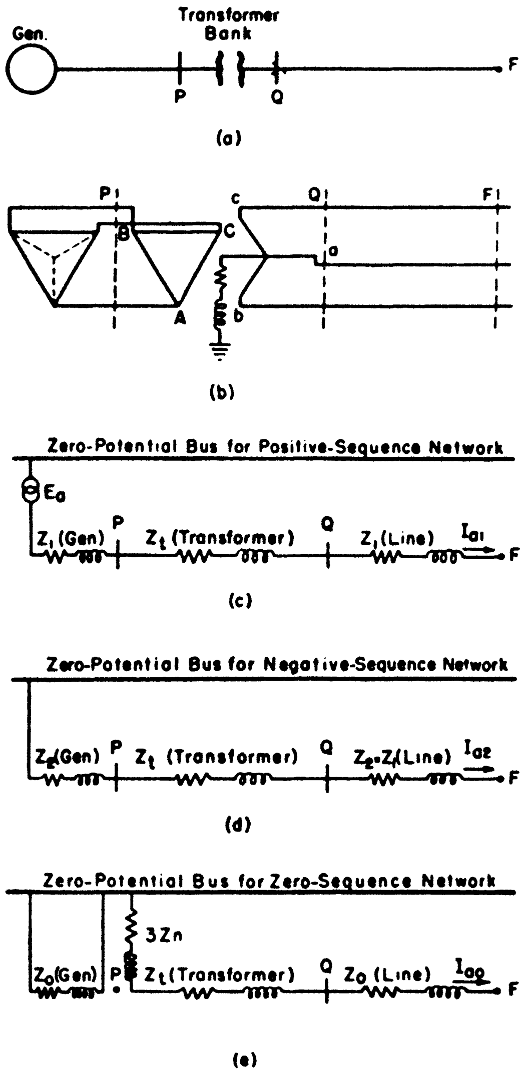


FIG. 20. Diagrams for Problem 6. (*a*) One-line diagram. (*b*) Three-line diagram. (*c*), (*d*), (*e*) Positive-, negative-, and zero-sequence networks, respectively.

circulating currents in the Δ-connected windings, but no zero-sequence currents could flow from the generator into the line and no zero-sequence voltages could exist across the Δ-connected windings. The generator zero-sequence impedance is represented as shorted to the zero-potential bus for the network, indicating that there may be zero-sequence currents in the Δ-connected windings but there will be no resultant zero-sequence voltages across them. For the case under consideration, that of a symmetrical generator with balanced generated voltages, there will be no generated zero-sequence voltages and no zero-sequence currents in the generator.

When the sequence impedance diagrams are expressed in per cent or per unit, the same diagrams can be used to determine fundamental-frequency currents and voltages throughout the system with a fault at any location. In sequence impedance diagrams of power system, impedances (or equivalent impedances) are per phase and therefore base kva in these diagrams is kva per phase and base voltage is line-to-neutral voltage.

Problem 6. With a line-to-ground fault at F in the system shown in Fig. 20(a), determine line-to-ground voltages at F and voltages across the Δ-connected windings of the generator and transformer bank. Draw a three-line diagram showing currents in amperes in the fault, in the transmission line, at the generator terminals, and in the Δ-connected windings of the generator and transformer bank. Neglect resistance, transformer exciting current, and line capacitance.

Data. Three-phase generator rated 30,000 kva, 13.8 kv. Positive- and negative-sequence reactances are 35% and 50%, respectively, based on the generator rating. Step-up transformer bank of three single-phase units, each unit rated 10,000 kva, 13.2 – 66.4/115 Y kv, leakage reactance 10%, neutral of Y grounded through a reactance of 15 ohms. Three-phase transmission line 25 miles in length, with positive- and negative-sequence reactances of 0.8 ohm per mile and zero-sequence reactance of 2.7 ohms per mile. *Operating condition:* The line-to-line voltage in the transmission line was 115 kv before the fault.

Solution. Calculations will be made in per unit on a 30,000 kva base with a base voltage of 115 kv in the transmission circuit.

Base three-phase kva = 30,000 kva (10,000 kva per phase)

Base line-to-line voltage in transmission line = 115 kv line-to-line ($115/\sqrt{3}$ = 66.4 kv line-to-neutral)

$$\text{Base line current in transmission line} = \frac{10{,}000}{66.4} = 151 \text{ amp}$$

Base line-to-line voltage at P (base voltages in Δ circuits) =

$$115 \times \frac{13.2}{115} = 13.2 \text{ kv}$$

$$\text{Base line-to-neutral voltage at } P = \frac{13.2}{\sqrt{3}} = 7.62 \text{ kv}$$

$$\text{Base line current at } P = \frac{10{,}000}{7.62} = 1313 \text{ amp}$$

$$\text{Base current in Δ-connected circuits} = \frac{10{,}000}{13.2} = 758 \text{ amp}$$

At no load, the internal voltage of the generator is the same as its terminal voltage. For the given operating condition, a terminal voltage of 13.2 kv is required to produce 115 kv on the transmission line at no load. For this operating condition the generator terminal voltage is less than its rated voltage. The equivalent line-to-neutral voltage of the generator to be used in the positive-sequence network with the Δ-Y transformer banks replaced by its equivalent Y-Y bank is

$$E_a = \frac{13.2}{\sqrt{3}} \text{ kv} = 7.62 \text{ kv} = 1.0 \text{ in per unit}$$

Note. In the work which follows the numerical values of per unit quantities will be written without the words *per unit.*

Applying [27] and [31] of Chapter I, the system reactances expressed in per unit on the chosen base quantities are

$$x_1 \text{ (line)} = x_2 \text{ (line)} = \frac{25 \times 0.8 \times 30{,}000}{(115)^2 \times 10^3} = 0.045$$

$$x_0 \text{ (line)} = \frac{25 \times 2.7 \times 30{,}000}{(115)^2 \times 10^3} = 0.153$$

$$3x_n = \frac{3 \times 15 \times 30{,}000}{(115)^2 \times 10^3} = 0.102$$

$$x_1 \text{ (generator)} = 0.35 \times \left(\frac{13.8}{13.2}\right)^2 = 0.382$$

$$x_2 \text{ (generator)} = 0.50 \times \left(\frac{13.8}{13.2}\right)^2 = 0.545$$

$$x_t \text{ (transformer)} = 0.10$$

With a fault at F, the terminals of the transmission line, positive- and negative-sequence currents flowing from the generator to the fault meet the impedances of the generator, transformer bank, and line in series. Zero-sequence currents, starting at the grounded reactance of the transformer bank, flow through this reactance, the reactance of the transformer bank, and the reactance of the line in series to the fault and return through the ground. The total positive-, negative-, and zero-sequence impedances are

$$Z_1 = j(0.382 + 0.10 + 0.045) = j0.527$$

$$Z_2 = j(0.545 + 0.10 + 0.045) = j0.690$$

$$Z_0 = j(0.102 + 0.10 + 0.153) = j0.355$$

Applying [30] for a line-to-ground fault at the terminals of an unloaded generator, the per unit components of current of phase a in the transmission line with E_a as reference vector are

$$I_{a1} = I_{a2} = I_{a0} = \frac{E_a}{Z_1 + Z_2 + Z_0} = \frac{1.0}{j1.572} = -j0.636$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = -j1.908$$

$$I_b = I_c = 0$$

$$I_{\text{reactor}} = 3I_{a0} = -j1.908$$

In amperes: $I_a = I_{\text{reactor}} = -j1.908 \times 151 = -j288$ amp.

The line currents are also the currents in the Y-connected transformer windings. When expressed in per unit with transformer exciting currents neglected, they are also the per unit currents in the Δ-connected transformer windings (with directions indicated by arrows in Fig. 21).

$$I_{CB} = I_a = -j1.908$$

$$I_{BA} = I_c = 0$$

$$I_{AC} = I_b = 0$$

In amperes: $I_{CB} = -j1.908 \times 758 = -j1446$ amp; $I_{BA} = I_{AC} = 0$.

There are no zero-sequence components of line current at P. The transformer connection diagram given in Fig. 20(b) corresponds to that of Fig. 19(b), in which the positive- and negative-sequence components of line current and line-to-neutral voltage on the Δ side of the Δ-Y transformer bank are 90° behind and 90° ahead, respectively, of those on the Y side of the bank. The per unit components of line current at P are therefore

$$I_{A0} = 0$$

$$I_{A1} = -jI_{a1} = -j(-j0.636) = -0.636$$

$$I_{A2} = +jI_{a2} = +j(-j0.636) = 0.636$$

The per unit line currents at P are

$$I_A = I_{A1} + I_{A2} = 0$$

$$I_B = a^2I_{A1} + aI_{A2} = j1.103$$

$$I_C = aI_{A1} + a^2I_{A2} = -j1.103$$

In amperes: $I_B = -I_C = j1.103 \times 1313 = j1446$ amp.

The per unit positive- and negative-sequence components of the current I_{BC} in the Δ-connected generator are the positive- and negative-sequence components of the line current I_A flowing from the Δ turned 90° forward and 90° backward, respectively. (See [62a].)

$$I_{BC1} \text{ (generator)} = jI_{A1} = -j0.636$$

$$I_{BC2} \text{ (generator)} = -jI_{A2} = -j0.636$$

From [58],

$$I_{BC} \text{ (generator)} = I_{BC1} + I_{BC2} = -j1.272$$

$$I_{CA} \text{ (generator)} = a^2I_{BC1} + aI_{BC2} = j0.636$$

$$I_{AB} \text{ (generator)} = aI_{BC1} + a^2I_{BC2} = j0.636$$

In amperes: $I_{BC} = -j1.272 \times 758 = -j964$ amp; $I_{CA} = I_{AB} = j482$ amp.

Figure 21 is a three-line current diagram for Problem 6 showing the currents in amperes with E_a as reference vector. Arrows in Fig. 21 show directions of currents which correspond to their calculated values. The direction of any calculated current may be reversed by reversing the assumed direction of current flow and changing the phase of the calculated current by 180° (i.e., multiplying it by −1). If the directions of I_{CA} and I_{AB} in the generator and I_B in the line calculated above had been reversed, their calculated values would have been multiplied by −1.

From [8]–[10], the components of V_a at the fault are

$$V_{a1} = E_a - I_{a1}Z_1 = 1 - (-j0.636)(j0.527) = 0.664$$
$$V_{a2} = -I_{a2}Z_2 = -(-j0.636)(j0.690) = -0.438$$
$$V_{a0} = -I_{a0}Z_0 = -(-j0.636)(j0.355) = -0.226$$

The line-to-ground voltages at the fault from [7], [25], and [26] of Chapter II are

$$V_a = 0.664 - 0.436 - 0.226 = 0$$
$$V_b = -0.226 - \tfrac{1}{2}(0.226) + j\frac{\sqrt{3}}{2}(1.102) = -0.339 + j0.955 = 1.02\angle 109.5^\circ$$
$$V_c = -0.339 - j0.955 = 1.02\angle\overline{70.5^\circ}$$

In kilovolts: $V_a = 0$; $V_b = 1.02\angle 109.5^\circ \times 66.4 = 67.7\angle 109.5^\circ$ kv; $V_c = 67.7\angle\overline{70.5^\circ}$ kv.

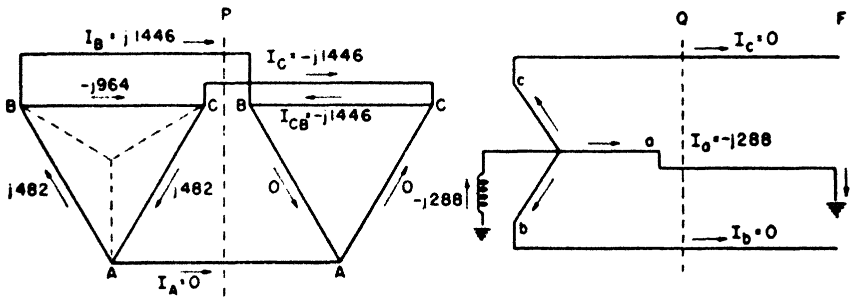


FIG. 21. Three-line diagram showing currents in amperes for Problem 6.

There are no zero-sequence voltages at P. The positive- and negative- sequence components of the voltage to ground of phase a at P, calculated on an equivalent Y–Y basis, can be obtained from the positive- and negative-sequence networks. Let V'_{A1} and V'_{A2} represent these components. Then at the generator terminals from [8] and [9],

$$V'_{A1} = E_{a1} - I_{a1}Z_1 = 1 - (-j0.636)(j0.382) = 0.757$$
$$V'_{A2} = -I_{a2}Z_2 = -(-j0.636)(j0.545) = -0.346$$

For the connection diagram of Fig. 20(b), positive- and negative-sequence components of line-to-neutral voltages on the Δ side of a Δ–Y bank are 90° behind and 90° ahead, respectively, of their components calculated on an equivalent Y–Y basis. Therefore at P,

$$V_{A0} = 0$$
$$V_{A1} = -jV'_{A1} = -j0.757$$
$$V_{A2} = jV'_{A2} = -j0.346$$

From [7], [25], and [26], of Chapter II,

$$V_A = -j1.103 = 1.103\angle\overline{90^\circ}$$
$$V_B = -0.356 + j0.552 = 0.657\angle 122.8$$
$$V_C = 0.356 + j0.552 = 0.657\angle 32.8$$

In kv: $|V_A| = 1.103 \times 7.62 = 8.41$ kv; $|V_B| = |V_C| = 5.04$ kv.

The voltages across the deltas of the generator and transformer bank are the line-to-line voltages at P which can be determined in per unit of *line-to-neutral voltage* by substituting the above values of V_A, V_B, and V_C in [11]. An alternate solution will be given in terms of per unit line-to-line symmetrical components. From [55],

$$V_{BC0} = 0$$

$$V_{BC1} = jV_{A1} = 0.757$$

$$V_{BC2} = -jV_{A2} = -0.346$$

From [54],

$$V_{BC} = 0.411$$

$$V_{CA} = -0.205 - j0.956 = 0.98\overline{/102.1^\circ}$$

$$V_{AB} = -0.205 + j0.956 = 0.98/102.1^\circ$$

In kilovolts: $|V_{BC}| = 0.411 \times 13.2 = 5.42$ kv; $|V_{CA}| = V_{AB} = 12.9$ kv.

If the Δ-Y transformer bank had been connected as in Fig. 19(a), or if the connection diagram of the Δ-Y bank had not been given and the connection of Fig. 19(a) had been assumed, the positive- and negative-sequence voltages at P would be determined from V'_{A1} and V'_{A2}, the positive- and negative-sequence voltages at A calculated on the equivalent Y-Y basis, by the equations

$$V_{A1} = jV'_{A1} = j0.757$$

$$V_{A2} = -jV'_{A2} = j0.346$$

The phase voltages at A would then become

$$V_A = j1.103$$

$$V_B = 0.356 - j0.552$$

$$V_C = -0.356 - j0.552$$

These voltages differ from those calculated for the given connection diagram by 180°. This is true also for the line currents and the currents and voltages in the Δ-connected windings.

If the line-to-ground fault is at point Q, the procedure is similar to that with a fault at F except that the transmission line impedances are not included in calculating the total sequence impedances between the generator and fault. If the fault is at P, the zero-sequence impedance is infinite and no current will flow into the fault. The phase voltages at P are the same as those of the ungrounded Y-connected generators with a line-to-ground fault at its terminals.

Problem 7. Solve Problem 2 for a double line-to-ground fault: (a) generator Y-connected with neutral solidly grounded; (b) generator Δ-connected.

Problem 8. With phase a as reference phase, derive equations relating the symmetrical components of I_a flowing into the fault and the line-to-ground voltage V_a at the fault for: (1) a line-to-ground fault on phase b; (2) a line-to-line fault between phases a and c; (3) a double line-to-ground fault on phases a and b. (For check see Table I, Chapter VII.)

Problem 9. A generator having a solidly grounded neutral and rated 10,000 kva, 13.8 kv has positive-, negative-, and zero-sequence reactances of 30, 40, and 5%, respectively. (*a*) What reactance must be placed in the generator neutral so that the fault current for a line-to-ground fault of zero fault impedance will not exceed rated line current? (*b*) What value of resistance in the neutral will serve the same purpose? Express resistance and reactance values in per unit and also in ohms.

Problem 10. (*a*) What reactance must be placed in the neutral of the generator of Problem 9 to reduce the ground current to rated line current for a double line-to-ground fault? (*b*) What will be the magnitudes of the line currents when the ground current is so reduced? (*c*) As the reactance in the neutral is indefinitely increased, what are the limiting minimum values of the line currents?

Problem 11. Solve Problem 4 for a line-to-ground fault, assuming the generator Δ-connected.

Problem 12. Solve Problem 6, Fig. 20, for (1) a line-to-line fault at F, (2) a line-to-ground fault at Q, (3) a double line-to-ground fault at P, using the transformer connection diagram of Fig. 19(*a*).

CHAPTER IV

UNSYMMETRICAL FAULTS ON NORMALLY BALANCED THREE-PHASE SYSTEMS

In a large three-phase power transmission system there may be circuits which are unsymmetrical, and therefore do not carry balanced currents even under normal operation. However, transmission systems as a whole are approximately symmetrical, and the main transmission circuits under normal operation have substantially balanced voltages and currents. In this chapter, the circuits discussed are assumed symmetrical; therefore, under normal operating conditions, the currents and voltages are balanced. In cases where an unsymmetrical circuit appreciably affects the operation of the rest of the system, or when the performance of the circuit itself is under consideration, it becomes necessary to investigate such circuits. Chapter VIII is devoted to the discussion of unsymmetrical three-phase circuits.

Two types of unsymmetrical faults on normally balanced power systems will be considered: short circuits and open conductors.

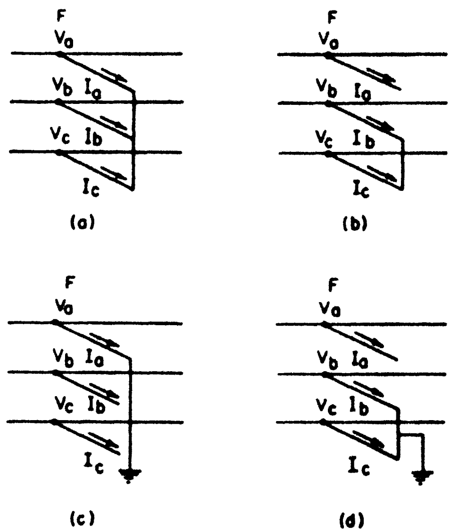


FIG. 1. Currents flowing into the fault at any point F of a three-phase system. (*a*) Three-phase fault. (*b*) Line-to-line fault. (*c*) Single line-to-ground fault. (*d*) Double line-to-ground fault.

SHORT CIRCUITS

I_a, I_b, and I_c, which were used to represent the line currents in Chapter III, will now be used to indicate the currents flowing into the fault from the three phases a, b, and c, respectively; V_a, V_b, and V_c as before will represent the voltages to ground at the point of fault F. In Fig. 1 the fault currents are represented as flowing in hypothetical stub connections from the conductor, so that they will not be confused with line currents. I_{a1}, I_{a2}, and I_{a0} are the positive-, negative-, and zero-sequence components, respectively, of the fault current I_a; and V_{a1}, V_{a2}, and V_{a0} are the corresponding components of V_a, the voltage to ground of phase a at F. Positive direction of current flow for fault currents and their sequence components will be taken from the conductors into the fault.

If the conditions at the fault for the four types of short circuits shown in Fig. 1 are expressed in equations, it will be seen that the equations are the same as those for faults at the terminals of an unloaded generator given in Chapter III. The relations between the symmetrical components of current and of voltage will therefore be the same, the only difference being that the symmetrical components of current are those for the fault current I_a instead of the line current I_a. The symmetrical components of voltage are, as before, components of the line-to-ground voltage of phase a at the point of fault.

Six equations are needed to determine the six unknown symmetrical components, I_{a1}, I_{a2}, I_{a0}, V_{a1}, V_{a2}, and V_{a0}. The conditions at the fault provide three equations involving phase currents and voltages from which three equations can be obtained connecting symmetrical components of current or of voltage of phase a at the point of fault. The other three equations should express the positive-, negative-, and zero-sequence components of voltage, respectively, of the reference phase a at the fault in terms of the corresponding sequence current flowing into the fault and the impedance associated with it. For a system with one unloaded symmetrical generator these three equations are given by [8]–[10] of Chapter III. For the general case, these three equations can be obtained from the three sequence networks, each considered separately. Figure 2(a) gives a one-line diagram of a symmetrical power system; the sequence networks of this system are given in parts (b), (c), and (d) of Fig. 2.

Positive-Sequence Network. In a large three-phase power system, there are usually several generating stations connected by transformers and transmission line, with load tapped off at various points. Under load, the relative angular positions of the internal voltages of the synchronous generators and motors of the system in the positive-sequence network will depend upon operating conditions. In Fig. 2(b), the internal voltages E_g, E_m, and E_n can be calculated when operating conditions are known. At no load, neglecting charging currents, all voltages, when expressed in per unit of their respective base voltages, will be equal and in phase. This statement is based upon the use of equivalent Y–Y transformer banks to replace Δ–Y or Y–Δ connected banks. The shift in phase because of Δ–Y or Y–Δ transformer banks can be taken into account when required. (See Chapter III.)

Before the occurrence of a fault, there is no positive-sequence fault current and the voltages at the point of fault F are balanced positive-

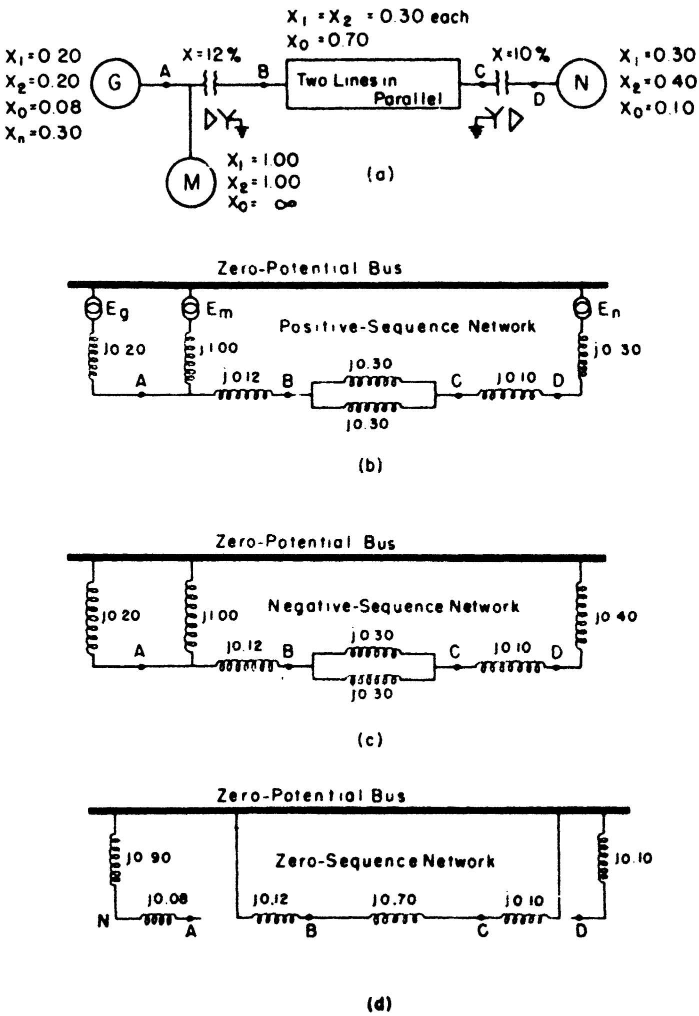


FIG. 2. (a) One-line diagram of a symmetrical three-phase power system. (b), (c), and (d) Positive-, negative-, and zero-sequence networks for system in (a).

sequence voltages. Let the voltage of phase a at F before the fault be V_f. I_{a1} is the positive-sequence current flowing from phase a into the fault and V_{a1} is the positive-sequence voltage of phase a at the fault.

The effect of a fault of any type on the positive-sequence network is to change the positive-sequence fault current from 0 to I_{a1} and the positive-sequence voltage at the point of fault F from V_f to V_{a1}. The change in voltage, or the voltage required to change the fault voltage from V_f to V_{a1}, is $-(V_f - V_{a1})$. By the principle of superposition, the rms initial symmetrical positive-sequence current at any point in the system can be determined by adding to the rms load current at that point the current resulting from the voltage $-(V_f - V_{a1})$ applied at F, with all other applied voltages equated to zero. If the voltage $-(V_f - V_{a1})$ is applied at point F in the positive-sequence network, with all generated voltages reduced to zero, the positive-sequence current ΔI_f flowing *into the network* from F, given by [52], Chapter I, is

$$\Delta I_f = \frac{-(V_f - V_{a1})}{A_{ff}} = \frac{-(V_f - V_{a1})}{Z_1}$$

where $Z_1 = A_{ff}$ = driving-point impedance at the fault in the positive-sequence network or the positive-sequence impedance of the system viewed from the fault.

I_{a1} has been defined as the positive-sequence current flowing into the fault. I_{a1} is therefore equal in magnitude and opposite in sign to ΔI_f. Replacing ΔI_f by $-I_{a1}$ in the above equation and solving for V_{a1},

$$V_{a1} = V_f - I_{a1}Z_1 \qquad [1]$$

An alternate method of obtaining [1] is by the superposition of voltages instead of currents. I_{a1} flowing from the zero-potential bus of the positive-sequence network through the network to the fault meets Z_1, the positive-sequence impedance viewed from the fault. The voltage drop produced is $I_{a1}Z_1$; the voltage rise is $-I_{a1}Z_1$. Adding $-I_{a1}Z_1$ or subtracting $I_{a1}Z_1$ from V_f, [1] is obtained.

V_f and Z_1 are known, or can be determined. V_{a1} and I_{a1} are unknown; but, regardless of their values, [1] gives the relation existing between them.

Negative-Sequence Network. When there are no negative-sequence voltages generated in a system, and none induced from outside sources, the only negative-sequence voltages associated with the network will be those resulting from the flow of negative-sequence currents. The negative-sequence voltage of phase a at the fault is zero before the occurrence of the fault and V_{a2} after the fault occurs; the *change in voltage* resulting from the fault will be V_{a2}. The negative-sequence current entering the negative-sequence network because of the fault may be determined by applying V_{a2} to the negative-sequence network between the fault point and the zero-potential bus, with no other voltages applied. The applied voltage V_{a2} will send current *into the network*; I_{a2} is defined as the negative-sequence current flowing *from the network into the fault.* If Z_2 is the negative-sequence impedance of the system viewed from the fault, or the driving-point impedance at the fault in the negative-sequence network,

$$-I_{a2} = \frac{V_{a2}}{Z_2}, \quad \text{or} \quad V_{a2} = -I_{a2}Z_2$$

The relation between V_{a2} and I_{a2} can also be obtained by adding the voltage rise, caused by I_{a2} flowing from the zero-potential bus of the negative-sequence network to the fault, to the negative-sequence voltage at the fault point before the fault occurred. The voltage rise is $-I_{a2}Z_2$, negative-sequence prefault voltage was zero; therefore

$$V_{a2} = 0 - I_{a2}Z_2 = -I_{a2}Z_2 \qquad [2]$$

In [2], Z_2 is known or can be determined; V_{a2} and I_{a2} are unknown, but [2] expresses the relation between them.

Zero-Sequence Network. Assuming no generated and no induced zero-sequence voltages, the relation between the fault current and voltage in the zero-sequence network is similar to that in the negative-sequence network. If V_{a0} is the zero-sequence voltage of phase a at the fault, I_{a0} the zero-sequence current from phase a flowing into the fault, and Z_0 the zero-sequence impedance of the system viewed from the fault, then

$$V_{a0} = -I_{a0}Z_0 \qquad [3]$$

In [3], Z_0 is known or can be determined; I_{a0} and V_{a0} are unknown, but [3] expresses the relation between them.

Solution of Simultaneous Symmetrical Component Equations. Equations [1], [2], and [3], together with the three equations expressing relations between the symmetrical components of current or of voltage of phase a at the fault, provide the six simultaneous equations needed for determining the six symmetrical components I_{a1}, I_{a2}, I_{a0}, V_{a1}, V_{a2}, and V_{a0}.

A simple way of solving these six equations is to select I_{a1} and V_{a1} as the unknowns which are first to be determined. This selection is made because in a normally balanced system the positive-sequence network is the only network in which there are generated voltages. To determine I_{a1} and V_{a1}, two equations are required. One of these equations is given by [1]. The other is obtained by eliminating the four unknowns V_{a2}, V_{a0}, I_{a2}, and I_{a0} from the five equations consisting of [2], [3], and the three fault equations. The remaining equation in terms of I_{a1}, V_{a1}, Z_2, and Z_0 can be written

$$V_{a1} = I_{a1}K \qquad [4]$$

where K is a function of Z_2 and Z_0, depending upon the type of fault.

From [1] and [4], V_{a1} is eliminated and I_{a1} obtained. Knowing I_{a1}, V_{a1} and the other components are readily obtained.

System Replaced by Equivalent Generator for Determining Initial Symmetrical Rms Fault Currents and Voltages. If [1], [2], and [3] are compared with [8], [9], and [10] of Chapter III, respectively, it will be noted that the relations between the symmetrical components of the fault voltage and current of phase a for each of the three sequences are the same for the system as for the unloaded generator. E_a in [8] is equal to V_f, the voltage of phase a at the fault before the fault occurred, and Z_1, Z_2, and Z_0 in [8], [9], and [10], respectively, are the positive-, negative-, and zero-sequence impedances viewed from the fault. Since the equations connecting the current and voltage at the point of fault in each of the three sequence networks is the same for a system as for an unloaded generator, the relations developed in Chapter III for a generator may be extended by analogy to a system. Thus, to determine the *initial symmetrical rms fault currents and voltages*, the system may be treated as an unloaded generator with internal voltage equal to V_f, the prefault voltage at the point of fault, and sequence impedances equal to those of the system viewed from the fault.

To determine the current and voltage distribution in the system, the distribution in each of the sequence networks must first be determined. The fault currents I_{a1}, I_{a2}, and I_{a0}, flowing from the zero-potential bus to the fault in their respective networks, divide among the various parallel paths inversely in proportion to the impedances of these paths. In the negative- or zero-sequence networks, the voltage rise from the zero-potential bus to any system point will give the negative- or zero-sequence voltage, respectively, at that point.

In the positive network the currents throughout the system *due to the fault* can be added to the load currents before the fault to give total positive-sequence currents. The positive-sequence voltage at any point P in the system may be determined by adding to the positive-sequence voltage at the fault the voltage drop between P and the fault caused by the *total* positive-sequence current; or by superposing upon the positive-sequence voltage which existed at P before the fault the voltage rise resulting from the flow of the positive-sequence *fault* current. The line currents in the three phases and the phase voltages to ground at any point in the system are obtained by substituting the symmetrical components of I_a and V_a, respectively, at that point in [19]–[21] and [7]–[9] of Chapter II.

Determination of Currents and Voltages Following Initial Values. When a power system is operating under normal conditions, neglecting fluctuations in load, the rotors of the various synchronous machines on the system have fixed angular positions with respect to each other, which depends upon the load and the excitations of the machines. In the positive-sequence network, the angular displacement of the rotors is represented as angular displacements between the internal voltages of the machines, and the magnitudes of the voltage vectors are determined by the excitations. Under normal operation there exists a condition of equilibrium, the electrical output of each machine being equal to its mechanical input less losses. This applies to a synchronous motor or condenser as well as to a generator, if the electrical output and mechanical input are both considered negative. When a disturbance occurs, such as a short circuit, the electrical outputs of some, if not all, of the machines of the system change instantly, the change depending upon the severity of the disturbance. For example, a three-phase fault at the terminals of a machine reduces its electrical output immediately to resistance losses. The mechanical input, controlled by governors, does not change immediately. A machine will attempt to speed up if its mechanical input exceeds its electrical output, and to slow down if the reverse is true. Its acceleration depends upon its inertia as well as upon the difference between mechanical input and electrical output. If the disturbance is not severe enough to cause the machines to lose synchronism, the rotors will eventually take up new fixed relative angular positions, and a condition of equilibrium will again exist. If the disturbance is a fault, followed by the switching out of service of a circuit, and it is required to determine the currents and voltages during the disturbance, transient reactances are used in the positive sequence network and generated voltages behind transient reactances. If it is known that the disturbance is such that the relative angular positions of the rotors change but slightly, as an approximation, the procedure given above for determining initial symmetrical rms currents and voltages can be used to determine rms transient currents and voltages of fundamental frequency, the only difference being the use of transient reactance viewed from the fault in [1] instead of subtransient reactance. Equations [2] and [3] for the negative- and zero-sequence networks, respectively, apply for subtransient, transient, or steady-state conditions.

When the change in relative angular positions of the rotors must be considered, as in the case of transient stability studies, [1], which gives the relation between V_{a1} and I_{a1} as a function of V_f, can be used if V_f is redefined as the voltage which would exist at the fault point with the fault removed and with the generated voltages of the machines given the angular positions which they have at the instant under consideration. Calculation of the relative angular positions of the generated voltages in the positive-sequence network following a fault is simplified if the fault is replaced by an equivalent circuit in the positive-sequence network.

EQUIVALENT CIRCUIT TO REPLACE FAULT IN POSITIVE-SEQUENCE NETWORK — CONNECTIONS OF SEQUENCE NETWORKS TO REPRESENT FAULT CONDITIONS

Line-to-Line Fault. Figure 1(*b*) shows a line-to-line fault between phases *b* and *c* at some point *F* of a previously balanced three-phase system. I_a, I_b, I_c, V_a, V_b, and V_c represent the three fault currents and the three voltages to ground of phases *a*, *b*, and *c*, respectively, at the point of fault *F*. The fault conditions are

$$I_a = 0 \qquad I_b = -I_c \qquad V_b = V_c$$

These conditions are the same as those for a line-to-line fault on an unloaded generator (Chapter III). Therefore, following the same procedure,

$$I_{a0} = 0,$$

and

$$I_{a1} = -I_{a2} \tag{5}$$

$$V_{a1} = V_{a2} \tag{6}$$

For a line-to-line fault through zero fault impedance, the positive- and negative-sequence components of fault current in the unfaulted phase are equal in magnitude but opposite in phase, while the positive- and negative-sequence components of voltage at the fault are equal both in magnitude and phase.

Replacing I_{a2} in [2] by $-I_{a1}$ from [5] and substituting [2] in [6],

$$V_{a1} = V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 \tag{7}$$

Solving [1] and [7] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2} \tag{8}$$

Equivalent Circuit to Replace the Line-to-Line Fault in the Positive-Sequence Network. Since $V_{a1} = I_{a1}Z_2$ in [7], it is evident that V_{a1} and I_{a1} will be correctly determined if the impedance Z_2 is inserted between the fault point and the zero-potential bus in the positive-sequence network. The equivalent circuit to replace a line-to-line fault at F in the positive-sequence network is therefore Z_2, the impedance of the negative-sequence network viewed from the fault, as shown in Fig. 3.

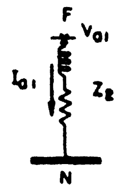


FIG. 3. Equivalent circuit to replace a line-to-line fault in the positive-sequence network.

An equivalent circuit of a single lumped impedance to replace the fault in the positive-sequence network is particularly useful in analytical calculations. When either an a-c or d-c calculating table is used to determine current and voltage distribution, the equivalent circuit consisting of the complete negative-sequence network may be used to advantage; then the current and voltage distribution for the negative- as well as for the positive-sequence network will be obtained. The currents and voltages so determined will be those for the reference phase a.

Connection of Sequence Networks for Solution of Line-to-Line Faults on an A-C Calculating Table. Assume a line-to-line fault to occur at B in Fig. 2(a). Since the positive- and negative-sequence components of voltage at the point of fault are equal from [6], the positive- and negative-sequence networks should be connected with their fault points together and also their zero-potential buses, as in Fig. 4. This

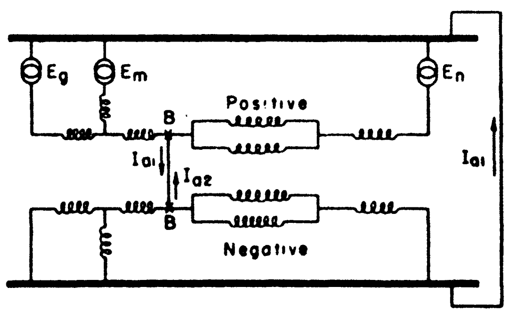


FIG. 4. Connection of positive- and negative-sequence networks of Fig. 2 for a line-to-line fault at B.

connection will also satisfy the condition of [5] that $I_{a1} = -I_{a2}$ in the fault, as may be seen by an inspection of Fig. 4. The current I_{a1} flows from the positive-sequence zero-potential bus through the network into the fault, then (as $-I_{a2}$) through the negative-sequence network from the fault to the zero-potential bus of the negative-sequence network, and finally back to the zero-potential bus for the positive-sequence network. The voltages in each sequence network are measured from the zero-potential bus for the network to the point under consideration.

Problem 1. The generator and motor described in Problem 2, Chapter I, Fig 18(*a*), have negative-sequence reactances of 40 and 210%, respectively, on the chosen kva base. The operating conditions previous to the fault are the same as those given in Problem 2, Chapter I. A line-to-line fault occurs on the bus T between phases b and c.

(*a*) Determine the initial symmetrical rms currents flowing from the three phases into the fault, the three line currents in the generator and in the motor, and the line-to-ground voltages at the point of fault.

(*b*) Draw a three-line diagram showing line and fault currents.

(*c*) The positive-sequence diagram is given in Chapter I, Fig. 18(*b*). Draw the negative-sequence diagram and indicate how the two sequence networks should be connected for use on an a-c calculating table to determine the sequence currents and voltages for the reference phase a.

Solution (*a*) From the given conditions,

$$V_f = V_t = 0.98 + j0$$

$$Z_1 = \frac{j0.40 \times j2.00}{j2.40} = j0.333$$

$$Z_2 = \frac{j0.40 \times j2.10}{j2.50} = j0.336$$

From [5] and [8],

$$I_{a0} = 0$$

$$I_{a1} = \frac{V_f}{Z_1 + Z_2} = \frac{0.98 + j0}{j0.669} = -j1.465$$

$$I_{a2} = -I_{a1} = j1.465$$

The three fault currents are

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_b = (I_{a0} + a^2 I_{a1} + a I_{a2}) = (a^2 - a)I_{a1} = -j\sqrt{3}I_{a1} = -2.537 + j0$$

$$I_c = -I_b = 2.537 + j0$$

The positive- and negative-sequence currents due to the fault will flow from generator and motor inversely in proportion to their respective impedances; therefore,

$$I_{a1} \text{ (from generator)} = -j1.465 \times \frac{2.00}{2.40} = -j1.221$$

$$I_{a1} \text{ (from motor)} = -j1.465 \times \frac{0.40}{2.40} = -j0.244$$

$$I_{a2} \text{ (from generator)} = j1.465 \times \frac{2.10}{2.50} = j1.231$$

$$I_{a2} \text{ (from motor)} = j1.465 \times \frac{0.40}{2.50} = j0.234$$

Adding the load currents in generator and motor to the positive-sequence currents due to the fault, initial symmetrical rms positive-sequence currents are obtained:

$$\text{Total } I_{a1} \text{ (from generator)} = 0.5 - j1.221$$

$$\text{Total } I_{a1} \text{ (from motor)} = -0.5 - j0.244$$

Combining the sequence currents in generator and motor by [19]–[21] of Chapter II, the following currents are obtained. From the generator:

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 + 0.5 - j1.221 + j1.231 = 0.5 + j0.010$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

$$= 0 + (-0.5 - j0.866)(0.5 - j1.221) + (-0.5 + j0.866)(j1.231)$$

$$= -2.373 - j0.438$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} = 1.873 + j0.428$$

From the motor:

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 - 0.5 - j0.244 + j0.234 = -0.5 - j0.010$$

$$I_b = a^2 I_{a1} + a I_{a2} = -0.164 + j0.438$$

$$I_c = a I_{a1} + a^2 I_{a2} = 0.664 - j0.428$$

In a symmetrical system with $I_{a0} = 0$, $V_{a0} = 0$. From [7],

$$V_{a1} = V_{a2} = I_{a1} Z_2 = (-j1.465)(j0.33\dot{6}) = 0.482 + j0$$

Combining the sequence voltages, the line-to-ground voltages at T are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0 + 0.482 + j0 + 0.482 + j0 = 0.964 + j0$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = (a^2 + a) V_{a1} = -V_{a1} = -0.482 + j0$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = (a + a^2) V_{a1} = -V_{a1} = -0.482 + j0$$

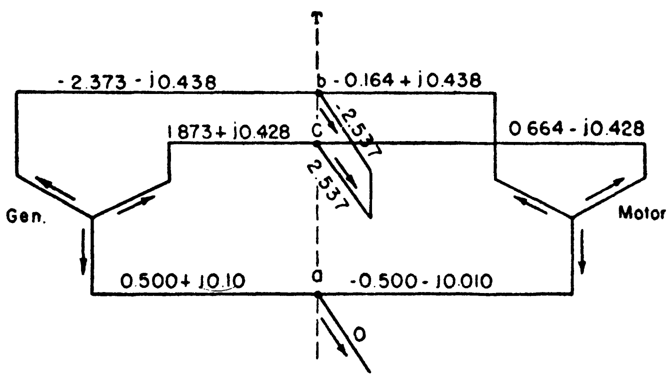


FIG. 5(*a*). Line currents for Problem 1. Line-to-line fault at T.

(*b*) A three-line diagram of the system showing the line currents is given in Fig. 5(*a*). Positive direction of line currents is taken from the machine neutrals towards the fault and is indicated by arrows. The voltage of phase *a* at T before the fault is reference vector.

(*c*) The connection of the sequence networks for solution on the a-c network analyzer is shown in Fig. 5(*b*). Since the fault points T of both networks are connected, at the fault $V_{a1} = V_{a2}$ and $I_{a1} = -I_{a2}$, positive direction of current flow being from the system into the fault in both networks. The internal voltages of the generator and motor in the positive-sequence network correspond to the operating condition before the fault given in Problem 2, Chapter I, with the voltage at T before the fault as reference vector.

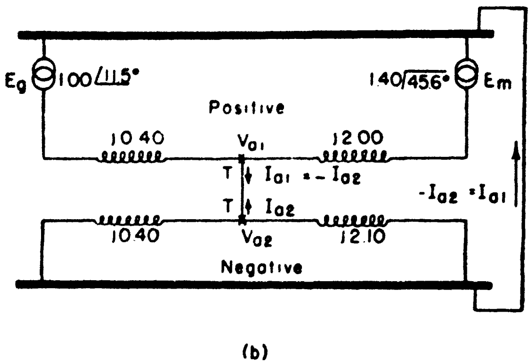


FIG. 5(*b*). Connection of the sequence networks for Problem 1.

Line-to-Ground Fault. Figure 1(*c*) shows a line-to-ground fault through zero impedance on phase a at some point F in a grounded system. I_a, I_b, and I_c represent fault currents, and V_a, V_b, and V_c voltages to ground. The fault conditions are

$$I_b = 0 \qquad I_c = 0 \qquad V_a = 0$$

These conditions are the same as those for a line-to-ground fault on an unloaded generator, given in Chapter III. Therefore, by the same procedure,

$$I_{a0} = I_{a1} = I_{a2} \tag{9}$$

$$V_{a1} = -(V_{a0} + V_{a2}) \tag{10}$$

For a line-to-ground fault of zero fault impedance on a grounded system, the zero-, positive-, and negative-sequence components of fault current in the faulted phase are equal in magnitude and phase, and the positive-sequence component of voltage to ground at the fault is equal in magnitude and opposite in phase to the sum of the zero- and negative-sequence components.

Substituting V_{a2} and V_{a0} from [2] and [3] in [10], then replacing I_{a2} and I_{a0} by I_{a1} from [9],

$$V_{a1} = -(V_{a0} + V_{a2}) = I_{a1}(Z_0 + Z_2) \tag{11}$$

Solving [1] and [11] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_0} \tag{12}$$

Equivalent Circuit to Replace a Line-to-Ground Fault in the Positive-Sequence Network. From [11], $V_{a1} = I_{a1}(Z_0 + Z_2)$. It follows, therefore, that the equivalent circuit to replace the fault in the positive-sequence network is the sum of Z_0 and Z_2, this impedance to be inserted between the point of fault and the zero-potential bus for the network, as indicated in Fig. 6. Instead of inserting a lumped impedance equal to $Z_0 + Z_2$, the negative- and zero-sequence networks can be connected in series between the point of fault and the zero-potential bus.

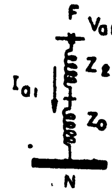


FIG. 6. Equivalent circuit to replace a single line-to-ground fault in the positive-sequence network.

Connections of Sequence Networks for Solution of a Line-to-Ground Fault on an A-C Calculating Table. Equations [9] and [10] determine the manner in which the networks must be connected to satisfy the conditions for the signs of the currents and voltages. For the system shown in Fig. 2(*a*) with a line-to-ground fault at *B*, the connection of the sequence networks is given in Fig. 7. This connection satisfies the condition that the fault currents in the three sequence networks are equal, since the same current flows into the fault from

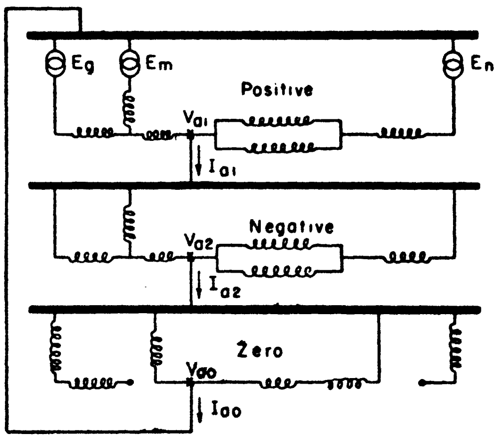


FIG. 7. Connection of sequence networks of Fig. 2 for line-to-ground fault at *B*.

all three networks. The voltage condition that $V_{a1} = -(V_{a0} + V_{a2})$ is also satisfied. The current and voltage distribution in the three sequence networks is that of the reference phase *a*.

Double Line-to-Ground Fault. Figure 1(*d*) shows a double line-to-ground fault through zero impedance on phases *b* and *c* at some point *F* in a grounded system. The fault conditions are

$$I_a = 0 \qquad V_b = 0 \qquad V_c = 0$$

These conditions are the same as those given in Chapter III; therefore, by the same procedure,

$$V_{a0} = V_{a1} = V_{a2} \tag{13}$$

$$I_{a1} = -(I_{a0} + I_{a2}) \tag{14}$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} \tag{15}$$

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} \tag{16}$$

For a double line-to-ground fault of zero fault impedance on a grounded system, the positive-, negative-, and zero-sequence components of voltage to ground of the unfaulted phase are equal in magnitude and phase, while the positive-sequence component of current in the fault is equal in magnitude and opposite in phase to the sum of the zero- and negative-sequence components, these components varying inversely as their sequence impedances viewed from the fault.

Substituting [16] in [2] and [2] in [13],

$$V_{a0} = V_{a1} = V_{a2} = I_{a1}\frac{Z_0Z_2}{Z_0 + Z_2} \tag{17}$$

Solving [1] and [17] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_0Z_2}{Z_0 + Z_2}} \tag{18}$$

Equivalent Circuit to Replace the Double Line-to-Ground Fault in the Positive-Sequence Network. From [17], $V_{a1} = I_{a1}\dfrac{Z_0Z_2}{Z_0 + Z_2}$. The equivalent circuit to replace the fault in the positive-sequence network is, therefore, the parallel value of the impedances Z_0 and Z_2 as shown in Fig. 8. If this value of impedance is inserted in the positive-sequence network between the point of fault and the zero-potential bus for the network, the positive-sequence current and voltage distribution will be correctly determined. Instead of inserting the impedance $Z_0Z_2/(Z_0 + Z_2)$, the same results will be obtained if the negative- and zero-sequence networks are con-

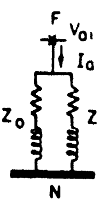


FIG. 8. Equivalent circuit to replace a double line-to-ground fault in the positive-sequence network.

nected in parallel between the point of fault and zero-potential bus for the positive-sequence network.

Connection of Sequence Networks for Solution of Double Line-to-Ground Faults on an A-C Calculating Table. Equations [13]–[17] determine the manner in which the networks should be connected to satisfy the conditions for the signs of the currents and voltages. The connection for a double line-to-ground fault at B in the system shown in Fig. 2(a) is given in Fig. 9. This connection satisfies the conditions

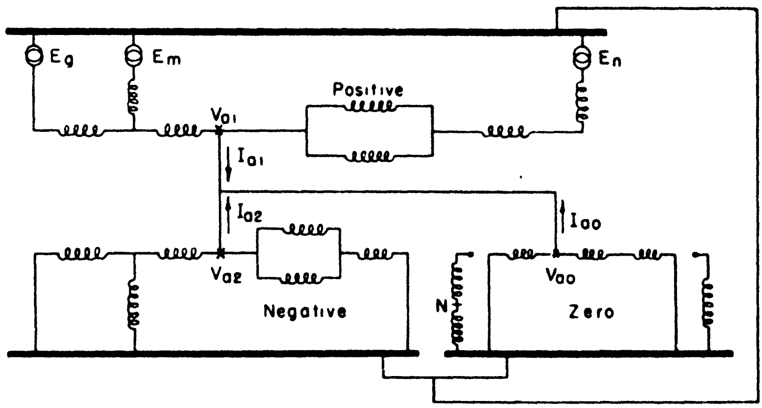


Fig. 9. Connection of sequence networks of Fig. 2 for a double line-to-ground fault at B.

that $V_{a1} = V_{a2} = V_{a0}$, that $I_{a1} = -(I_{a2} + I_{a0})$, and that I_{a0} and I_{a2} divide inversely in proportion to their sequence impedances viewed from the fault.

Determination of Fault Currents by D-C Calculating Table. A d-c calculating table may be used to determine initial symmetrical rms current distribution for unsymmetrical as well as for three-phase faults when resistance and capacitance are neglected and the system is operated at no load. In this case the internal voltages of the machines in the positive-sequence network behind subtransient reactance in per unit will be equal and in phase and therefore can be represented on a d-c table. It may also be used under load conditions, when resistance and capacitance are neglected, to determine the currents due to the fault, the applied voltage in the positive-sequence network being V_f, the prefault voltage at the point of fault. This voltage on the d-c table will be applied between the neutrals of the machines, connected at a common point, and the zero potential bus for the positive-sequence network. Load currents at any point P can be added to currents at P due to the fault to give total currents. Since load currents are usually small relative to short-circuit currents and are not in phase with them, their inclusion will not, in general, appreciably affect the magnitudes of the short-circuit currents.

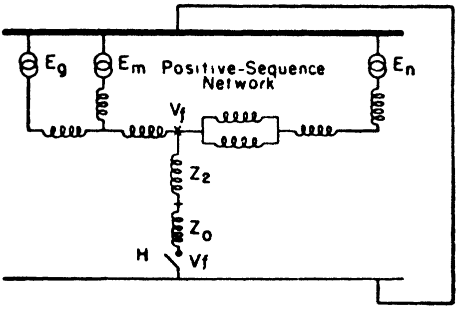
Two methods of determining system voltages and currents during short circuits have been given, both based upon *superposition*. The principles underlying these two methods are explained in Chapter I, and discussed in connection with their application to the calculation of system currents and voltages during three-phase short circuits. The method of superposition is a powerful tool, frequently used in a-c power transmission problems. For this reason, the two methods of calculation based on superposition, described in Chapter I, will be reviewed in their application to unsymmetrical faults.

Assume a line-to-ground fault to occur at B in the system of Fig. 2(a). It has been shown that the positive-sequence currents and voltages in the fault and in the positive-sequence network will be correctly determined if the impedance $Z_2 + Z_0$, the sum of the negative- and zero-sequence impedances viewed from the fault, is inserted in the positive-sequence network between the fault point B and the zero-potential bus for the network. If one terminal of the impedance $Z_2 + Z_0$ is connected at the fault point in the positive-sequence network with the other terminal H isolated as in Fig. 10(a), the system operating conditions before the fault are in no way affected. An open switch is indicated at H with a connection to the zero-potential bus of the positive-sequence network. The voltage rise across the switch measured from the zero-potential bus to H is V_f. The fault is replaced by its equivalent circuit if the switch is closed at H in Fig. 10(a).

By the method called the *first method* in Chapter I, the currents resulting from the voltages E_g, E_m, and E_n, applied one at a time in Fig. 10(a) with the switch closed, are superposed to give total currents in accordance with equations [33] of Chapter I. When calculations are made on an a-c calculating table, and the negative- and zero-sequence networks set up and connected as in Fig. 7, the currents and voltages in the three sequence networks are those resulting from all the applied voltages acting simultaneously. With the a-c calculating table, superposition is automatically applied.

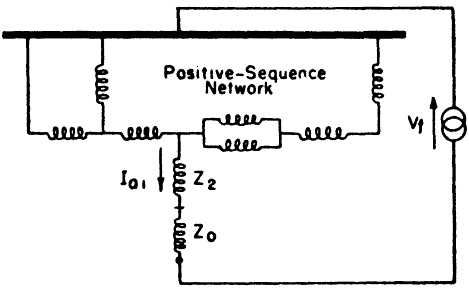
In the method called the *second method* in Chapter I, the *changes* in currents resulting from the fault are determined, not the total currents; load currents are not included. When the switch is closed in Fig. 10(a) to represent fault conditions, the voltage across it becomes zero. The voltage across the switch can be made zero, if a voltage $-V_f$ is applied across it. The currents resulting from the voltage $-V_f$ are the changes in the currents resulting from the fault; and, as indicated in equations

[52] of Chapter I, are determined with $-V_f$ acting alone. The voltage $-V_f$ is the only voltage applied to the network, but, for simplicity, the voltage rise $-V_f$ between the zero-potential bus and H is repre-



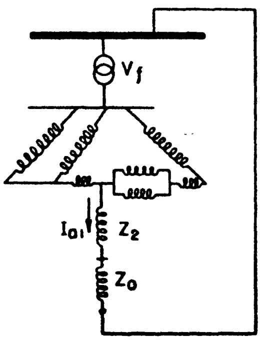
(a)

(*a*) Line-to-ground fault replaced by an equivalent circuit in the positive-sequence network by closing the switch at *H*.



(b)

(*b*) Connection of sequence networks to obtain changes in currents and voltages resulting from the fault.



(c)

(*c*) Connection for d-c calculating table, where Z_2 and Z_0 can be replaced by the negative- and zero-sequence networks, respectively.

FIG. 10.

sented in Fig. 10(*b*) as a voltage rise V_f in the opposite direction. In Fig. 10(*b*), V_f is shown outside the positive-sequence network. With this arrangement, fault currents will be correctly determined; but the positive-sequence voltage at the fault referred to the zero-potential bus for the network will be $-I_{a1}Z_1$ and not V_{a1}. If V_f is placed between the zero-potential bus for the positive-sequence network and the impedances of the synchronous machines connected at a common point as in Fig. 10(c), the voltage at the fault in the positive-sequence network will be V_{a1}. The connection of the synchronous machines shown in Fig. 10(c) is that used for the d-c calculating table.

It should be noted that the initial symmetrical rms currents in the fault, voltages at the fault, and currents and voltages in the negative- and zero-sequence networks, calculated by the two methods, are identical, provided V_f in Fig. 10(c) is the same as V_f in Fig. 10(a) before the fault is applied. Since the load currents and internal machine voltages are included in the first method and not in the second, the currents and voltages in the positive-sequence network will be different. The second method is preferable for an analytic solution, because the internal voltages of the various symmetrical machines on the system under load are not required. The voltage V_f under load, if not given, can usually be closely estimated by one familiar with the power system. The load currents can also be estimated and added to the currents due to the fault to obtain total currents.

In problems where part of the system is switched out of service following a fault, it is necessary to consider the internal voltages of the machines in both parts of the system. Problems of this nature frequently arise. For example, if one machine and a part of the system with an unsymmetrical fault are cut off from the rest of the system by switching, the currents and voltages in the fault and in the severed part of the system are determined by using the internal voltage of the machine and the sequence impedances of the part of the system connected to it.

The following problem illustrates the application of the second method to the analytic determination of initial symmetrical rms currents and voltages at the fault, negative- and zero-currents and voltages in the system, positive-sequence currents resulting from the fault, and phase currents in the system with load currents neglected.

Problem 2. A double line-to-ground fault occurs at B in the system shown by one-line diagram in Fig. 2(a). Determine the per unit initial symmetrical rms phase voltages and currents at the fault and the phase currents in the system resulting from the fault.

Solution. The operating conditions previous to the fault are not given; therefore load currents cannot be included. It will be assumed that the voltage V_f at B before the fault was equal to base voltage in the transmission circuit. Under this assumption V_f is unity in magnitude. This is a reasonable assumption. By the second method described above, currents and voltages in Figs. 10(c) or (b) vary directly with V_f; and therefore, if it is later learned that V_f differed from unity, the currents and voltages obtained with $V_f = 1$ can be accurately determined if they are multiplied by the known per unit value of V_f. Load currents can also be added later, if given.

With V_f as reference vector,

$$V_f = 1\,\underline{/0^\circ} = 1 + j0$$

The positive-sequence impedance viewed from the fault B in Fig. 2 (b) is

$$Z_1 = \frac{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right)(j0.15 + j0.10 + j0.30)}{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right) + (j0.15 + j0.10 + j0.30)} = j0.188$$

The negative-sequence impedance viewed from the fault B in Fig. 2 (c) is

$$Z_2 = \frac{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right)(j0.15 + j0.10 + j0.40)}{\left(\dfrac{j1.0 \times j0.20}{j1.20} + j0.12\right) + (j0.15 + j0.10 + j0.40)} = j0.199$$

The zero-sequence impedance viewed from the fault B in Fig. 2 (d) is

$$Z_0 = \frac{(j0.12)(j0.70 + j0.10)}{(j0.12 + j0.70 + j0.10)} = j0.104$$

From [15], [16], and [18], the sequence components of the fault currents are

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} = \frac{1.0 + j0}{j0.188 + \dfrac{j0.104 \times j0.199}{j0.303}} = -j3.900$$

$$I_{a2} = -I_{a1}\frac{Z_0}{Z_0 + Z_2} = j3.900\frac{j0.104}{j0.303} = j1.34$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_0 + Z_2} = j3.900\frac{j0.199}{j0.303} = j2.56$$

Substituting I_{a1}, I_{a2}, and I_{a0} in [19], [29], and [30] of Chapter II, the three fault currents are

$$I_a = -j3.90 + j1.34 + j2.56 = 0$$

$$I_b = -\tfrac{1}{2}(-j3.90 + j1.34) - j\frac{\sqrt{3}}{2}(-j3.90 - j1.34) + j2.56 = -4.54 + j3.84$$

$$I_c = -\tfrac{1}{2}(-j3.90 + j1.34) + j\frac{\sqrt{3}}{2}(-j3.90 - j1.34) + j2.56 = 4.54 + j3.84$$

The ground current at the fault is

$$I_g = 3I_{a0} = j7.68$$

The sequence voltages at the fault from [17] are

$$V_{a1} = V_{a2} = V_{a0} = I_{a1}\frac{Z_0Z_2}{Z_0 + Z_2} = -j3.90 \times j0.0684 = 0.267 + j0$$

The phase voltages to ground at the fault are

$$V_a = V_{a0} + V_{a1} + V_{a2} = 3V_{a0} = 0.801 + j0$$

$$V_b = V_c = 0$$

The sequence currents due to the fault will flow from the right and left of B inversely in proportion to the respective impedances in the two directions.

Positive-sequence current distribution in Fig. 2(b):

$$I_{a1} \text{ (from right of } B) = -j3.90 \times \frac{0.287}{0.837} = -j1.33$$

$$I_{a1} \text{ (from left of } B) = -j3.90 \times \frac{0.55}{0.837} = -j2.57$$

$$I_{a1} \text{ (from generator } G) = -j2.57 \times \frac{1.0}{1.20} = -j2.142$$

$$I_{a1} \text{ (from motor } M) = -j2.57 \times \frac{0.20}{1.20} = -j0.428$$

Negative-sequence current distribution in Fig. 2(c):

$$I_{a2} \text{ (from right of } B) = j1.34 \times \frac{0.287}{0.937} = j0.41$$

$$I_{a2} \text{ (from left of } B) = j1.34 \times \frac{0.65}{0.937} = j0.93$$

$$I_{a2} \text{ (from generator } G) = j0.93 \times \frac{1.0}{1.20} = j0.775$$

$$I_{a2} \text{ (from motor } M) = j0.93 \times \frac{0.20}{1.20} = j0.155$$

Zero-sequence current distribution in Fig. 2(d):

$$I_{a0} \text{ (from left of } B) = j2.56 \times \frac{0.80}{0.92} = j2.23$$

$$3I_{a0} = I_n \text{ in neutral connection} = j6.69$$

$$I_{a0} \text{ (from right of } B) = j2.56 \times \frac{0.12}{0.92} = j0.33$$

$$3I_{a0} = I_n \text{ in neutral connection} = j0.99$$

Figure 11(a) shows the flow of sequence currents resulting from the fault, determined analytically by applying V_f as in Fig. 10(b). Positive direction for current flow is indicated by arrows. If the direction of an arrow is reversed, the sign of the current to which the arrow is attached must also be reversed. The currents in Fig. 11(a) are the *changes* in the sequence currents because of the fault. Combining the components of current, using [19], [29], and [30] of Chapter II, the phase currents are obtained.

The currents flowing toward B from the right are the currents which flow in the Y-connected windings of the transformer bank at C and through the transmission lines in parallel. They are

$$I_a = -j1.33 + j0.41 + j0.33 = -j0.59$$

$$I_b = -\tfrac{1}{2}(-j1.33 + j0.41) - j\frac{\sqrt{3}}{2}(-j1.33 - j0.41) + j0.33 = -1.51 + j0.79$$

$$I_c = -\tfrac{1}{2}(-j1.33 + j0.41) + j\frac{\sqrt{3}}{2}(-j1.33 - j0.41) + j0.33 = 1.51 + j0.79$$

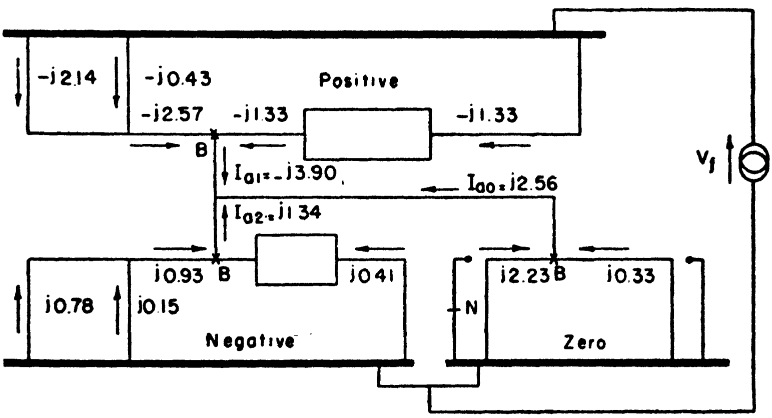
The currents flowing toward B from the left are the currents in the Y-connected windings of the transformer bank at B.

$$I_a = -j2.57 + j0.93 + j2.23 = j0.59$$

$$I_b = -\tfrac{1}{2}(-j2.57 + j0.93) - j\frac{\sqrt{3}}{2}(-j2.57 - j0.93) + j2.23 = -3.03 + j3.05$$

$$I_c = -\tfrac{1}{2}(-j2.57 + j0.93) + j\frac{\sqrt{3}}{2}(-j2.57 - j0.93) + j2.23 = 3.03 + j3.05$$

$$I_a + I_b + I_c = 3I_{a0} = j6.69$$



(a)

FIG. 11(a). Sequence currents for Problem 2. Double line-to-ground fault at B in system of Fig. 2 operating at no load.

There are no zero-sequence line currents on the Δ sides of the transformer bank. The positive- and negative-sequence currents of Fig. 11(a) in the three machines (assumed Y-connected) are given in terms of equivalent Y–Y transformer banks; the shift in phase in passing through the bank is not included. Turning the positive- and negative-sequence currents in the three machines, calculated on the basis of equivalent Y-Y transformer banks, 90° forward and 90° backward, respectively, the line currents are obtained. (See Chapter III.) In machine N:

$$I_{a1} = j(-j1.33) = 1.33$$
$$I_{a2} = -j(j0.41) = 0.41$$
$$I_a = 1.74$$
$$I_b = -\tfrac{1}{2}(1.33 + 0.41) - j\frac{\sqrt{3}}{2}(1.33 - 0.41) = -0.87 - j0.80$$
$$I_c = -0.87 + j0.80$$

In machine G:

$$I_{a1} = j(-j2.142) = 2.142$$
$$I_{a2} = -j(j0.775) = 0.775$$
$$I_a = 2.92$$
$$I_b = -1.46 - j1.18$$
$$I_c = -1.46 + j1.18$$

In machine M:

$$I_{a1} = j(-j0.428) = 0.428$$
$$I_{a2} = -j(j0.155) = 0.155$$
$$I_a = 0.583 + j0$$
$$I_b = -0.29 - j0.236$$
$$I_c = -0.29 + j0.236$$

Currents in the three phases of the system of Fig. 2(a) with a double line-to-ground fault at B are shown in Fig. 11(b). Currents in Δ-connected windings of the transformer banks, in per unit of base current in

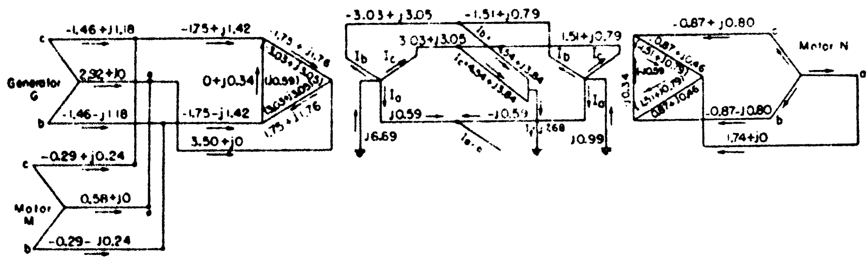


FIG. 11(b). Phase currents for Problem 2.

the Δ circuits, are equal in magnitude and phase to the currents in the Y-connected windings in per unit of base current in the Y flowing in the opposite directions. If Δ currents are expressed in per unit of base Δ current and line currents in per unit of base line current, the sum of the per unit currents flowing into a Δ terminal will not add to zero. Per unit Δ currents are converted to per unit line currents if they are divided by $\sqrt{3}$; and per unit line currents to per unit Δ currents if they are multiplied by $\sqrt{3}$. In Fig. 11(b), Δ currents in per unit of base Δ currents are given in parentheses inside the Δ's; Δ currents in per unit of base line currents are given outside the Δ's. Using the latter, the sum of the currents flowing into a Δ terminal is zero. To satisfy this condition, currents must be expressed in amperes, or in per unit of the same base current.

FAULTS THROUGH IMPEDANCE

The most common type of ground faults occur on transmission circuits when there is a flashover between one or two conductors and a tower. In such cases, the impedances in the arc or arcs, the tower, and the tower footing influence the currents and voltages obtained. The impedance of the tower itself is negligible but the tower footing resistance, which is the resistance between the tower footing and true ground, may vary from a low value of 3 to 20 ohms in exceptionally moist earth, or when care has been taken to secure low resistance, up to 300 ohms or more in rocky soil where nothing has been done to lower the resistance. A line-to-line fault may occur directly between two conductors, or through an arc when there is a flashover between conductors not involving ground.

Figure 12 shows the currents I_a, I_b, and I_c flowing from the three phases a, b, and c, respectively, through hypothetical stub connections into the fault for the four types of faults. Z_f is the fault impedance. V_a, V_b, and V_c are the phase voltages to ground at the fault. In Fig. 12, the fault impedance is shown as an equal impedance in the three phases for a three-phase fault; for the line-to-line fault, the impedance is shown between conductors; for the single line-to-ground fault, it is between the conductor and ground; for the double line-to-ground fault, it is placed between the conductors and ground, the impedance between conductors being neglected. The fault impedances of Fig. 12 are practical, but others are possible. With fault

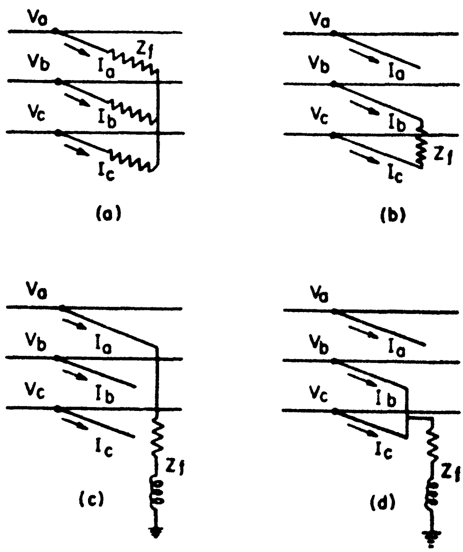


FIG. 12. Currents flowing into impedance faults. (a) Three-phase fault. (b) Line-to-line fault. (c) Single line-to-ground fault. (d) Double line-to-ground fault.

impedances as indicated in Fig. 12, if V_f is the prefault voltage at the fault point F, Z_1, Z_2, and Z_0 the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, and Z_f the fault impedance, the initial symmetrical rms currents flowing into the fault and the voltages to ground at the fault can be obtained in a manner similar to that used for faults through zero fault impedance.

Three-Phase Fault. In Fig. 12(a), the fault impedance Z_f is assumed equal in the three phases. The system, therefore, is not unbalanced by the three-phase fault, the currents and voltages remain of positive sequence, and

$$\begin{aligned} I_{a1} &= \frac{V_f}{Z_1 + Z_f} \\ V_{a1} &= I_{a1}Z_f \end{aligned} \qquad [19]$$

From [19] the equivalent circuit to replace the fault in the positive-sequence network is the fault impedance Z_f. This impedance is connected between the fault point and the zero-potential bus for the network as indicated in Fig. 13(a).

Line-to-Line Fault through Impedance Z_f. The fault conditions from Fig. 12(b) are

$$I_a = 0 \qquad I_b = -I_c \qquad V_b - V_c = I_bZ_f$$

Substituting $I_a = 0$ and $I_b = -I_c$ in [22]–[24] of Chapter II,

$$\begin{aligned} I_{a0} &= 0 \\ I_{a2} &= -I_{a1} \end{aligned} \qquad [20]$$

Replacing V_b and V_c in the fault equation by [8] and [9] of Chapter II, respectively,

$$V_b - V_c = (a^2 - a)V_{a1} - (a^2 - a)V_{a2} = I_bZ_f \qquad [21]$$

Replacing I_b in [21] by $a^2I_{a1} + aI_{a2} = I_{a1}(a^2 - a)$, and dividing the equation by $(a^2 - a)$,

$$V_{a1} = V_{a2} + I_{a1}Z_f \qquad [22]$$

Replacing I_{a2} in [2] by $-I_{a1}$, and then substituting [2] in [22],

$$V_{a1} = I_{a1}(Z_2 + Z_f) \qquad [23]$$

Substituting [1] in [23], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_f} = -I_{a2} \qquad [24]$$

From [23], $V_{a1} = I_{a1}(Z_2 + Z_f)$. It follows, therefore, that the equivalent circuit to replace the fault in the positive-sequence network is the impedance $(Z_2 + Z_f)$, this impedance to be inserted between the fault point and the zero-potential bus for the network.

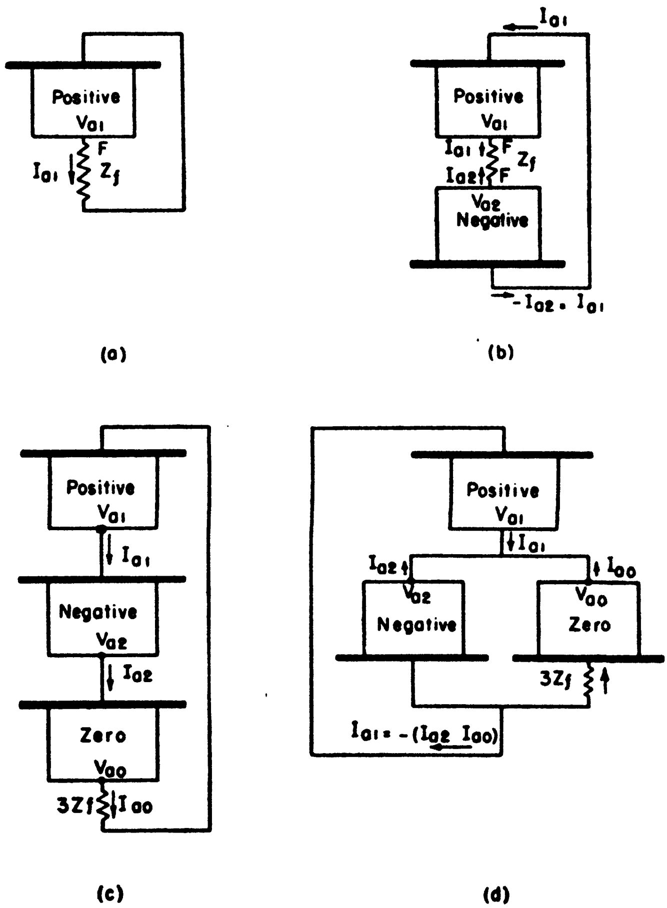


FIG. 13. Connections of sequence networks for faults through impedance as indicated in Figs. 12 (*a*), (*b*), (*c*), and (*d*), respectively.

Instead of using an equivalent circuit in the positive-sequence network to obtain positive-sequence currents and voltages, the positive- and negative-sequence networks can be connected to satisfy [20] and [23], as shown in Fig. 13(*b*).

Line-to-Ground Fault through Impedance Z_f. From Fig. 12(*c*), the fault conditions are

$$I_b = 0 \qquad I_c = 0 \qquad V_a = I_a Z_f$$

Substituting $I_b = I_c = 0$ in [22]–[24] of Chapter II,

$$I_{a1} = I_{a2} = I_{a0} \qquad [25]$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = 3I_{a1}$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = I_aZ_f = 3I_{a1}Z_f$$

Therefore

$$V_{a1} = -V_{a2} - V_{a0} + I_{a1}(3Z_f) \qquad [26]$$

Substituting [2] and [3] in [26], and replacing I_{a2} and I_{a0} by I_{a1},

$$V_{a1} = I_{a1}(Z_2 + Z_0 + 3Z_f) \qquad [27]$$

Substituting [1] in [27], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_2 + Z_0 + 3Z_f} \qquad [28]$$

From [27], the equivalent circuit to replace the fault in the positive-sequence network is the impedance $(Z_2 + Z_0 + 3Z_f)$, this impedance to be inserted between the fault point and the zero-potential bus for the network.

The manner of connecting the sequence networks to satisfy [25] and [26] is shown in Fig. 13(c).

Double Line-to-Ground Fault through Impedance Z_f. From Fig. 12(d), the fault conditions are

$$I_a = 0 \qquad V_c = V_b = (I_b + I_c)Z_f$$

With $I_a = 0$,

$$I_{a1} + I_{a2} + I_{a0} = 0$$

or

$$I_{a1} = -(I_{a2} + I_{a0}) \qquad [29]$$

Substituting $I_a = 0$ in [22] of Chapter II,

$$I_{a0} = \tfrac{1}{3}(I_b + I_c)$$

or

$$I_b + I_c = 3I_{a0} \qquad [30]$$

With $V_b = V_c$ in [10]–[12] of Chapter II,

$$V_{a0} = \tfrac{1}{3}(V_a + 2V_b) \qquad [31]$$

$$V_{a1} = V_{a2} = \tfrac{1}{3}(V_a - V_b) \qquad [32]$$

Subtracting [32] from [31], and replacing V_b by $(I_b + I_c)Z_f = 3I_{a0}Z_f$,

$$V_{a0} - V_{a1} = V_b = 3I_{a0}Z_f$$

or

$$V_{a1} = V_{a0} - 3I_{a0}Z_f \qquad [33]$$

Replacing V_{a2} and V_{a0} in [32] and [33] by their values from [2] and [3],

$$V_{a1} = V_{a2} = -I_{a2}Z_2$$

$$V_{a1} = -I_{a0}(Z_0 + 3Z_f)$$

Therefore

$$I_{a2} = -\frac{V_{a1}}{Z_2} \tag{34}$$

and

$$I_{a0} = -\frac{V_{a1}}{Z_0 + 3Z_f} \tag{35}$$

Substituting [34] and [35] in [29],

$$I_{a1} = V_{a1}\left[\frac{1}{Z_2} + \frac{1}{Z_0 + 3Z_f}\right]$$

Therefore

$$V_{a1} = I_{a1}\frac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f} \tag{36}$$

Substituting [36] in [34] and [35], I_{a2} and I_{a0} are expressed in terms of I_{a1}:

$$I_{a2} = -I_{a1}\frac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f} \tag{37}$$

$$I_{a0} = -I_{a1}\frac{Z_2}{Z_2 + Z_0 + 3Z_f} \tag{38}$$

Replacing V_{a1} in [36] by its value from [1], and solving for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} \tag{39}$$

From [36], the equivalent circuit to replace the fault in the positive-sequence network is the impedance $Z_2(Z_0 + 3Z_f)/(Z_2 + Z_0 + 3Z_f)$, this impedance to be connected in the positive-sequence network between the fault point and the zero-potential bus for the network. In Fig. 13(*d*), the sequence networks are connected to satisfy equations [29], [32], and [33].

When the symmetrical components of current flowing into the fault and voltages to ground at the fault for the reference phase *a* have been determined for any type of fault, then by the use of [7]–[9] and [19]–[21] of Chapter II, the fault currents and voltages to ground at the fault

TABLE I

Initial Symmetrical RMS Fault Currents and Voltages at the Fault and Their Symmetrical Components for Various Types of Faults

The voltage to ground of phase a at the point of fault F before the fault occurred is V_f. The positive-, negative-, and zero-sequence impedances viewed from the fault are Z_1, Z_2, and Z_0, respectively. Z_f is the fault impedance. Z_{eq} is the equivalent impedance to replace the fault in the positive-sequence network.

	Three-phase fault through three-phase fault impedance, Z_f	Line-to-line fault. Phases b and c shorted through fault impedance, Z_f	Line-to-ground fault. Phase a grounded through fault impedance, Z_f	Double line-to-ground fault. Phases b and c shorted, then grounded through fault impedance, Z_f
I_{a1}	$I_{a1} = \dfrac{V_f}{Z_1 + Z_f}$	$I_{a1} = -I_{a2} = \dfrac{V_f}{Z_1 + Z_2 + Z_f}$	$I_{a1} = I_{a2} = I_{a0}$ $= \dfrac{V_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$I_{a1} = -(I_{a2} + I_{a0})$ $= \dfrac{V_f}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}}$
I_{a2}	$I_{a2} = 0$	$I_{a2} = -I_{a1}$	$I_{a2} = I_{a1}$	$I_{a2} = -I_{a1} \dfrac{Z_0 + 3Z_f}{Z_2 + Z_0 + 3Z_f}$
I_{a0}	$I_{a0} = 0$	$I_{a0} = 0$	$I_{a0} = I_{a1}$	$I_{a0} = -I_{a1} \dfrac{Z_2}{Z_2 + Z_0 + 3Z_f}$
V_{a1}	$V_{a1} = I_{a1}Z_f$	$V_{a1} = V_{a2} + I_{a1}Z_f$ $= I_{a1}(Z_2 + Z_f)$	$V_{a1} = -(V_{a2} + V_{a0}) + I_{a1}(3Z_f)$ $= I_{a1}(Z_0 + Z_2 + 3Z_f)$	$V_{a1} = V_{a2} = V_{a0} - 3I_{a0}Z_f$ $= I_{a1} \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$
V_{a2}	$V_{a2} = 0$	$V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2$	$V_{a2} = -I_{a2}Z_2 = -I_{a1}Z_2$	$V_{a2} = -I_{a2}Z_2$ $= I_{a1} \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$
V_{a0}	$V_{a0} \doteq 0$	$V_{a0} = 0$	$V_{a0} = -I_{a0}Z_0$ $= -I_{a1}Z_0$	$V_{a0} = -I_{a0}Z_0$ $= I_{a1} \dfrac{Z_0Z_2}{Z_2 + Z_0 + 3Z_f}$
Z_{eq}	$Z_{eq} = Z_f$	$Z_{eq} = Z_2 + Z_f$	$Z_{eq} = Z_0 + Z_2 + 3Z_f$	$Z_{eq} = \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}$

TABLE I (*Continued*)

	Three-phase fault through three-phase fault impedance, Z_f	Line-to-line fault. Phases b and c shorted through fault impedance, Z_f	Line-to-ground fault. Phase a grounded through fault impedance, Z_f	Double line-to-ground fault. Phases b and c shorted, then grounded through fault impedance, Z_f
I_a	$\frac{V_f}{Z_1 + Z_f}$	0	$\frac{3V_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	0
I_b	$\frac{a^2V_f}{Z_1 + Z_f}$	$-j\sqrt{3}\frac{V_f}{Z_1 + Z_2 + Z_f}$	0	$-j\sqrt{3}V_f\frac{Z_0 + 3Z_f - aZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
I_c	$\frac{aV_f}{Z_1 + Z_f}$	$j\sqrt{3}\frac{V_f}{Z_1 + Z_2 + Z_f}$	0	$j\sqrt{3}V_f\frac{Z_0 + 3Z_f - a^2Z_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_a	$V_f\frac{Z_f}{Z_1 + Z_f}$	$V_f\frac{2Z_2 + Z_f}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3Z_f}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{3Z_2(Z_0 + 2Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_b	$V_f\frac{a^2Z_f}{Z_1 + Z_f}$	$V_f\frac{a^2Z_f - Z_2}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3a^2Z_f - j\sqrt{3}(Z_2 - aZ_0)}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{-3Z_fZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_c	$V_f\frac{aZ_f}{Z_1 + Z_f}$	$V_f\frac{aZ_f - Z_2}{Z_1 + Z_2 + Z_f}$	$V_f\frac{3aZ_f + j\sqrt{3}(Z_2 - a^2Z_0)}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$V_f\frac{-3Z_fZ_2}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_{bc}	$j\sqrt{3}V_f\frac{Z_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{Z_f}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{3Z_f + Z_0 + 2Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	0
V_{ca}	$j\sqrt{3}V_f\frac{a^2Z_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{a^2Z_f - j\sqrt{3}Z_2}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{a^2(3Z_f + Z_0) - Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$\sqrt{3}V_f\frac{\sqrt{3}Z_2(Z_0 + 3Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$
V_{ab}	$j\sqrt{3}V_f\frac{aZ_f}{Z_1 + Z_f}$	$j\sqrt{3}V_f\frac{aZ_f + j\sqrt{3}Z_2}{Z_1 + Z_2 + Z_f}$	$j\sqrt{3}V_f\frac{a(3Z_f + Z_0) - Z_2}{Z_0 + Z_1 + Z_2 + 3Z_f}$	$-\sqrt{3}V_f\frac{\sqrt{3}Z_2(Z_0 + 3Z_f)}{Z_1Z_2 + (Z_1 + Z_2)(Z_0 + 3Z_f)}$

for the three phases can be obtained. The line-to-line voltages at the fault can be obtained from [11] of Chapter III, or by taking the differences of the line-to-ground voltages.

Fault Currents and Voltages from Formulas. For convenience in determining fault currents and voltages, Table I of this chapter has been prepared. This table gives I_{a1}, the positive-sequence component of initial symmetrical rms current flowing from the reference phase a into the fault, for various types of fault in terms of V_f, the voltage at the fault before the fault occurred, Z_1, Z_2, Z_0, the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, and Z_f, the fault impedance. The other symmetrical components of fault current and the symmetrical components of voltage to ground at the fault are given in terms of I_{a1}. The currents flowing into the fault from the three phases and the line-to-ground and line-to-line voltages at the fault are expressed in terms of V_f, Z_1, Z_2, Z_0, and Z_f.

Table I can also be used for determining transient or sustained currents and voltages of fundamental frequency at the fault in cases where the rotors of the machines on the system do not materially change their relative angular positions. If this is the case, transient or equivalent steady-state impedance, respectively, is used for Z_1, the positive-sequence impedance viewed from the fault.

A study of Table I shows that the effect of fault impedance is to reduce the fault current. It may either increase or decrease fault voltages, depending upon its magnitude relative to the magnitudes of the sequence impedances and the amount of resistance in the system exclusive of the fault resistance.

Table I can be used for determining fault currents and voltages when there is no impedance in the fault if Z_f is replaced by zero.

OPEN CONDUCTORS

When circuits are controlled by fuses or any device which does not open all three phases, one or two phases of the circuit may be open while the other phases or phase is closed. This condition may also occur with one or two broken conductors. The case of a conductor breaking and one end falling to ground is considered in Chapters VII and X. If both ends fall to ground, the condition is that of a line-to-ground fault.

When the system impedances and admittances are constant, i.e., do not vary with the voltages and currents associated with them, the method of symmetrical components can be used to determine fundamental-frequency currents and voltages in the system with one or two open conductors. The exciting impedance of a transformer is an example of an impedance which is not constant, but varies with the applied voltage. In transformers under load, the exciting impedances can usually be neglected. An open conductor in a circuit supplying an unloaded or lightly loaded transformer bank is discussed in Vol. II. Constant system impedances and admittances are assumed in the following discussion.

One Open Conductor. Figure 14(a) shows a section of a three-phase system with phase a open between points P and Q. Let I_a, I_b, I_c be the *line currents* in phases a, b, c, respectively, with positive direction from P to Q; V_a, V_b, V_c, the voltages to ground at P, and V'_a, V'_b, V'_c the voltages to ground at Q. From Fig. 14(a),

$$I_a = 0$$

$$V_a - V'_a = v_a = \text{series voltage drop between } P \text{ and } Q \text{ in phase } a$$

$$V_b - V'_b = v_b = 0 = \text{series voltage drop between } P \text{ and } Q \text{ in phase } b$$

$$V_c - V'_c = v_c = 0 = \text{series voltage drop between } P \text{ and } Q \text{ in phase } c$$

In Chapter II, the fundamental symmetrical component equations for three-phase voltage and current vectors are given by [7]–[12] and [19]–[24]. It was pointed out that in these equations the voltage vectors V_a, V_b, and V_c and the current vectors I_a, I_b, and I_c can be used to represent any three voltage vectors and any three current vectors revolving at the same rate which are associated with the three phases of a three-phase system. These voltage equations will now be applied to v_a, v_b, and v_c, the series voltage drops between P and Q in phases a, b, and c, respectively, and the current equations to the line currents I_a, I_b, and I_c flowing in phases a, b, and c, respectively, from P to Q.

Resolving the series voltage drops v_a, v_b, v_c into their symmetrical components by [10]–[12] of Chapter II, with $v_b = v_c = 0$,

$$v_{a0} = V_{a0} - V'_{a0} = \tfrac{1}{3}(v_a + v_b + v_c) = \tfrac{1}{3}v_a$$

$$v_{a1} = V_{a1} - V'_{a1} = \tfrac{1}{3}(v_a + av_b + a^2v_c) = \tfrac{1}{3}v_a$$

$$v_{a2} = V_{a2} - V'_{a2} = \tfrac{1}{3}(v_a + a^2v_b + av_c) = \tfrac{1}{3}v_a$$

Therefore

$$v_{a0} = v_{a1} = v_{a2} \qquad [40]$$

Since $I_a = 0 = I_{a1} + I_{a2} + I_{a0}$,

$$I_{a1} = -(I_{a2} + I_{a0}) \qquad [41]$$

From [40], the open conductor introduces equal *series voltage drops* into each of the sequence networks at the opening in the direction of current flow, i.e., from P to Q. Stated in another way, the opening introduces equal series voltage rises in the three sequence networks in

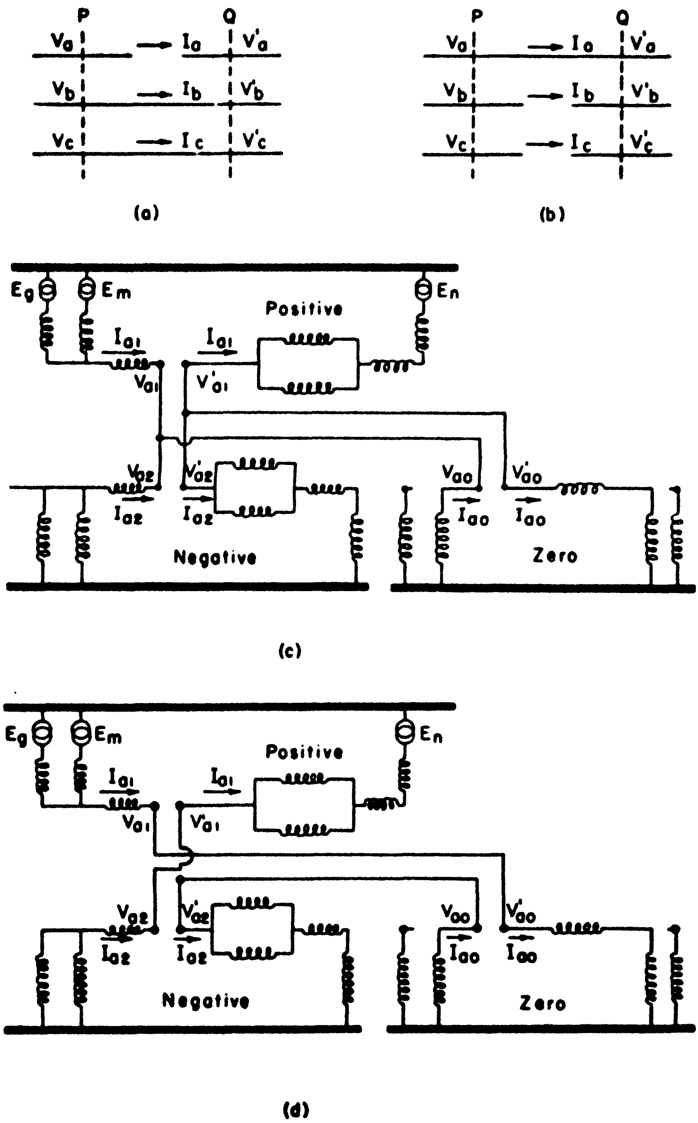


FIG. 14. Section of three-phase system between P and Q. (a) Phase a open. (b) Phases b and c open. Connection of sequence networks of Fig. 2 for open conductors at B. (c) Phase a open. (d) Phases b and c open.

the direction QP, which tend to send currents from Q to P. These generated voltages meet *series impedances*, as distinguished from impedances to neutral or to ground.

In a three-phase system with linear impedances, symmetrical except for an open conductor, let z_0 and z_2, respectively, represent the series impedances in the zero- and negative-sequence networks viewed from the opening. Consider, for example, the system shown by the one-line diagram of Fig. 2(a) with the sequence networks given by parts (b), (c), and (d) of Fig. 2. The negative-sequence series impedance viewed from any point B can be determined from Fig. 2(c) if a series voltage is inserted at B and the current read, the ratio of the series voltage at B to the current at B being the series impedance z_2 viewed from the opening at B. From Fig. 2(c), with the opening at B, C, or D,

$$z_2 = \left(j\,\frac{1.00 \times 0.20}{1.20} + j0.12 + j0.15 + j0.10 + j0.40\right) = j0.936$$

If the opening is at A in Fig. 2(c),

$$z_2 = j0.20 + j\,\frac{1.00(0.12 + 0.15 + 0.10 + 0.40)}{1.00 + 0.77} = j0.635$$

The zero-sequence series impedance z_0 is determined from Fig. 2(d) in a manner similar to that used to determine z_2. With the opening at A or D, $z_0 = \infty$. With the opening at B, or C,

$$z_0 = j(0.12 + 0.70 + 0.10) = j0.92$$

With balanced generated voltages, there are no negative- or zero-sequence voltages applied to the system. The only voltages causing negative- and zero-sequence currents to flow are those resulting from the opening. The sum of the voltage drops in any closed circuit is zero; therefore, the series negative- (or zero-) sequence voltage drop between P and Q plus the negative- (or zero-) sequence voltage drop in the negative- (or zero-) sequence network is zero. The equations are

$$v_{a0} + I_{a0}z_0 = 0 \qquad [42]$$

$$v_{a2} + I_{a2}z_2 = 0 \qquad [43]$$

From [40], [42], and [43],

$$I_{a0} = I_{a2}\frac{z_2}{z_0} \qquad [44]$$

From [41] and [44],

$$I_{a1} = -(I_{a2} + I_{a0}) = -\frac{z_0 + z_2}{z_0}I_{a2}$$

Therefore

$$I_{a2} = -\frac{z_0}{z_0 + z_2} I_{a1} \qquad [45]$$

$$I_{a0} = -\frac{z_2}{z_0 + z_2} I_{a1} \qquad [46]$$

From [40], [43], and [45],

$$v_{a1} = v_{a2} = v_{a0} = -I_{a2}z_2 = \frac{z_2 z_0}{z_0 + z_2} I_{a1} \qquad [47]$$

From [47] it follows that v_{a1}, the positive-sequence series voltage drop between P and Q, will be obtained if the impedance $z_2z_0/(z_0 + z_2)$ is inserted in the positive-sequence network between points P and Q. This impedance, which is the negative- and zero-sequence series impedances connected in parallel, is the equivalent circuit which replaces the opening in the positive-sequence network. Instead of inserting the lumped impedance $z_2z_0/(z_0 + z_2)$, the negative- and zero-sequence networks can be connected in parallel between P and Q in the positive-sequence network.

Figure 14(c) shows the connection of the sequence networks of Fig. 2 with one conductor open at point B for solution on the a-c network analyzer. This connection satisfies [40] and [41].

Two Open Conductors. Figure 14(b) shows a section of a three-phase system with phases b and c open between points P and Q. With the notation used for one open conductor,

$$I_b = I_c = 0$$

$$V_a - V_a' = v_a = 0$$

Then by resolving the line currents into their symmetrical components by equations [22]–[24] of Chapter II,

$$I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3} \qquad [48]$$

Since $v_a = 0$,

$$v_{a1} = -(v_{a2} + v_{a0}) \qquad [49]$$

Replacing v_{a2} and v_{a0} in [49] by their values $-I_{a0}z_0$ and $-I_{a2}z_2$ from [42] and [43], and substituting I_{a1} for I_{a2} and I_{a0},

$$v_{a1} = V_{a1} - V_{a1}' = -(v_{a2} + v_{a0}) = I_{a0}z_0 + I_{a2}z_2 = I_{a1}(z_0 + z_2) \qquad [50]$$

From [50], the positive-sequence series voltage drop v_{a1} across the opening will be obtained if the impedance $(z_0 + z_2)$ is inserted in the opening in the positive-sequence network. This impedance, which is the negative- and zero-sequence series impedances connected in series, is the equivalent circuit which replaces the opening in the positive-sequence network.

Figure 14(d) shows the connection of the sequence networks of Fig. 2 with two conductors open at B for solution on the a-c network analyzer. This connection satisfies [48] and [49]. If the zero-sequence series impedance viewed from the opening is infinite, the impedance across the opening will be infinite, and no currents will flow between P and Q when there are two open conductors.

Since one or two open conductors introduce an impedance in the positive-sequence network, the effect of the fault is to increase the transfer impedances between machines on opposite sides of the opening and therefore to decrease the power which can be transferred between them for given internal voltages with constant angular displacement. At the first instant, the rotors of the machines do not change their relative angular positions. Initial symmetrical rms currents and voltages of fundamental frequency can therefore be determined by using subtransient machine reactances in the positive-sequence network and the voltages behind these reactances corresponding to the operating conditions previous to the occurrence of the fault. When an analytic solution is made, the positive-sequence currents due to the internal voltage of each machine can be determined separately; then, by superposition, the total positive-sequence currents will be the sum of the several separate currents. From the positive-sequence line currents at P and Q (see Figs. 14(a) and (b)) the negative- and zero-sequence line currents at P and Q are obtained from [44] and [45] or from [48].

When an a-c calculating table is available, the negative- and zero-sequence series systems are connected in parallel for one open conductor and in series for two open conductors and inserted in the positive-sequence network at the point where the disturbance occurs, as in Figs. 14(c) and (d). The sequence line currents for phase a are then read directly.

When machines are operating at no load, all internal voltages are equal and in phase and no currents will flow. With the d-c calculating table, internal voltages are assumed in phase and of unit or 100% magnitude. It cannot therefore be used in the usual manner to determine the currents with one or two open conductors. But it can be used if the internal voltages in the machines are applied one at a time and the currents recorded, the currents so determined to be multiplied by the per unit vector values of the corresponding applied internal voltages and added vectorially in each of the sequence networks.

Following the opening of a conductor or two conductors, the machines will change their relative angular positions, depending upon their inertias and the difference between their mechanical inputs and electrical outputs. For example, if a generator is supplying a motor through a transmission circuit and one conductor is opened, the power received by the motor becomes less, and it will fall back in phase relative to the generator. If there is a phase displacement between generator and motor internal voltages at which the electrical input to the motor equals its mechanical output under sustained conditions (with steady-state impedances and the voltages behind them), the motor will remain in synchronism with the generator, provided it did not fall out of step during the transient disturbance. (See Problem 3.)

Single-Phase Switching. In the case of single-phase switching, a faulted conductor is cut out of service by opening it at both ends. If capacitance is negligible, there is no current in an open conductor, whether it is open at both ends or only at one point, and its effect upon the current and voltages of the other two conductors is the same. In fact, as far as the other two conductors are concerned, it would make no difference whether the open conductor were removed entirely or left in place.

The discussion given above for one and two open conductors can be applied to single-phase switching if capacitance is negligible and it is assumed that the conductor or conductors are open at one point only, chosen at some convenient location in the line. It must be remembered however, that the voltage along the open conductor itself would depend upon the location of the opening.

Problem 3. In the system shown in Fig. 2, with motor M disconnected and base power numerically equal to base kva, 40% of base system power is supplied to motor N at unity power factor and unit voltage on the line side of the transformer at C.

(*a*) One lead on the line side of the transformer at B is accidentally disconnected.

(*b*) Two leads are disconnected.

Assumin· .he motor N and generator G do not lose synchronism during the transient disturbance, will they remain in synchronism after the disturbance is over? The equivalent steady-state positive-sequence reactances of generator and motor are approximately 55% and 100%, respectively, based on system kva and voltage. The load on the motor is a constant power load.

Solution. Expressed in per unit, with the voltage at C as reference vector,

$$V_c = 1 + j0$$

$$I = 0.4 + j0$$

The internal voltages in generator and motor behind equivalent steady-state reactances, determined in per unit from Fig. 2 (*b*) with motor M disconnected and per unit positive-sequence generator and motor reactances of 0.55 and 1.00, respectively, are

$$E_g = 1 + (0.4)(j0.82) = 1 + j0.328 = 1.052\underline{/18.2°}$$
$$E_n = 1 - (0.4)(j1.10) = 1 - j0.440 = 1.097\overline{\backslash 23.8°}$$

The series negative- and zero-sequence impedances z_2 and z_0, determined from Figs. 2 (*c*) and (*d*) with motor *M* disconnected, are

$$z_2 = j(0.20 + 0.12 + 0.15 + 0.10 + 0.40) = j0.97$$
$$z_0 = j0.92$$

The equivalent impedances to replace one and two open conductors in the positive-sequence network are:

(*a*) With one open phase at *B*,

$$\frac{z_2 z_0}{z_2 + z_0} = j\frac{0.97 \times 0.92}{1.89} = j0.47$$

(*b*) With two open phases open at *B*,

$$z_2 + z_0 = j(0.97 + 0.92) = j1.89$$

During normal operation the transfer impedance (see Chapter I) in the positive-sequence network between internal voltages behind steady-state impedances in machines *G* and *N* is

$$Z_{gn} = j1.92$$

(*a*) With one phase open, the transfer impedance is

$$Z_{gn} = j(1.92 + 0.47) = j2.39$$

(*b*) With two phases open, it is

$$Z_{gn} = j(1.92 + 1.89) = j3.81$$

With the transfer impedance increased, the angle between the internal machine voltages must be increased to transfer the same power. With resistance neglected, there is no power loss in the negative- and zero-sequence networks and the power out of the generator and into the motor is determined from the positive-sequence network.

Let δ = angle between E_g and $E_n = \delta_g - \delta_n$; then, with E_n as reference vector,

$$E_n = 1.097\underline{/0°} = 1.097$$
$$E_g = 1.052\underline{/\delta} = 1.052\ (\cos\delta + j\sin\delta)$$

The current *I*, flowing from the generator into the motor, is

$$I = \frac{E_g - E_n}{jx_{gn}} = \frac{1.052\sin\delta}{x_{gn}} + j\frac{(1.097 - 1.052\cos\delta)}{x_{gn}}$$

The power component of *I* referred to E_n is

$$\frac{1.052\sin\delta}{x_{gn}}$$

The power delivered by the generator and received by the motor with no resistance in the circuit is

$$P = (1.097)\,\frac{1.052 \sin \delta}{x_{gn}} = \frac{1.154 \sin \delta}{x_{gn}}$$

The constant per unit power load on the motor is 0.4. To deliver this power under steady-state conditions:

(*a*) With one phase open at *B*,

$$P = 0.4 = \frac{1.154 \sin \delta}{2.39} = 0.483 \sin \delta$$

$$\delta = \sin^{-1} \frac{0.4}{0.483} = 56.0°$$

If the motor and generator do not lose synchronism during the transient disturbance they will remain in synchronism and the constant per unit power load of 0.4 will be delivered to the motor; the angle between internal voltages of generator and motor will be 56° with one phase open at *B*.

(*b*) With two phases open at *B*,

$$P = 0.4 = \frac{1.154 \sin \delta}{3.81} = 0.303 \sin \delta$$

$$\delta = \sin^{-1} \frac{0.4}{0.303} = \sin^{-1} 1.32$$

Sin δ cannot be 1.32; and therefore, assuming the machines did not lose synchronism during the transient disturbance, their rotors cannot take up relative positions such that the power supplied to the motor is equal to its mechanical load. The machines lose synchronism.

Problem 4. The given system is shown in Fig. 2(*a*) with sequence networks in Figs. 2(*b*), (*c*), and (*d*). Determine the currents flowing from the three phases into fault and the phase voltages to ground at the fault for (1) a line-to-ground fault at *A*, (2) a double line-to-ground fault at *C*, (3) a line-to-line fault at *D*.

Problem 5. Solve Problem 2 for a line-to-ground fault at *D*.

CHAPTER V

TWO COMPONENT NETWORKS FOR THREE-PHASE SYSTEMS

Equations for Voltages and Currents in a Three-Phase System. The equations for the phase voltages to ground and line currents in terms of the symmetrical components of voltage and current, respectively, of phase a developed in Chapter II are

$$V_a = V_{a1} + V_{a2} + V_{a0} = (V_{a1} + V_{a2}) + V_{a0} \qquad [1]$$

$$\begin{aligned} V_b &= a^2 V_{a1} + a V_{a2} + V_{a0} \\ &= -\tfrac{1}{2}(V_{a1} + V_{a2}) - j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) + V_{a0} \end{aligned} \qquad [2]$$

$$\begin{aligned} V_c &= a V_{a1} + a^2 V_{a2} + V_{a0} \\ &= -\tfrac{1}{2}(V_{a1} + V_{a2}) + j\frac{\sqrt{3}}{2}(V_{a1} - V_{a2}) + V_{a0} \end{aligned} \qquad [3]$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = (I_{a1} + I_{a2}) + I_{a0} \qquad [4]$$

$$\begin{aligned} I_b &= a^2 I_{a1} + a I_{a2} + I_{a0} \\ &= -\tfrac{1}{2}(I_{a1} + I_{a2}) - j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) + I_{a0} \end{aligned} \qquad [5]$$

$$\begin{aligned} I_c &= a I_{a1} + a^2 I_{a2} + I_{a0} \\ &= -\tfrac{1}{2}(I_{a1} + I_{a2}) + j\frac{\sqrt{3}}{2}(I_{a1} - I_{a2}) + I_{a0} \end{aligned} \qquad [6]$$

where V and I refer to voltage and current, subscripts a, b, and c refer to the three phases a, b, and c, and 0, 1, 2, to zero, positive, and negative sequence, respectively. The operator $a = \left(-\tfrac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 1\underline{/120°}$; $a^2 = \left(-\tfrac{1}{2} - j\frac{\sqrt{3}}{2}\right) = 1\overline{/120°}$.

In [1]–[6] phase voltages to ground and line currents are expressed in terms of the symmetrical components of phase a, selected as reference phase, and also in terms of positive-plus-negative, positive-minus-negative, and zero-sequence components* of phase a. Equations [1]–[6] provide a simple transition from symmetrical components to other related components and can be used to advantage when there is a single dissymmetry, such as an unsymmetrical fault, provided the positive- and negative-sequence impedances in the system can be assumed equal. This assumption, while never strictly true for rotating machines, in many cases leads to but slight error when the impedances involved are transmission lines and transformers in series with rotating machines.

POSITIVE-PLUS-NEGATIVE, POSITIVE-MINUS-NEGATIVE, AND ZERO-SEQUENCE COMPONENTS OF CURRENT AND VOLTAGE

The use of the sum and difference of the positive- and negative-sequence components of currents and voltages, together with zero-sequence components of current and voltage, will be illustrated for the case of unsymmetrical short circuits on an otherwise symmetrical system in which the positive- and negative-sequence impedances are assumed equal; and the connections between the component networks to satisfy fault conditions will be determined for the three types of unsymmetrical short circuits.

Unsymmetrical Short Circuits. In a balanced three-phase power system, neither negative- nor zero-sequence voltages are generated. Assuming a system, balanced before the fault, let V_f represent the prefault voltage of phase a at the fault point F. Let I_a, I_b, and I_c be the currents from phases a, b, and c, respectively, flowing into the fault; and V_a, V_b, and V_c the voltages to ground of phases a, b, and c at the fault. If Z_1, Z_2, and Z_0 are the positive-, negative-, and zero-sequence impedances, respectively, viewed from the fault, then the positive-, negative-, and zero-sequence components of the voltage of phase a at the fault, with $Z_2 = Z_1$, will be

$$V_{a1} = V_f - I_{a1}Z_1 \qquad [7]$$

$$V_{a2} = \quad - I_{a2}Z_1 \qquad [8]$$

$$V_{a0} = \quad - I_{a0}Z_0 \qquad [9]$$

where positive direction for the symmetrical components of current is from the system into the fault.

Equations [7]–[9] are the same as [1]–[3] of Chapter IV except that Z_2 has been replaced by Z_1. V_f in [7] represents the prefault voltage of phase a at F for any given operating condition. If the system is

* Determination of Voltages and Currents during Unbalanced Faults, by Edith Clarke, *Gen. Elec. Rev.*, November, 1937, pp. 511–513.

operating at no load, neglecting charging current, the prefault voltage V_f in per cent will be the internal generated voltage E_a of phase a, which is the same in all synchronous machines of the system. Under load, V_f is determined by the internal generated voltages and the load distribution in the system corresponding to the given operating condition.

Adding equations [7] and [8], and subtracting [8] from [7],

$$(V_{a1} + V_{a2}) = V_f - (I_{a1} + I_{a2})Z_1 \qquad [10]$$

$$(V_{a1} - V_{a2}) = V_f - (I_{a1} - I_{a2})Z_1 \qquad [11]$$

Equations [10] and [11] express the positive-plus-negative and positive-minus-negative components of voltage at the fault in terms of their respective components of fault current, prefault voltage, and impedance viewed from the fault. V_f is a positive-sequence voltage: as no negative-sequence voltages are generated, both positive-plus-negative and positive-minus-negative *generated voltages* are positive-sequence voltages. From [10] and [11], the impedance met by both $(I_{a1} + I_{a2})$ and $(I_{a1} - I_{a2})$ is Z_1, a positive-sequence impedance: in a system with equal positive- and negative-sequence impedances, the impedances met by currents consisting of positive- and negative-sequence components are positive-sequence impedances. With the same generated voltages and the same impedances, the positive-plus-negative network and the positive-minus-negative network are identical; and each is the same as the positive-sequence network. The positive-sequence network can therefore be used to determine both the positive-plus-negative and the positive-minus-negative components of current and voltage, provided the proper connections are made at the fault point. The positive-sequence network, representing both the positive-plus-negative and the positive-minus-negative networks, will be used twice with different connections at the fault. The use of the sum and difference of the positive- and negative-sequence components, instead of the positive- and negative-sequence components themselves, is of special advantage when a short-circuit study is made on a d-c calculating table or an a-c network analyzer, because it is necessary to set up only the positive- and zero-sequence networks, and numerical calculations do not require the use of the operator a. This allows a larger system to be set up on a given calculating table and simplifies numerical work.

Line-to-Ground Fault. Conditions at the fault (fault on phase a):

$$V_a = 0 \qquad [12]$$

$$I_b = I_c = 0 \qquad [13]$$

Substituting [12] in [1] and replacing I_b and I_c in [5] and [6] by zero, the following relations between the components at the fault are obtained:

$$(V_{a1} + V_{a2}) = -V_{a0} \tag{14}$$

$$(I_{a1} + I_{a2}) = 2I_{a0} \tag{15}$$

$$(I_{a1} - I_{a2}) = 0 \tag{16}$$

Substituting [16] in [11],

$$V_{a1} - V_{a2} = V_f \tag{17}$$

Replacing I_{a0} in [9] by $\frac{1}{2}(I_{a1} + I_{a2})$ from [15], then substituting [9] in [14],

$$(V_{a1} + V_{a2}) = -V_{a0} = (I_{a1} + I_{a2})\frac{Z_0}{2} \tag{18}$$

Equating $(V_{a1} + V_{a2})$ in [18] and [10] and solving for $(I_{a1} + I_{a2})$,

$$(I_{a1} + I_{a2}) = 2I_{a0} = \frac{V_f}{Z_1 + \dfrac{Z_0}{2}} = \frac{2V_f}{2Z_1 + Z_0} \tag{19}$$

Phase Currents and Voltages at the Fault. From the equation above, and [1]–[6], the currents flowing from the three phases into the fault and the line-to-ground voltages at the fault are

$$\begin{aligned} I_a &= 3I_{a0} = \frac{3V_f}{2Z_1 + Z_0} \\ I_b &= I_c = 0; \quad V_a = 0 \\ V_b &= \tfrac{3}{2}V_{a0} - j\frac{\sqrt{3}}{2}V_f = V_f\left[-\frac{3}{2}\left(\frac{Z_0}{2Z_1 + Z_0}\right) - j\frac{\sqrt{3}}{2}\right] \\ V_c &= \tfrac{3}{2}V_{a0} + j\frac{\sqrt{3}}{2}V_f = V_f\left[-\frac{3}{2}\left(\frac{Z_0}{2Z_1 + Z_0}\right) + j\frac{\sqrt{3}}{2}\right] \end{aligned} \tag{20}$$

where V_f is the prefault line-to-neutral voltage of phase a at the fault point F.

Equivalent Circuit for Line-to-Ground Fault. Dividing [18] by $(I_{a1} + I_{a2}) = 2I_{a0}$, and replacing V_{a0} by $-I_{a0}Z_0$ from [9],

$$\frac{(V_{a1} + V_{a2})}{(I_{a1} + I_{a2})} = \frac{-V_{a0}}{2I_{a0}} = \frac{Z_0}{2} \tag{21}$$

In [21], $(V_{a1} + V_{a2})$ is the voltage at the point of fault in the positive-plus-negative network, and $(I_{a1} + I_{a2})$ is the current flowing from this network into the fault. The impedance therefore to be placed between the fault point and the zero-potential bus for the network to give the correct ratio of $(V_{a1} + V_{a2})$ to $(I_{a1} + I_{a2})$ is $Z_0/2$. Instead of the lumped impedance $Z_0/2$, the zero-sequence network with all zero-sequence impedances divided by two can be used as the equivalent circuit. The current flowing into the equivalent circuit is $(I_{a1} + I_{a2})$, which from [15] is $2I_{a0}$. Twice zero-sequence current flowing through one-half zero-sequence impedance produces zero-sequence voltage.

If all zero-sequence impedances are divided by two, and the network then connected in series with the positive-plus-negative network (which

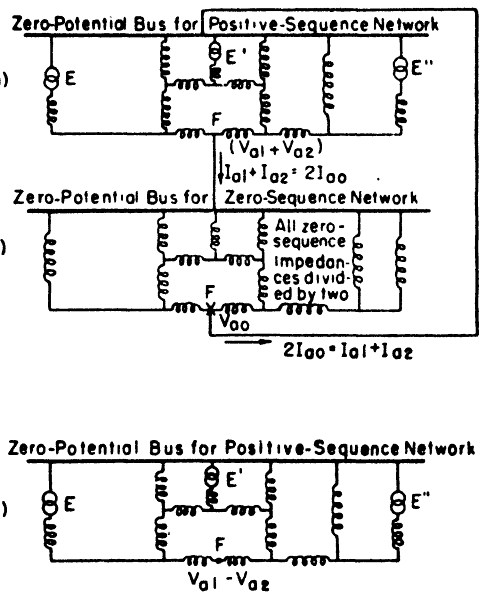


Fig. 1 Equivalent circuit for line-to-ground fault. (*a*) Positive-plus-negative network. (*b*) Zero-sequence network with all impedances divided by two. (*c*) Positive-minus-negative network. The positive-sequence network is used for both (*a*) and (*c*).

is also the positive-sequence network) as in Fig. 1, an equivalent circuit is obtained which satisfies [18] and [19] at the fault. Part (*a*) of Fig. 1 gives positive-plus-negative components of current and voltage. The zero-sequence voltages and twice the zero-sequence currents are given by part (*b*) of Fig. 1. When an a-c network analyzer is used, positive-plus-negative currents and voltages, and zero-sequence voltages and twice zero-sequence currents, can be read directly from parts (*a*) and (*b*), respectively, of Fig. 1 at any point P in the system. If the three-phase system was operating under load before the fault, and the generated voltages were adjusted to give the operating conditions which existed before the fault, the effect of load at any point in the system is included in part (*a*) of Fig. 1.

From [16] and [17], $(I_{a1} - I_{a2}) = 0$ and $(V_{a1} - V_{a2}) = V_f$. There is no current flowing into the fault from the positive-minus-negative network, and no change in voltage at the fault; consequently, there are no currents and no voltage drops in the network resulting from the fault. The positive-minus-negative network, which is unaffected by the line-to-ground fault, is shown in part (*c*) of Fig. 1. It is the positive-sequence network before the fault occurred. If load currents are considered, there will be currents in the network and voltage drops due to these currents. Load currents under normal operation are positive-sequence currents and cause positive-sequence voltage drops. They are determined from the positive-sequence network before the fault occurs, with generated voltages adjusted to give the operating conditions which existed before the fault.

At any point P in the system, the positive-minus-negative components of current and voltage are (both before and after the fault)

$$(I_{a1} - I_{a2}) \text{ at } P = I_a \text{ at } P \text{ before the fault} \tag{22}$$

$$(V_{a1} - V_{a2}) \text{ at } P = V_f \text{ at } P \text{ before the fault} \tag{23}$$

If load currents are neglected and there is no capacitance,

$$(I_{a1} - I_{a2}) \text{ at } P = 0 \tag{24}$$

Substituting zero-sequence components and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]–[6], phase currents and voltages at any point P in the system are obtained.

Line-to-Line Fault. Conditions at the fault (fault on phases b and c):

$$I_a = 0 \tag{25}$$

$$I_b = -I_c \tag{26}$$

$$V_b = V_c \tag{27}$$

From [25]–[27] and [2]–[6], the relations between the components at the fault are

$$I_{a0} = 0 \tag{28}$$

$$(I_{a1} + I_{a2}) = 0 \tag{29}$$

$$(V_{a1} - V_{a2}) = 0 \tag{30}$$

Substituting [28] in [9], [29] in [10], and [30] in [11].

$$V_{a0} = 0 \qquad [31]$$

$$(V_{a1} + V_{a2}) = V_f \qquad [32]$$

$$(I_{a1} - I_{a2}) = \frac{V_f}{Z_1} \qquad [33]$$

Phase Currents and Voltages at the Fault. Substituting [28]–[33] in [1]–[6], the phase currents flowing into the fault and the phase voltages to ground at the fault are

$$\begin{aligned} I_a &= 0 \\ I_b &= -j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) \\ I_c &= j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) \\ V_a &= V_f \\ V_b &= V_c = -\tfrac{1}{2}V_f \end{aligned} \qquad [34]$$

Equivalent Circuit for Line-to-Line Fault. From [28] and [31], $I_{a0} = 0$ and $V_{a0} = 0$; therefore the zero-sequence network is not involved. From [30] and [33], $V_{a1} - V_{a2} = 0$ and $(I_{a1} - I_{a2}) = V_f/Z_1$. These two equations are satisfied if in the positive-minus-negative network (which is also the positive-sequence network) the fault point F is shorted to the zero-potential bus as in Fig. 2, part (a). It will be noted that with a three-phase fault, $V_{a1} = 0$ and $I_{a1} = V_f/Z_1$. The positive-minus-negative components of current and voltage for a line-to-line fault between phases b and c with phase a as reference phase are just the same as those of phase a for the three-phase short circuit. Positive-minus-negative currents and volt-

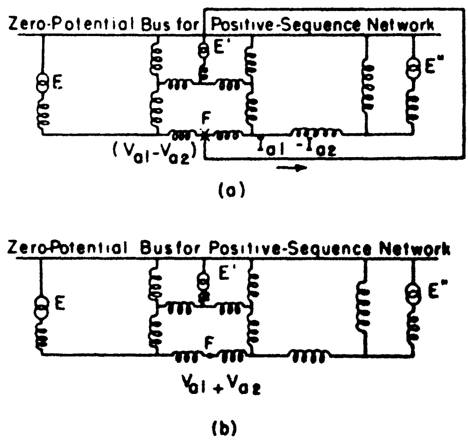


FIG. 2. Equivalent circuit for line-to-line fault. (a) Positive-minus-negative network. (b) Positive-plus-negative network. The positive-sequence network is used for both (a) and (b).

ages at any point in the system can be calculated from Fig. 2(a), or read directly if a calculating table is used.

From [29] and [32], $(I_{a1} + I_{a2}) = 0$ and $V_{a1} + V_{a2} = V_f$ at the fault. The positive-plus-negative network (which is also the positive-sequence network) is unaffected by the line-to-line fault. This is shown in part (b) of Fig. 2. At any point P in the system, the positive-plus-negative components of currents and voltages are

$$(I_{a1} + I_{a2}) \text{ at } P = I_a \text{ at } P \text{ before the fault} \quad [35]$$

$$(V_{a1} + V_{a2}) = V_a \text{ at } P \text{ before the fault} \quad [36]$$

If loads are neglected and there is no capacitance,

$$(I_{a1} + I_{a2}) \text{ at } P = 0 \quad [37]$$

Substituting $V_{a0} = 0$, $I_{a0} = 0$, and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]-[6], phase currents and voltages at any desired point P of the system are obtained.

Double Line-to-Ground Fault. Conditions at the fault (fault on phases b and c):

$$I_a = 0 \quad [38]$$

$$V_b = V_c = 0 \quad [39]$$

Substituting [38] in [4], and [39] in [2] and [3], the following relations between the components at the fault are obtained:

$$(I_{a1} + I_{a2}) = -I_{a0} \quad [40]$$

$$(V_{a1} + V_{a2}) = 2V_{a0} \quad [41]$$

$$(V_{a1} - V_{a2}) = 0 \quad [42]$$

Substituting [42] in [11],

$$(I_{a1} - I_{a2}) = \frac{V_f}{Z_1} \quad [43]$$

Replacing $-I_{a0}$ in [9] by $(I_{a1} + I_{a2})$ given in [40], and substituting [9] in [41],

$$(V_{a1} + V_{a2}) = 2V_{a0} = -I_{a0}(2Z_0) = (I_{a1} + I_{a2})(2Z_0) \quad [44]$$

Solving [10] and [44] for $(I_{a1} + I_{a2})$,

$$(I_{a1} + I_{a2}) = -I_{a0} = \frac{V_f}{Z_1 + 2Z_0} \quad [45]$$

Phase Currents and Voltages at the Fault. From the above equation and [1]–[6], the line currents flowing into the fault and the voltages to ground at the fault are

$$
\begin{aligned}
I_a &= 0 \\
I_b &= \tfrac{3}{2}I_{a0} - j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) = V_f\left[-\frac{\tfrac{3}{2}}{Z_1 + 2Z_0} - j\frac{\sqrt{3}}{2Z_1}\right] \\
I_c &= \tfrac{3}{2}I_{a0} + j\frac{\sqrt{3}}{2}\left(\frac{V_f}{Z_1}\right) = V_f\left[-\frac{\tfrac{3}{2}}{Z_1 + 2Z_0} + j\frac{\sqrt{3}}{2Z_1}\right] \qquad [46] \\
V_a &= 3V_{a0} = V_f\left(\frac{3Z_0}{Z_1 + 2Z_0}\right) \\
V_b &= V_c = 0
\end{aligned}
$$

Equivalent Circuit for Double Line-to-Ground Fault. Dividing [44] by $(I_{a1} + I_{a2}) = -I_{a0}$, and replacing V_{a0} by $-I_{a0}Z_0$ from [9],

$$\frac{(V_{a1} + V_{a2})}{(I_{a1} + I_{a2})} = \frac{2V_{a0}}{-I_{a0}} = 2Z_0 \qquad [47]$$

From [47], the equivalent circuit to be placed between the fault point and the zero-potential bus of the positive-plus-negative network to give the correct ratio of $(V_{a1} + V_{a2})$ to $(I_{a1} + I_{a2})$ is $2Z_0$. The zero-sequence network with all impedances multiplied by two can be used as the equivalent circuit. The current $(I_{a1} + I_{a2})$ flowing into the equivalent circuit from [40] is $-I_{a0}$. Zero-sequence current flowing through twice zero-sequence impedance produces twice zero-sequence voltage. The (arbitrary) positive direction for phase current and their components is from the network into the fault; therefore the zero-sequence network must be connected so that $(I_{a1} + I_{a2})$ flowing from the positive-plus-negative network into the fault, traverses the zero-sequence network in the negative direction.

If all zero-sequence impedances are multiplied by two and the network then connected in series with the positive-plus-negative network as in Figs. 3(a) and (b), an equivalent circuit is obtained which satisfies [40], [41], and [47] at the fault. If a calculating table is used, at any point P of the system, the positive-plus-negative components of current and voltage and twice the zero-sequence voltage and the zero-sequence current can be read directly from parts (a) and (b), respectively, of Fig. 3.

From [42] and [43], $(V_{a1} - V_{a2}) = 0$ and $(I_{a1} - I_{a2}) = V_f/Z_1$. This condition is satisfied if the fault point in the positive-minus-negative network is shorted to the zero-potential bus of the network, as in Fig. 3(*c*). Figure 3(*c*) is just the same as Fig. 2(*a*). Positive-minus-negative currents and voltages at any point *P* of the system can be calculated from Fig. 3(*c*), or read directly if a calculating table is used.

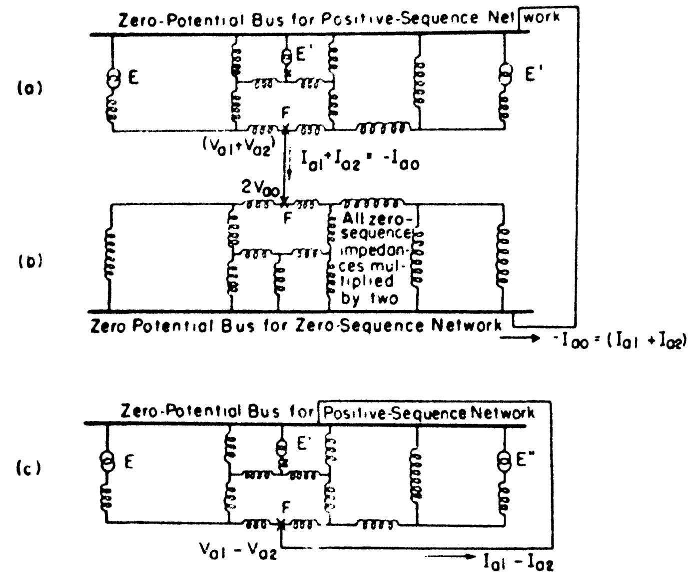


FIG. 3. Equivalent circuit for double line-to-ground fault. (*a*) Positive-plus-negative network. (*b*) Zero-sequence network with all impedances multiplied by two. (*c*) Positive-minus-negative network. The positive-sequence network is used for both (*a*) and (*c*).

Substituting zero-sequence components and the sum and difference of the positive- and negative-sequence components of current and voltage in [1]-[6], phase currents and voltages at any point *P* in the system are obtained.

Average Power during Unsymmetrical Faults. The average power at any point in a system in per unit of three-phase base power (see Chapter II) is

$$\begin{aligned} P &= V_{a1} \cdot I_{a1} + V_{a2} \cdot I_{a2} + V_{a0} \cdot I_{a0} \\ &= \tfrac{1}{2}[(V_{a1} + V_{a2}) \cdot (I_{a1} + I_{a2}) + (V_{a1} - V_{a2}) \cdot (I_{a1} - I_{a2})] \\ &\qquad + V_{a0} \cdot I_{a0} \end{aligned} \quad [48]$$

From [48], it follows that the average power at any given point in a three-phase system is the zero-sequence power at the point plus one-half the sum of the power in the positive-plus-negative and the positive-minus-negative networks at the given point.

Problem 1. Determine the connection of the positive-plus-negative and zero-sequence networks for one and two open conductors in a symmetrical system with equal positive- and negative-sequence impedances. Is the positive-minus-negative network affected by an open conductor in phase *a*? by open conductors in phases *b* and *c*? (*Note.* This problem is solved in Chapter X by α, β, and 0 components, but positive-plus-negative, positive-minus-negative, and zero-sequence components can be used as well.)

Problem 2. Check the phase currents and voltages at the fault given by equations [20], [34], and [46] with those given in Table I of Chapter IV when $Z_1 = Z_2$ and the fault impedance $Z_f = 0$.

CHAPTER VI

TRANSMISSION CIRCUITS WITH DISTRIBUTED CONSTANTS

In system studies in which transmission circuits are of short or moderate length, capacitance can frequently be neglected without appreciable error. This is usually the case in short-circuit calculations in which currents and voltages of fundamental frequency only are to be determined. On the other hand, there are certain problems in which even small capacitances must be taken into consideration. In the systems discussed in preceding chapters, capacitance has not been included. The connections of the sequence networks to represent unsymmetrical faults will be the same with capacitance included as with it neglected, but the equivalent circuits which replace an actual circuit in the sequence networks will be different in the two cases.

EQUIVALENT CIRCUITS FOR SYMMETRICAL TRANSMISSION CIRCUITS WITH DISTRIBUTED CONSTANTS

Let Fig. 1(*a*) indicate one phase of a symmetrical single-phase or three-phase transmission circuit, with or without ground wires. The terminals are S and R. S is the sending end under normal operation, and R the receiving end. The equivalent circuit which replaces the transmission line between the points S and R in any one of the sequence networks is a three-terminal circuit. One terminal is at S, one at R, and one is connected to zero potential for the network. In the positive- and negative-sequence networks all neutral points are at zero potential. In the zero-sequence network of a grounded system, the ground at any given point is the zero potential from which voltages at that point are measured.

A three-terminal circuit can be replaced by an equivalent Y or an equivalent Δ for calculating conditions at its terminals. (See Chapter I.) The equivalent circuits for transmission circuits, developed[1] by Dr. A. E. Kennelly, are called T-lines and Π-lines. The T-line is an equivalent Y, and the Π-line an equivalent Δ. Positive- and negative-sequence self-impedances and admittances of transmission lines are equal; therefore the positive- and negative-sequence equivalent circuits are the same. Zero-sequence equivalent circuits for the transmission line are similar to the positive, but, since zero-sequence impedances and admittances differ from those of positive sequence, the branches of the equivalent T or Π of the zero-sequence equivalent circuit will differ from those of the positive. Let

$$Z = \ell z = \ell(r + jx) = \ell(r + j2\pi fL)$$
$$= \text{total series impedance per phase}$$

$$Y = \ell y = \ell(g + jb) = \ell(g + j2\pi fC)$$
$$= \text{total shunt admittance per phase}$$

where ℓ is length of line in miles; r, L, C, and g are resistance, inductance, capacitance, and leakance, respectively, per mile; f is frequency

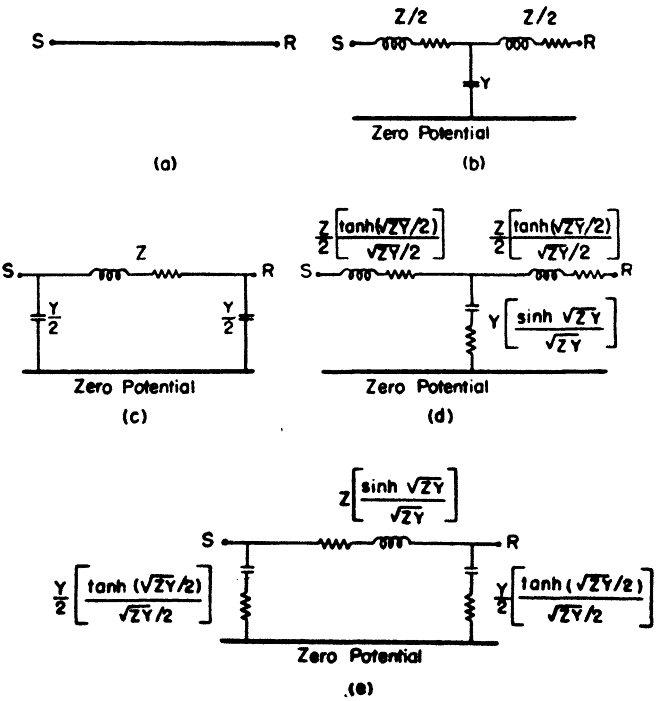


FIG. 1. (*a*) One-line diagram of single-phase or three-phase transmission circuit, with or without ground wires. (*b*) Nominal T. (*c*) Nominal Π. (*d*) Equivalent T. (*e*) Equivalent Π.

in cycles per second. C is in farads per mile; L is in henries per mile; r, x, and z are in ohms per mile; g, b, and y are in mhos per mile. The subscripts 1, 2, and 0 with the above symbols will be used to indicate positive, negative, and zero sequence, respectively.

Methods for determining the sequence constants per unit length of transmission circuits are given in later chapters. For the present, let it be assumed that these constants are known or can be readily determined. In the usual overhead transmission line during normal operation there is no corona, and leakance across insulators is negligible; therefore $g = 0$ and $Y = j\ell b = j2\pi f\ell C$, where $b = 2\pi fC$ = capacitive susceptance in mhos per mile.

Nominal T or Π. As a first approximation, the transmission line can be represented by a nominal T or Π as shown in Figs. 1(*b*) and (*c*). For the nominal T as shown in Fig. 1(*b*), one-half the total series impedance Z is placed in each arm of the T and the total shunt admittance Y in the staff. For the nominal Π as shown in Fig. 1(*c*), the total series impedance Z is placed in the architrave of the Π and one-half the total shunt admittance Y in each of the pillars of the Π.

Equivalent T or Π. The equivalent T or Π can be obtained from the nominal T or Π by applying[1] correcting factors as indicated in Figs. 1(*d*) and (*e*). The correcting factors are hyperbolic functions of $\sqrt{ZY}$ which depend upon frequency, length of line, and line constants. In addition to the correcting factors indicated in Figs. 1(*d*) and (*e*), $\cosh\sqrt{ZY}$ is also used in transmission calculations. $\cosh\sqrt{ZY}$ and the correcting factors $\dfrac{\sinh\sqrt{ZY}}{\sqrt{ZY}}$ and $\dfrac{\tanh\frac{\sqrt{ZY}}{2}}{\frac{\sqrt{ZY}}{2}}$ are complex numbers which may be calculated or read from charts[2,3,4]. The calculation of complex hyperbolic functions by the use of mathematical tables of hyperbolic and circular functions is discussed later and illustrated in Problem 3. An alternate method is by substitution of Z and Y in following series.*

$$\cosh\sqrt{ZY} = 1 + \frac{ZY}{2} + \frac{Z^2Y^2}{24} + \frac{Z^3Y^3}{720} + \cdots \qquad [1]$$

$$\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = 1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5040} + \cdots \qquad [2]$$

$$\frac{\tanh\frac{\sqrt{ZY}}{2}}{\frac{\sqrt{ZY}}{2}} = 1 - \frac{ZY}{12} + \frac{Z^2Y^2}{120} - \frac{17Z^3Y^3}{20{,}160} + \cdots \qquad [3]$$

* *A Short Table of Integrals*, by B. O. Peirce, equations 790–792, Ginn and Company.

Length of line, frequency, and required degree of precision determine the number of terms which must be retained in the above series. In many problems, only the first two terms need be used. If more than three terms are required, the method illustrated in Problem 3 will be found less laborious.

Hyperbolic Functions from Charts. Figures 2(*a*) and (*b*) of this chapter, taken from reference 3, give the real and imaginary parts of the hyperbolic functions of $\sqrt{ZY}$ listed in [1]–[3]. Let

$$\cosh \sqrt{ZY} = A = |A| \underline{/\phi_a} = a_1 + ja_2 \qquad [4]$$

$$\frac{\sinh \sqrt{ZY}}{\sqrt{ZY}} = \beta = |\beta| \underline{/\phi_\beta} = \beta_1 + j\beta_2 \qquad [5]$$

$$\frac{\tanh \dfrac{\sqrt{ZY}}{2}}{\dfrac{\sqrt{ZY}}{2}} = \gamma = |\gamma| \underline{/\phi_\gamma} = \gamma_1 + j\gamma_2 \qquad [6]$$

$|A|$, $|\beta|$, and $|\gamma|$ in [4]–[6] are the magnitudes of the hyperbolic functions when expressed in polar form; ϕ_a, ϕ_β, ϕ_γ are the corresponding angles. Expressed in complex form, a_1, β_1, γ_1 are the real parts and a_2, β_2, and γ_2 the corresponding imaginary parts. (The subscripts 1 and 2 have no connection here with positive and negative sequence.) The total series impedance Z can be written

$$Z = \ell(r + jx) = j\ell x\left(1 - j\frac{r}{x}\right) = j2\pi f\ell L\left(1 - j\frac{r}{x}\right) \qquad [7]$$

With leakance g neglected, the total shunt admittance Y becomes $Y = j\ell b = j2\pi f\ell C$. Then

$$\sqrt{ZY} = j2\pi f\ell\sqrt{LC}\sqrt{1 - j\frac{r}{x}} \qquad [8]$$

From [8], $\sqrt{ZY}$ and the hyperbolic functions of $\sqrt{ZY}$ in [4]–[6] are functions of:

(1) $f\ell$, the product of frequency in cycles per second and length of line in miles.

(2) r/x, the ratio of resistance to reactance.

(3) $\sqrt{LC}$, the square root of the product of inductance and capacitance.

The curves of Figs. 2(*a*) and (*b*) are based on a constant value of $LC = 30 \times 10^{-12}$, which is a representative or average value of the product of the positive-sequence inductance L and the positive-sequence capacitance C of conventional overhead lines, where L is in henries per mile and C in farads per mile. The parameter is r/x. To make the curves applicable to all values of the product LC, whether of positive or of zero sequence, an abscissa labeled $(f\ell)'$ is used which is the product of frequency in cycles per second, length of line in miles, and a factor determined as follows. Equation [8] may be rewritten

$$\sqrt{ZY} = j2\pi\sqrt{30 \times 10^{-12}}\sqrt{1 - j\frac{r}{x}}\left(f\ell\sqrt{\frac{LC}{30 \times 10^{-12}}}\right) \qquad [9]$$

$$= j2\pi\sqrt{30 \times 10^{-12}}\sqrt{1 - j\frac{r}{x}}\,(f\ell)' \qquad [10]$$

From [9] and [10]

$$(f\ell)' = f\ell\sqrt{\frac{LC}{30 \times 10^{-12}}} = f\ell\sqrt{\frac{\frac{x}{2\pi f}\,\frac{b}{2\pi f}}{30 \times 10^{-12}}} = f\ell\sqrt{\frac{xb10^6}{4.26}\left(\frac{60}{f}\right)^2} \qquad [11]$$

In terms of an equivalent 60-cycle circuit, in which $LC = 30 \times 10^{-12}$, or $xb = 4.26 \times 10^{-6}$,

$$(f\ell)' = 60\ell\sqrt{\frac{xb10^6}{4.26}} \qquad [12]$$

where $x = 2\pi fL$ and $b = 2\pi fC$, and f is the given frequency.

In Figs. 2(*a*) and (*b*), $(f\ell)'$ is abscissa. The real parts of the hyperbolic functions α_1, β_1, and γ_1 are given for several values of the parameter r/x, and may be read directly corresponding to calculated values of r/x and $(f\ell)'$; the imaginary parts α_2, β_2, and γ_2 are determined from $\frac{\alpha_2}{r/x}$, $\frac{\beta_2}{r/x}$, and $\frac{\gamma_2}{r/x}$, read directly corresponding to $(f\ell)'$, and multiplied by r/x.

Problem 1. Find A and the positive-sequence correcting factors β and γ for a three-phase, 60-cycle transmission circuit, 250 miles in length; $z_1 = 0.12 + j0.80$ ohm per mile; $y_1 = j5.1 \times 10^{-6}$ mho per mile.

Solution. $\frac{r}{x} = \frac{0.12}{0.80} = 0.15$; from [12],

$$(f\ell)' = 60 \times 250\sqrt{\frac{(0.8)(5.1)}{4.26}} = 15{,}000 \times 0.98 = 14{,}700$$

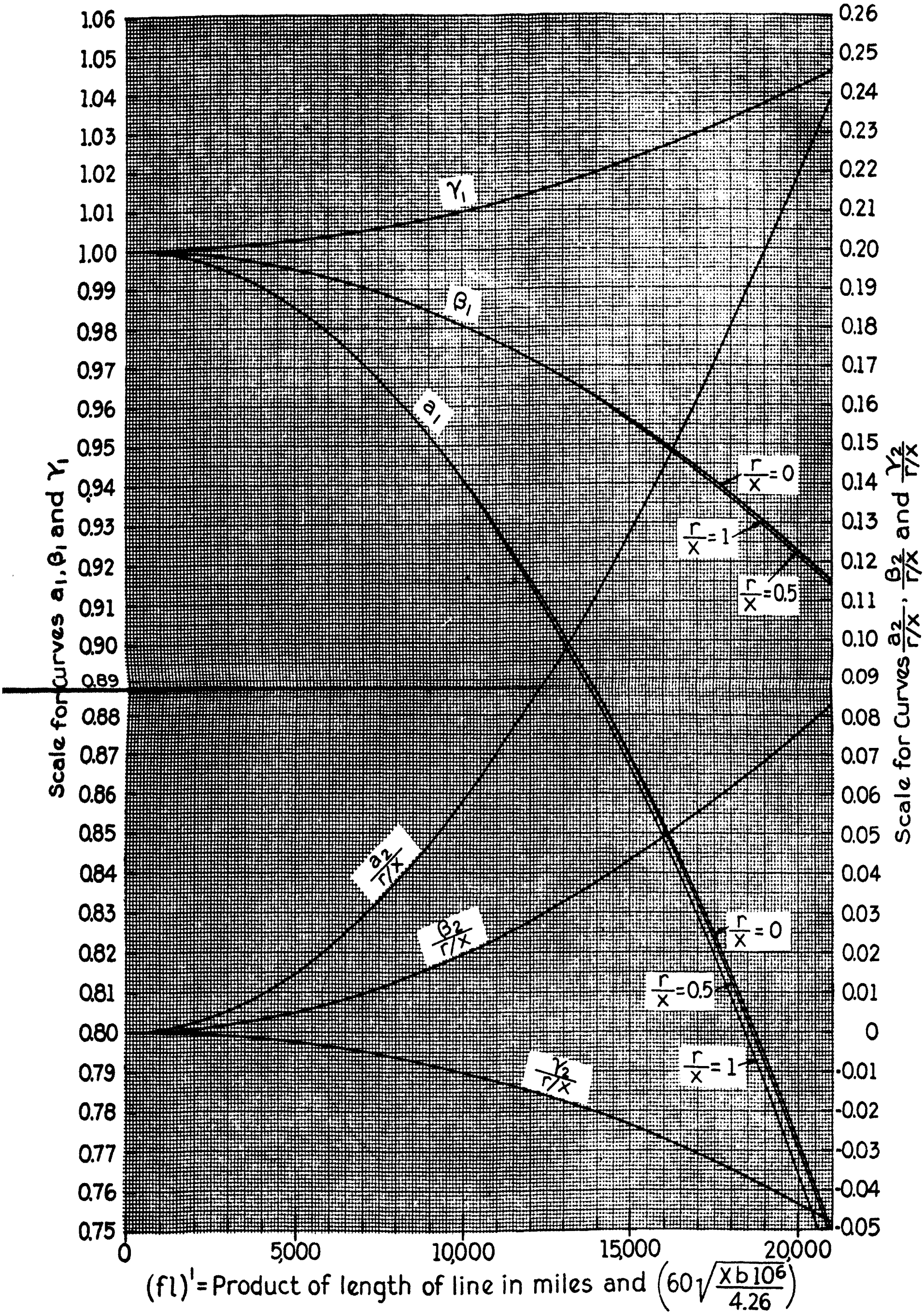


FIG. 2(*a*). Complex hyperbolic functions defined in [4]-[6]. x is inductive reactance in ohms per mile, and b is capacitive susceptance in mhos per mile, at the given frequency.

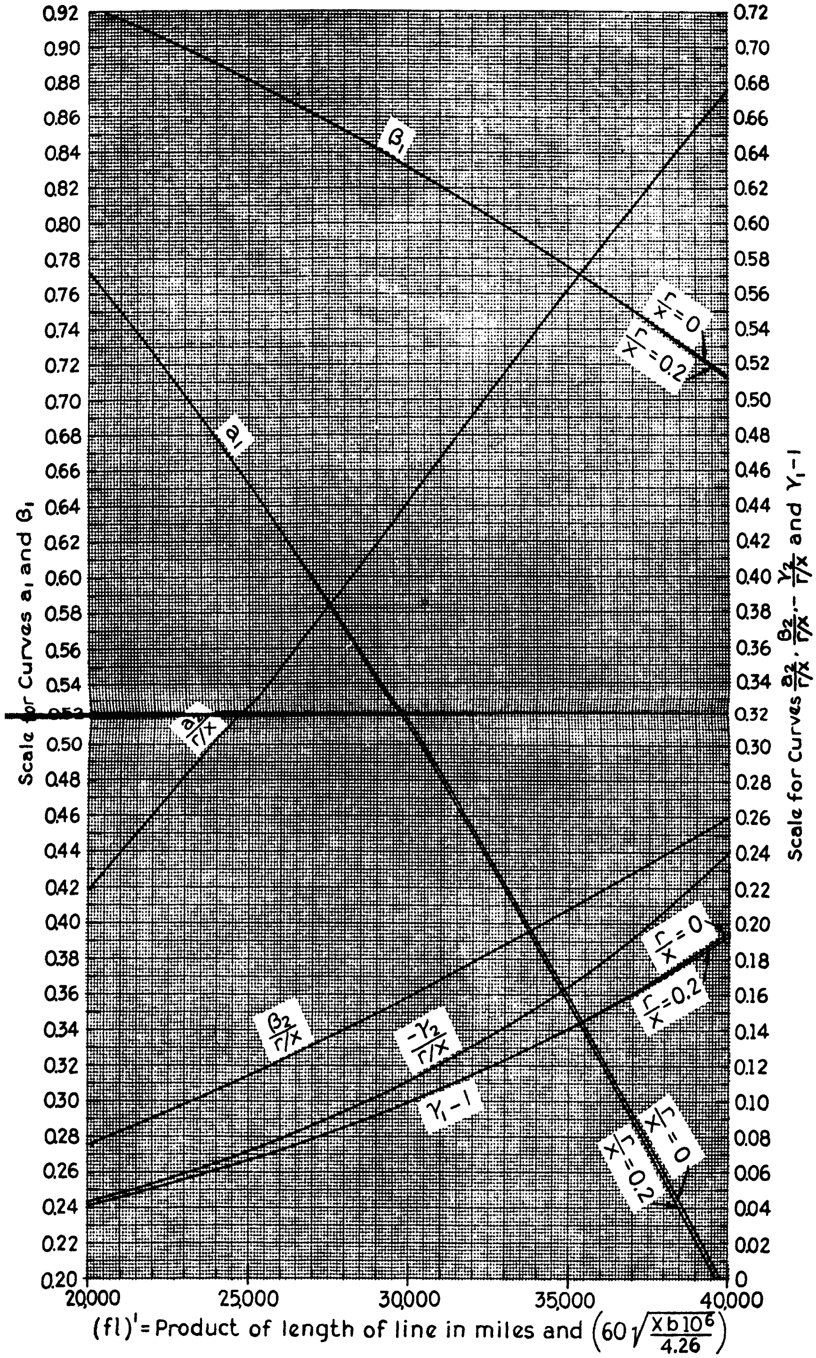


FIG. 2(*b*). Complex hyperbolic functions defined in [4]–[6]. *x* is inductive reactance in ohms per mile, and *b* is capacitive susceptance in mhos per mile, at the given frequency.

Read from Fig. 2(a) with $r/x = 0.15$ and $(f\ell)' = 14{,}700$,

$$A = a_1 + ja_2 = 0.875 + j(0.1220 \times 0.15) = 0.875 + j0.0183 = 0.875 \underline{/1.3^\circ}$$

$$\beta = \beta_1 + j\beta_2 = 0.958 + j(0.0415 \times 0.15) = 0.958 + j0.0062 = 0.958 \underline{/0.4^\circ}$$

$$\gamma = \gamma_1 + j\gamma_2 = 1.022 - j(0.0225 \times 0.15) = 1.022 - j0.0034 = 1.022 \overline{\backslash 0.2^\circ}$$

In this problem, $LC = 28.7 \times 10^{-12}$ for the given line — not 30×10^{-12}, the value upon which the curves of Figs. 2(a) and (b) are based. Yet the correcting factors β and γ for the positive-sequence network would have but slight error if read at the value of $f\ell$ ($= 15{,}000$) corresponding to the actual product of frequency and length of line. Except for very long lines, the correcting factors β and γ corresponding to the actual value of $f\ell$ are approximately correct for positive and negative sequence.

Zero-Sequence Equivalent Circuit. The curves of Figs. 2(a) and (b) can be used to determine the correcting factors β and γ to be used in the equivalent T or Π to replace the transmission circuit in the zero-sequence network. The correcting factors, as for the positive-sequence network, are functions of $\sqrt{ZY}$, where Z and Y are the total zero-sequence series impedance and shunt admittance, respectively. But, whereas for the positive-sequence network $LC = 30 \times 10^{-12}$ is a representative value, for the zero-sequence network this is not the case. The zero-sequence inductance of overhead transmission circuits varies approximately from two to four times the positive-sequence inductance, while the zero-sequence capacitance may be less than half or approximately 75% of the positive-sequence capacitance. However, if r/x is calculated as the ratio of the zero-sequence resistance to reactance, and $(f\ell)'$ is determined from [12], using zero-sequence inductance and capacitance, the curves of Figs. 2(a) and (b) are applicable.

Problem 2. Given a three-phase, 60-cycle transmission circuit, 250 miles in length; $z_0 = 0.40 + j2.2$ ohms per mile; $y_0 = j3.12 \times 10^{-6}$ mho per mile. Find the zero-sequence correcting factors β and γ from Fig. 2.

Solution. $\dfrac{r}{x} = \dfrac{0.40}{2.2} = 0.18$; from [12],

$$(f\ell)' = 60 \times 250 \sqrt{\frac{2.2 \times 3.12}{4.26}} = 19{,}000$$

Read from Fig. 2(a), with $r/x = 0.18$ and $(f\ell)' = 19{,}000$,

$$\beta = 0.930 + j(0.068 \times 0.18) = 0.930 + j0.012 = 0.930 \underline{/0.7^\circ}$$

$$\gamma = 1.037 - j(0.039 \times 0.18) = 1.037 - j0.007 = 1.037 \overline{\backslash 0.4^\circ}$$

In Problems 1 and 2, the imaginary parts β_2 and γ_2 of the correcting factors are small, resulting in small angles with β and γ in polar form. The error in neglecting β_2 and γ_2 in Fig. 2(*a*) is small. For the longer lines given by Fig. 2(*b*), the imaginary parts become larger relative to the real parts. For values of $(f\ell)'$ higher than those given in Fig. 2(*b*), or where greater precision than that obtainable from reading curves is desired, the correcting factors can be calculated either from the series given in [2] and [3] or from hyperbolic and circular functions. The latter method, illustrated in Problem 3, is simpler for very high values of $(f\ell)'$.

Cable Circuits. The curves of Figs. 2(*a*) and (*b*) may also be used to determine the correcting factors to be used in the equivalent T or Π to replace cable circuits in the sequence networks if leakance can be neglected. The procedure is then analogous to that used for the zero-sequence network illustrated in Problem 2. If leakance g cannot be neglected, the correcting factors can be calculated with $Y = \ell(g + j2\pi fC)$.

Capacitive Impedances versus Admittances in Equivalent Circuits. In analytic calculations, as contrasted with calculations on an a-c calculating table, it is convenient to replace capacitive admittances by capacitive impedances. In the nominal T of Fig. 1(*b*), $Y = j\ell b$ is replaced by $Z_c = -j\dfrac{x_c}{\ell}$; in the nominal Π line of Fig. 1(*c*), $\dfrac{Y}{2} = j\dfrac{\ell b}{2}$ is replaced by $2Z_c = -j\dfrac{2x_c}{\ell}\cdot$ In these equations $x_c = \dfrac{1}{b} = \dfrac{1}{2\pi fC}\cdot$ Z_c and $2Z_c$ are *divided* by the correcting factors β and γ, respectively, to obtain the equivalent T and Π of Figs. 1(*d*) and (*e*) with capacitive impedances replacing capacitive admittances.

Nominal versus Equivalent T or Π. There are many problems in which capacitance is appreciable but correcting factors need not be applied, the nominal T or Π being adequate. The degree of precision required in calculations will determine whether nominal or equivalent circuits should be used. A glance at Fig. 2 shows the error in neglecting positive-sequence correcting factors. With $f\ell = 12{,}000$ (corresponding to a 200-mile line at 60 cycles, a 240-mile line at 50 cycles, or a 480-mile line at 25 cycles), the error in neglecting β is approximately 3%, and in neglecting γ less than 1.5%. With $f\ell = 30{,}000$ (500-mile line at 60 cycles), the departure of β and γ from unity is pronounced: $\beta_1 = 0.832$; $\gamma_1 = 1.099$. Comparing β and γ in Problems 1 and 2, the error in neglecting correcting factors in the zero-sequence network is larger than in the positive for the same conventional transmission circuit.

Sometimes in analytic calculations, and more frequently in calculations on the a-c network analyzer, it is convenient to replace a long line by two or more equal sections of such length that each section can be represented by its nominal T or Π without correcting factors. This method of representation has the advantage of providing points along the line at which currents and voltages can be calculated or measured.

Equations for Currents and Voltages in Symmetrical Transmission Circuits with Uniformly Distributed Constants

The following equations[1] in which E_r and E_s represent voltages to neutral or to ground and I_r and I_s line currents at the circuit terminals R and S, respectively, positive direction for currents being from S to R, are applicable to voltages and currents in the positive-, negative-, and zero-sequence networks of symmetrical three-phase transmission circuits at constant frequency f.

$$E_s = E_r \cosh\theta + I_r \sqrt{\frac{z}{y}} \sinh\theta \tag{13}$$

$$I_s = I_r \cosh\theta + E_r \sqrt{\frac{y}{z}} \sinh\theta \tag{14}$$

$$E_r = E_s \cosh\theta - I_s \sqrt{\frac{z}{y}} \sinh\theta \tag{15}$$

$$I_r = I_s \cosh\theta - E_s \sqrt{\frac{y}{z}} \sinh\theta \tag{16}$$

In [13]–[16],

$\theta = l\alpha$ = hyperbolic angle of the line

$\alpha = \sqrt{zy}$ = normal attenuation constant per unit length of line

$Z_S = \sqrt{\frac{z}{y}} = \sqrt{\frac{r + jx}{g + jb}}$ = surge impedance to neutral or to ground

With mile as the unit of length,

$$\alpha = \sqrt{(r + jx)(g + jb)} = \alpha_1 + j\alpha_2$$
$$= \text{normal attenuation constant per mile} \tag{17}$$

Normal attenuation occurs when the impedance of the load is equal to the surge impedance. The real part α_1 determines the attenuation in magnitude of voltage and current per mile along the line, while α_2 determines the attenuation in phase. The distance in which the normal phase attenuation amounts to 360° or 2π radians is one wave length λ. With $\lambda\alpha_2 = 2\pi$,

$$\text{The wave length } \lambda = \frac{2\pi}{\alpha_2} \text{ miles}$$

$$\text{The apparent velocity of propagation } v = f\lambda$$

$$= \frac{2\pi f}{\alpha_2} \text{ miles per second} \qquad [18]$$

The surge impedance $Z_S = \sqrt{z/y}$ is independent of length of line. It may be written $\sqrt{z\ell/y\ell} = \sqrt{Z/Y} = Z/\sqrt{ZY}$, where $z\ell$ and $y\ell$ are replaced by Z and Y, the total impedance and admittance, respectively, per phase of ℓ miles of line. Likewise, the surge admittance $\sqrt{y/z}$ may be written $Y/\sqrt{ZY}$; and θ may be written $\sqrt{\ell z \ell y} = \sqrt{ZY}$. Making these substitutions, equations [13]-[16] become

$$E_s = E_r \cosh\sqrt{ZY} + I_r Z \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = E_r A + I_r B \qquad [19]$$

$$I_s = I_r \cosh\sqrt{ZY} + E_r Y \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = I_r D + E_r C \qquad [20]$$

$$E_r = E_s \cosh\sqrt{ZY} - I_s Z \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = E_s D - I_s B \qquad [21]$$

$$I_r = I_s \cosh\sqrt{ZY} - E_s Y \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = I_s A - E_s C \qquad [22]$$

The *circuit constants* A, B, C, D for symmetrical transmission circuits used in the construction of *circle diagrams*[5] are

$$A = D = \cosh\sqrt{ZY} = \cosh\theta = a_1 + ja_2$$

$$B = Z\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = Z\frac{\sinh\theta}{\theta} = Z(\beta_1 + j\beta_2) \qquad [23]$$

$$C = Y\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = Y\frac{\sinh\theta}{\theta} = Y(\beta_1 + j\beta_2)$$

The above equations on a per phase basis apply to either positive- (and negative-) or zero-sequence systems. As the positive- and zero-sequence line constants are different, their surge impedances, hyperbolic angles, wave lengths, and apparent velocities of propagation are different.

Third Method of Determining Complex Hyperbolic Functions. Expressed in complex form,

$$\theta = \sqrt{ZY} = \theta_1 + j\theta_2$$

With θ_1 and θ_2 in radians, the following equations* are applicable

$$\cosh \theta = \cosh \theta_1 \cos \theta_2 + j \sinh \theta_1 \sin \theta_2 \tag{24}$$

$$\sinh \theta = \sinh \theta_1 \cos \theta_2 + j \cosh \theta_1 \sin \theta_2 \tag{25}$$

$$\tanh \frac{\theta}{2} = \frac{\cosh \theta - 1}{\sinh \theta} = \frac{\sinh \theta}{\cosh \theta + 1} \tag{26}$$

The application of these equations is illustrated in the following problem, in which tables of circular† functions to hundredths of a degree, and of circular and hyperbolic‡ functions to thousandths of a radian, are used.

Problem 3. A three-phase line 65 miles long is used to transmit signals at 600 cycles. The positive-sequence impedance and admittance per mile at 600 cycles are $z = 0.8 + j8.0$ ohms and $y = 0 + j52 \times 10^{-6}$ mho, respectively. Calculate Z_S, α, θ, λ, v, and the hyperbolic functions to be used in equations [19]–[22] and in the correcting factors defined in [5] and [6].

Solution.

$$Z_S = \sqrt{\frac{z}{y}} = 10^3\sqrt{\frac{0.8 + j8.0}{j52}} = 10^3\sqrt{\frac{8.040 \,/84.294^\circ}{52 \,/90^\circ}} = 393 \,/\overline{2.85^\circ} \text{ ohms}$$

$$\alpha = \sqrt{zy} = 10^{-3}\sqrt{(0.8 + j8.0)(j52)} = 20.447(10^{-3}) \,/87.147^\circ$$
$$= 10^{-3}\,(1.0177 + j20.422)$$

$$\alpha_1 = 1.018 \times 10^{-3};\ \alpha_2 = 20.42 \times 10^{-3}$$

$$\theta = \theta_1 + j\theta_2 = l(\alpha_1 + j\alpha_2) = 0.06615 + j1.3274$$

$$\lambda = 308 \text{ miles};\ v = 185{,}000 \text{ miles per second}$$

The surge impedance of a conventional transmission line has a small negative angle which depends upon resistance. The magnitude of the surge impedance is but slightly affected by resistance and frequency; it is approximately equal to $\sqrt{L/C}$. The real part α_1 of the normal attenuation constant is influenced by resistance, but α_2 is approximately equal to $2\pi f\sqrt{LC}$, and v to $1/\sqrt{LC}$. From the Smithsonian Tables,

$$\cosh 0.06615 = 1.0022; \qquad \sinh 0.06615 = 0.06620$$
$$\cos 1.3274 = 0.2410; \qquad \sin 1.3274 = 0.9705$$

* *A Short Table of Integrals*, by B. O. Peirce, equations 660, 661, and 668, Ginn and Company.

† *Five Place Table of Natural Trigonometric Functions to Hundredths of a Degree*, by Amelia DeLella, John Wiley and Sons.

‡ *Smithsonian Mathematical Tables — Hyperbolic Functions*, prepared by George F. Becker and C. E. Van Orstrand, Washington, D. C., published by the Smithsonian Institution, 1909.

Substituting these values in [24] and [25],

$$A = \cosh\theta = \cosh\sqrt{ZY} = 0.2415 + j0.0642$$

$$\sinh\theta = \sinh\sqrt{ZY} = 0.01595 + j0.9726$$

$$\beta = \frac{\sinh\theta}{\theta} = \frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = 0.7320 + j0.0245$$

From [26], knowing $\cosh\theta$ and $\sinh\theta$,

$$\gamma = \frac{\tanh\frac{\theta}{2}}{\frac{\theta}{2}} = \frac{2\beta}{\cosh\theta + 1} = 1.178 - j0.0215$$

Read from Fig. 2 (*b*), with $r/x = 0.1$ and $(fl)' = 60 \times 65\sqrt{(8 \times 52)/4.26} = 38{,}500$

$$A = 0.241 + j0.0642$$

$$\beta = 0.732 + j0.0244$$

$$\gamma = 1.177 - j0.0215$$

Comparing the calculated correcting factors with those read from Fig. 2 (*b*), the differences are only those resulting from the scale used in plotting the curves. With the curves plotted to a larger scale, the agreement to any desired degree of precision can be obtained.

As a check on the calculation of Z_S and $\theta = \theta_1 + j\theta_2$ (and for those who prefer rectangular coordinates to polar) the following equations may be useful:

$$|z| = \sqrt{r^2 + x^2}$$

Let $\phi = \cos^{-1} x/|z|$; then*

$$\cos\frac{\phi}{2} = \sqrt{\frac{|z| + x}{2|z|}} \quad \text{and} \quad \sin\frac{\phi}{2} = \frac{r}{\sqrt{2|z|(|z| + x)}}$$

The surge impedance Z_S

$$= \sqrt{\frac{z}{y}} = \sqrt{\frac{|z|}{b}}\left(\cos\frac{\phi}{2} - j\sin\frac{\phi}{2}\right)$$

$$= \sqrt{\frac{|z| + x}{2b}} - j\frac{r}{\sqrt{2b(|z| + x)}} \qquad [27]$$

The hyperbolic angle

$$\theta = \theta_1 + j\theta_2 = l\sqrt{zy}$$

$$= l\sqrt{|z|b}\left(\sin\frac{\phi}{2} + j\cos\frac{\phi}{2}\right)$$

$$= l\left(r\sqrt{\frac{b}{2(|z| + x)}} + j\sqrt{\frac{b(|z| + x)}{2}}\right) \qquad [28]$$

* *A Short Table of Integrals*, by B. O. Peirce, equations 576–578, Ginn and Company.

Calculation of Currents and Voltages in Systems with Appreciable Capacitance

There is an important difference to be noted between the paralleling of two inductive impedances or two capacitive impedances and the paralleling of an inductive impedance and a capacitive impedance. For example, if $Z_a = j2$ and $Z_b = j8$, $Z_aZ_b/(Z_a + Z_b) = j1.6$. The impedance of $-j2$ and $-j8$ in parallel is $-j1.6$. But, if $Z_a = j2$ and $Z_b = -j8$, $Z_aZ_b/(Z_a + Z_b) = 16/(-j6) = j2.667$; if $Z_a = -j2$ and $Z_b = j8$, their parallel value is $-j2.667$. The equivalent impedance of an inductive and a capacitive impedance in parallel has the sign of the smaller of the two impedances and is larger in magnitude than the smaller. If $Z_a = j2$ and $Z_b = -j2$, their parallel value is infinite. The following problem illustrates the effects of capacitance upon system calculations.

Problem 4. Figure 3(*a*) shows a large three-phase 60-cycle power system, replaced for purposes of calculation by a single equivalent synchronous machine A. The positive sequence subtransient reactance of the large system viewed from F, based on 100,000 kva and 110 kv in the line, is 10%. The negative- and zero-sequence reactances are 10% and 5%, respectively. Power is supplied to a second power system over one 110 kv three-phase transmission circuit, 200 miles in length. The line constants are

$$z_1 = 0.235 + j0.79 \text{ ohm per mile; } b_1 = 5.35 \times 10^{-6} \text{ mho per mile}$$

$$z_0 = 0.515 + j2.65 \text{ ohms per mile; } b_0 = 3.20 \times 10^{-6} \text{ mho per mile}$$

Loads are supplied from the 110-kv line at various points through Δ-Δ transformer banks with synchronous condensers at some of the stations to regulate line voltages. At the time of a line-to-ground fault at F, the system was lightly loaded and only a 15,000-kva condenser at C was in operation. The voltage at F before the fault was 110 kv. The positive- and negative-sequence transient reactances of the condenser at C viewed from the line through the Δ-Δ transformer bank are approximately 30% based on its rating, or 200% based on 100,000 kva. The equivalent per unit excitation voltage E_a of the condenser (voltage behind transient reactance) is 0.85. The condenser is underexcited to prevent voltage rise at P. The sequence networks and their connection for a line-to-ground fault on phase a at F are shown in Fig. 3(*b*), with loads neglected. The negative-sequence network is the same as the positive with E_a and E_a' equated to zero. The zero-sequence network is open at P. The positive- and negative-sequence equivalent circuits for the transmission line are equivalent Π lines. The equivalent T-line is used in the zero-sequence network. The choice of a T or Π is arbitrary. For this particular problem, T-lines in all three networks would be preferable.

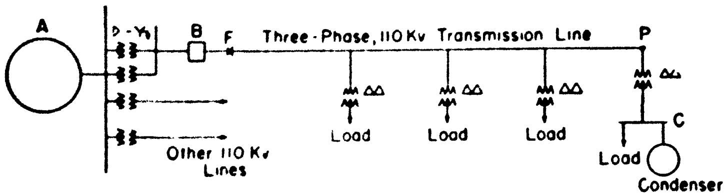
The positive- and negative-sequence equivalent Π lines shown in Fig. 3(*b*) with impedances in per unit are determined as follows:

$$\frac{r}{x} = \frac{0.235}{0.79} = 0.30; \quad (fl)' = 12{,}000\sqrt{\frac{0.79 \times 5.35}{4.26}} = 12{,}000 \text{ approximately}$$

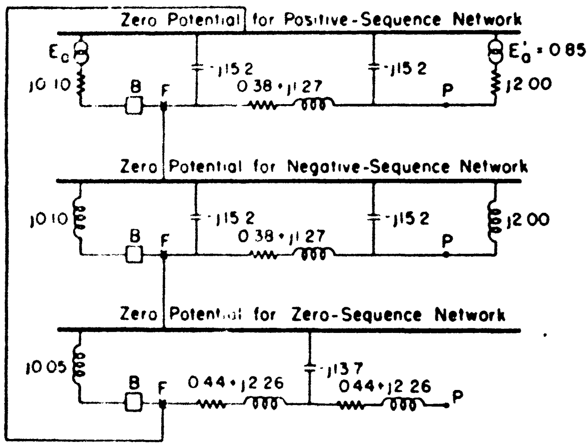
From Fig. 2(*a*), neglecting β_2 and γ_2, $\beta = 0.972$ and $\gamma = 1.014$.

$$\beta Z = 200(0.235 + j0.79)0.972 = 45.7 + j153.5 \text{ ohms}$$

$$\gamma \frac{Y}{2} = 100(j5.35 \times 10^{-6})1.014 = j544 \times 10^{-6} \text{ mho}$$



(a)



(b)

Fig. 3. (*a*) One-line system diagram. (*b*) Connection of sequence networks of (*a*) for a line-to-ground fault at *F*. Impedances are in per unit based on 100,000/3 kva per phase and a base line-to-neutral voltage of $110/\sqrt{3}$ kilovolts in the transmission line. Loads are neglected.

To express impedances, given in ohms, in per unit based on 100,000/3 kva per phase and a line-to-neutral voltage of $110/\sqrt{3}$ kv (see [27], Chapter I), the multiplier is $100{,}000/(110^2 \times 10^3) = 0.826 \times 10^{-2}$.

$$\beta Z = (45.7 + j153.5) \times 0.826 \times 10^{-2} = 0.38 + j1.27 \text{ per unit}$$

$$\frac{2}{\gamma Y} = -j1840 \times 0.826 \times 10^{-2} = -j15.2 \text{ per unit}$$

The zero-sequence equivalent T-line is determined as follows:

$$\frac{r}{x} = \frac{0.515}{2.65} = 0.194; \quad (fl)' = 12{,}000\sqrt{\frac{2.65 \times 3.20}{4.26}} = 17{,}000$$

From Fig. 2(a), neglecting β_2 and γ_2, $\beta = 0.944$ and $\gamma = 1.03$. In per unit,

$$\gamma \frac{Z}{2} = 100(0.515 + j2.65)1.03 \times (0.826 \times 10^{-2}) = 0.438 + j2.26$$

$$\frac{1}{\beta Y} = \frac{10^6 \times (0.826 \times 10^{-2})}{200(j3.20) \times 0.944} = -j13.7$$

Determine the initial symmetrical rms current in the fault and voltages to ground at the fault on phases b and c (with the circuit breaker at B closed). Assume as subtransient reactances for the condenser the given transient values.

Solution. The equivalent excitation voltage E_a is not given nor is it required, as the prefault line-to-line voltage at F is given. The line-to-neutral voltage $V_f = 110 \text{ kv}/\sqrt{3}$. In per unit with V_f as reference vector,

$$V_f = 1 \underline{/0°}.$$

The positive- and negative-sequence subtransient impedances viewed from F are equal and are determined by paralleling the impedances viewed from the fault in the two directions. The impedance Z_1' to the left of F is $j0.10$; that to the right is

$$Z_1'' = \frac{\left[\frac{j2.00(-j15.2)}{j(2.00-15.2)} + 0.38 + j1.27\right](-j15.2)}{(j2.30 + 0.38 + j1.27) - j15.2} = \frac{(0.38 + j3.57)(-j15.2)}{0.38 - j11.63}$$

$$= \frac{(3.60 \underline{/83.9°})(15.2 \overline{\backslash 90°})}{11.64 \overline{\backslash 88.1°}} = 4.70 \underline{/82.0°} = 0.654 + j4.65$$

$$Z_1 = Z_2 = \frac{Z_1' \times Z_1''}{Z_1' + Z_1''} = \frac{(j0.10)(4.70 \underline{/82.0°})}{0.654 + j4.75(=4.79\underline{/82.1°})} = 0.098 \underline{/89.9°} = 0.00 + j0.098$$

= per unit positive- or negative-sequence impedance viewed from the fault

Viewed from F in the zero-sequence network,

$$Z_0' = j0.05 \quad \text{and} \quad Z_0'' = 0.44 + j2.26 - j13.7 = 0.44 - j11.44 = 11.45 \overline{\backslash 87.8°}$$

$$Z_0 = \frac{Z_0'Z_0''}{Z_0' + Z_0''} = \frac{(0.05 \underline{/90°})(11.45 \overline{\backslash 87.8°})}{0.44 - j11.39(=11.40 \overline{\backslash 87.8°})} = j0.0502 =$$ per unit zero-sequence impedances viewed from the fault

It will be noted that Z_0 viewed from the fault is not appreciably affected by the impedance Z_0'' because of its high magnitude relative to $Z_0' = j0.05$. $Z_1 = Z_2$ is likewise but little affected by paralleling Z_1'' with $Z_1' = j0.10$. The impedances viewed towards the large system in this problem determine conditions at the fault. In cases such as this, where it is apparent that exact values of the sequence impedances in one direction are unimportant, they may be roughly estimated. Using transient reactance instead of subtransient for the synchronous condenser at C has negligible effect on the impedances Z_1 and Z_0 viewed from the fault.

Substituting V_f, Z_1, Z_2, and Z_0 in the equations of Table I, Chapter IV, with $R_f = 0$,

$$I_{a1} = I_{a2} = I_{a0} = \frac{1\,\underline{/0^\circ}}{j0.098 + j0.098 + j0.050} = -j4.06$$

$$I_f = 3I_{a0} = -j12.18$$

$$V_{a2} = -I_{a2}Z_2 = -0.398;\ V_{a0} = -I_{a0}Z_0 = -0.203;\ V_{a1} = -(V_{a2} + V_{a0}) = 0.601$$

$$V_b = -0.305 - j0.866 = 0.920\,\overline{\backslash 109.4^\circ}$$

$$V_c = -0.305 + j0.866 = 0.920\,\underline{/109.4^\circ}$$

V_b and V_c are both below normal line-to-neutral voltage.

Zero-Sequence Voltages at Points Distant from the Fault. In a system where all reactances are inductive, the zero-sequence voltage has its highest value at the fault. In circuits with capacitance this may not be the case. In Problem 4, the zero-sequence voltage V_{a0} at F is -0.203. Applying this voltage in the zero-sequence network between F and the zero-potential bus for the network, the zero-sequence voltage at P (neglecting resistance) is

$$V_{a0}\ (\text{at } P) = \frac{-0.203}{j(2.26 - 13.7)}(-j13.7) = -0.243$$

The zero-sequence voltage at P is approximately 20% higher than at F.

A Ground-Fault Neutralizer (Petersen Coil). When, because of lightning or other causes, a flashover between a conductor and tower occurs, the arc offers a relatively low impedance to fundamental-frequency currents, which will be inductive or capacitive, depending upon whether the system is operated with grounded or ungrounded neutral. A ground-fault neutralizer is a reactance, placed between neutral and ground in a system otherwise ungrounded, of such magnitude that the fundamental-frequency zero-sequence capacitive currents in a ground fault are neutralized by zero-sequence inductive currents passed by the ground-fault neutralizer. With little resultant fundamental-frequency current in the arc, it rapidly dies out, and switching operations are unnecessary.

A reactance in the neutral has an effective value of three times its actual value. (See equation [7], Chapter III.) Placed between the neutral of Y-connected transformer windings and ground, with its effective reactance $3X_L$ plus the transformer reactance x_t equal to the system capacitive reactance X_c (neglecting resistance and line reactance), the zero sequence impedance viewed from any system point will be infinite.

$$Z_0 = \frac{j(3X_L + x_t)(-jX_c)}{0} = \infty$$

Actually, resistance is present, and the effects of line reactance may be appreciable. But as a first approximation in determining the required neutral reactance, the total capacitive reactance X_c of all the lines is equated to $3X_L + x_t$. Thus,

$$3X_L + x_t = X_c \quad \text{and} \quad X_L = \tfrac{1}{3}(X_c - x_t)$$

When a line-to-ground fault occurs and $Z_0 = \infty$, the zero sequence voltage at the fault is $-V_a$, the voltage of phase a before the fault. (See Chapter III, equation [32] and Fig. 9.) No fundamental-frequency current will flow in the positive and negative sequence networks; but, with $V_{a0} = -V_a$ at the fault, both capacitive and inductive currents will flow in the zero-sequence network. In the fault these currents neutralize each other, except for their components resulting from resistance and imperfect tuning.

A neutralizer, in tune for a fault at one system point, is in tune for faults at other points as well. Therefore, the problem of locating the neutralizer on the system becomes that of determining the position where it will not be disconnected during system disturbances. Sometimes a system can be subdivided into areas, and each area provided with its own neutralizer. With this arrangement, the areas can be interconnected or separated and the system as a whole will still have protection.

When some of the lines are long, and the voltage high (110 kv or above), a rise in zero-sequence voltage at points distant from neutralizer may occur during ground faults, as explained in the above section. The use of two or more coils judiciously placed in the system will limit this voltage rise to a reasonable value.

CHARTS OF FUNDAMENTAL FREQUENCY LINE-TO-GROUND VOLTAGES DURING A LINE-TO-GROUND FAULT

During a line-to-ground fault, the voltages of the two unfaulted phases may be higher or lower than normal, depending upon the system impedances. Figures 4(a) and (b), taken from a paper[6] by Messrs. Hunter, Pragst, and Light, show the magnitudes of the fundamental-frequency voltages to ground of phases b and c, respectively, at the fault (on phase a) in terms of the positive- and zero-sequence system impedances viewed from the fault. The positive- and negative-sequence impedances are assumed equal, and the effects of corona and saturation are neglected.

The curves give only the fundamental-frequency dynamic voltages. Harmonic voltages resulting from unbalanced currents in synchronous machines with unequal reactances in the direct and quadrature axes

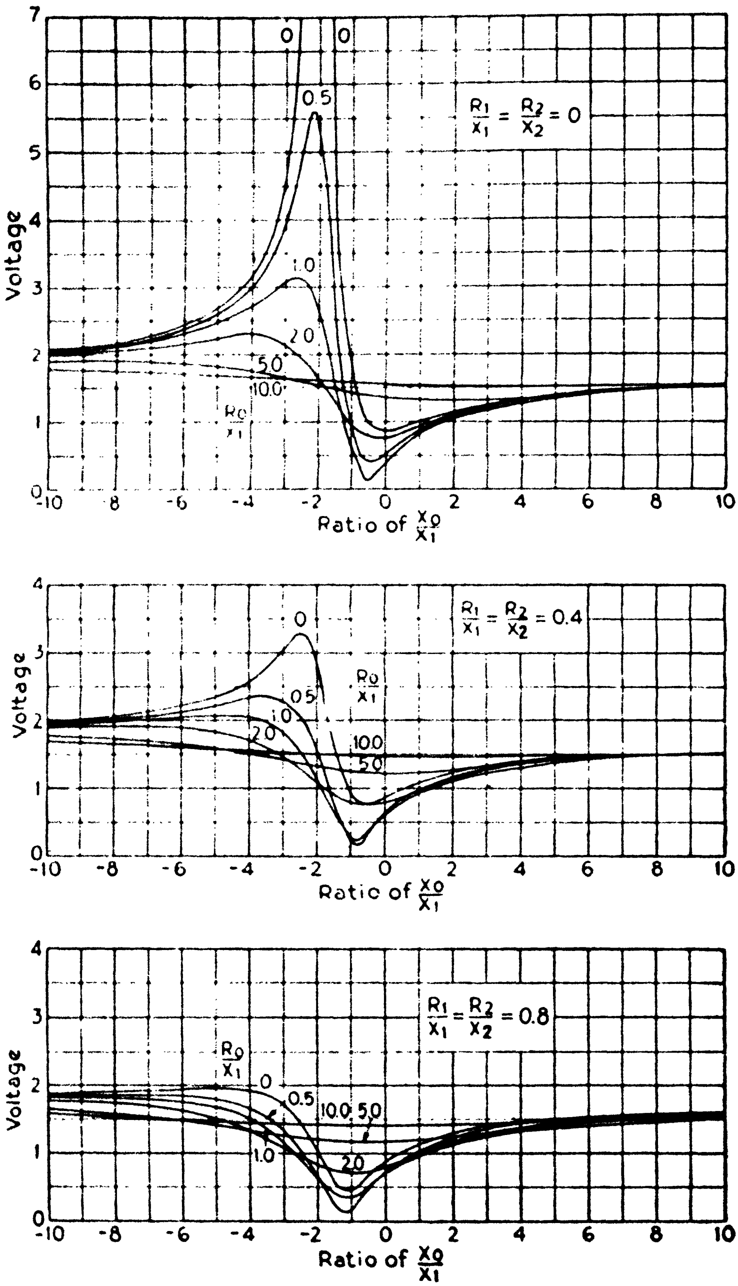


FIG. 4(a). Phase *b* voltage to ground at point of fault in per unit of normal voltage to neutral versus X_0/X_1 with conductor-to-ground fault on phase *a*.

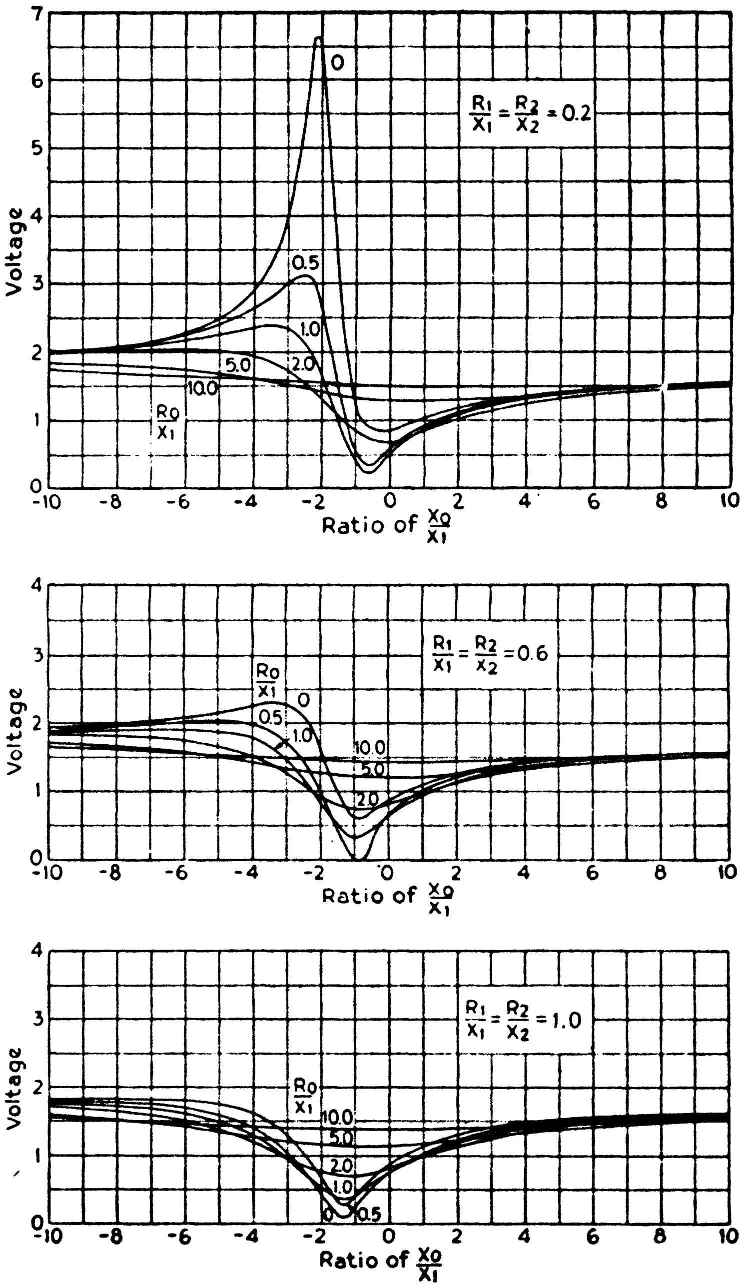


Fig. 4(*a*) *continued*

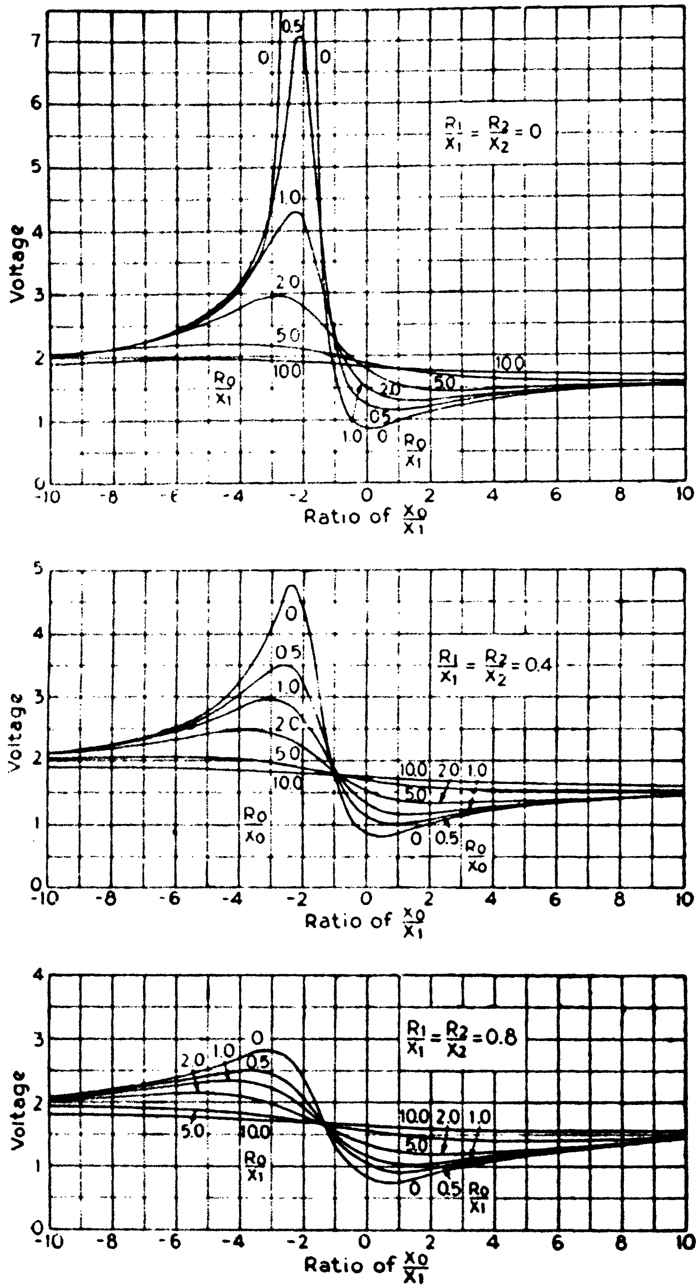


FIG. 4(*b*). Phase *c* voltage to ground at point of fault in per unit of normal voltage to neutral versus X_0/X_1 with conductor-to-ground fault on phase *a*.

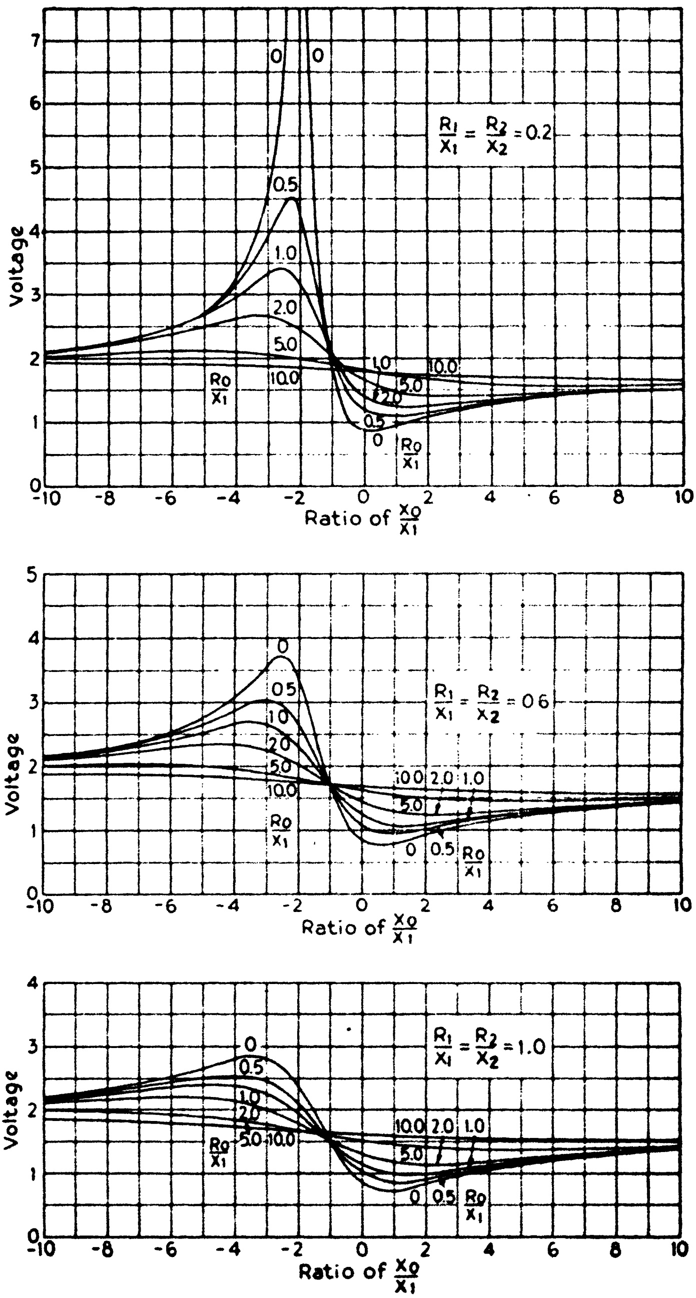


Fig. 4(*b*) *continued*

and transient voltages from switching and arcs to ground are not included. Only the voltages to ground which exist at the point in the system where the ground fault is located are given. In general, it may be stated that the voltages will be lower or higher at other points on the system, depending upon whether the system neutrals are grounded or isolated. The notation used is

$$Z_1 = Z_2 = R_1 + jX_1$$

$$Z_0 = R_0 + jX_0$$

where X_0 may be positive or negative. In an ungrounded system, the zero-sequence impedance viewed from the fault is a capacitive impedance, and X_0 is intrinsically negative. For an assumed ratio $R_1/X_1 = R_2/X_2$ with R_0/X_1 as parameter, X_0/X_1 is varied from -10 to $+10$ and the voltages of phases b and c plotted in per unit of V_f, the prefault line-to-neutral voltage at the point of fault F. With X_0/X_1 extended to $+\infty$ or $-\infty$, V_b and V_c in per unit of normal line-to-neutral voltage become 1.73. When the system neutral is grounded through a ground-fault neutralizer, the ratio of X_0/X_1 is very high, approaching infinity with the neutralizer tuned to the system.

Equations for V_b and V_c in terms of V_f, Z_1, and Z_0 without fault resistance can be obtained from Table I of Chapter IV if Z_f is equated to zero and Z_2 is replaced by its equivalent Z_1.

Fault Resistance. The curves of Figs. 4(a) and (b), as drawn, do not include fault resistance. In general, fault resistance, except in low-resistance systems, has a tendency to reduce the magnitude of the unbalanced voltage over that which would be obtained if the fault resistance were zero. The voltages that exist with resistance in the fault can be obtained from the curves by making the following substitutions:

The equations for the unbalanced voltages during a line-to-ground fault in Table I, Chapter IV, which include fault resistance may be written

$$V_a = \frac{V_f(3R_f)}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

$$V_b = a^2V_f - \frac{V_f[(R_0' - R_1') + j(X_0 - X_1)]}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

$$V_c = aV_f - \frac{V_f[(R_0' - R_1') + j(X_0 - X_1)]}{(2R_1' + R_0') + j(2X_1 + X_0)}$$

where

$$R_1' = R_1 + R_f$$

$$R_0' = R_0 + R_f$$

$$R_f = \text{fault resistance}$$

The curves are expressed as functions of the positive resistance R_1 and the zero resistance R_0. However, if values of R_1'/X_1 and R_0'/X_1 are obtained and reference is made to curves of R_1/X_1 and R_0/X_1 which are numerically equal to those values obtained with fault resistance, these particular curves will give the voltages to be expected with resistance in the fault.

The curves of Figs. 4(*a*) and (*b*) not only are useful in reducing calculations, but they also indicate clearly the relative voltage trend under different conditions of system grounding. The voltages which occur during ground faults are of particular interest to application and operating engineers because they have a direct influence on the serviceableness of system insulation and connected apparatus and on the size of the neutral grounding impedance to be used. One example of this is the selection of the correct lightning arrester for a given system. These protective devices have a maximum permissible line-to-ground voltage rating which specifies the voltage across the arrester that must not be exceeded if arrester troubles are to be avoided. As most arresters are connected between the line conductors and ground, the dynamic voltage rating of the arresters should not be lower than the voltages that exist during ground faults at the proposed arrester locations.

Use of the Curves of Figs. 4(*a*) and (*b*). In Problem 4, with connection of the sequence networks for a line-to-ground fault at F, the positive- and zero-sequence impedances viewed from the fault are $Z_1 = 0 + j0.098$ and $Z_0 = 0 + j0.050$, respectively.

$$\frac{R_1}{X_1} = \frac{R_2}{X_2} = \frac{R_0}{X_1} = 0; \quad \frac{X_0}{X_1} = \frac{0.050}{0.098} = 0.51$$

From the first of the curves in Fig. 4(*a*) and in Fig. 4(*b*), V_b and V_c are both less than normal.

Problem 5. In Problem 4, and Fig. 3(*a*), assume that the breakers at B are open but the line-to-ground fault at F remains on the system. Determine the voltages to ground at F of phases b and c, assuming that the condenser does not lose synchronism with the large system while the breakers are closed and therefore its speed is not appreciably different from synchronous speed at the instant the breakers are opened. It will be assumed further that the transient reactance of the condenser and the voltage $E'_a = 0.85$ behind this reactance can be used to calculate rms symmetrical voltages, immediately following the opening of the breakers.

Solution. The diagram of Fig. 3(*b*) can be used if the three networks are all opened at B. There are two ways of determining the fault current and voltages V_b and V_c. One way is to determine the positive-sequence voltage V_f of phase a at F with the fault removed and the voltage $E'_a = 0.85$ in the condenser behind transient reactance. The fault is then applied and calculations are made as in Problem 4. A second method is to replace the fault in the positive sequence network by an equivalent circuit with impedance $Z_0 + Z_2$ between F and the zero-potential bus of the positive-sequence network. Figure 3(*b*) with the circuit breakers open at B satisfies this condition. The former method will be used; the latter is reserved for a problem. (See Problem 14.)

The positive-sequence impedance viewed from the condenser neutral with the fault *removed* and the breaker *open* is

$$j2.00 + \frac{-j15.2(0.38 - j13.93)}{0.38 - j29.13} = 0.10 - j5.28$$

The current I in the shunt at F with $E'_a = 0.85\,\underline{/0^\circ}$ is

$$I = \frac{0.85}{0.10 - j5.28} \times \frac{-j15.2}{0.38 - j29.13} = 0.00 + j0.084$$

The voltage at F is

$$V_f = (-j15.2)(j0.084) = 1.27 \text{ per unit}$$

At P the voltage is 1.17. With the breaker open and no fault, there is a voltage rise through the condenser and along the line because of charging current flowing through reactance.

The sequence impedances viewed from the fault with the breaker open are Z_1'' and Z_0'', calculated in Problem 4.

$$Z_1 = Z_2 = 0.345 + j4.69; \quad R_1 = 0.345; \quad X_1 = 4.67$$

$$Z_0 = 0.44 - j11.44. \quad R_0 = 0.44; \quad X_0 = -11.44$$

$$\frac{R_1}{X_1} = 0.074; \quad \frac{R_0}{X_1} = 0.094; \quad \frac{X_0}{X_1} = -2.45$$

Read roughly from the curves of Figs. 4(*a*) and (*b*) with $R_1/X_1 = 0.2$ (an approximation) and R_0/X_1 = between 0 and 0.5, but with $X_0/X_1 = -2.45$, V_b is higher than $3V_f$, and V_c higher than $4V_f$, where V_f as calculated above is 1.27 times normal line-to-neutral voltage. These are the approximate voltages which would exist at the fault if there were *no corona* on the line and *no saturation* in the Δ–Δ transformer banks along the line.

Voltages calculated with corona and saturation neglected if appreciably above corona starting voltage for the line (see Fig. 8, Appendix B) or rated voltages across transformer windings along the line, even though too high in magnitude, serve to indicate conditions and locations where high voltages are to be expected. This is the case for the system discussed in Problem 5. The calculations made in this problem indicate definitely that voltages to ground above $\sqrt{3}$ times line-to-neutral voltages will occur. To determine exactly how high the voltages will be requires a more comprehensive study or, preferably, field tests.

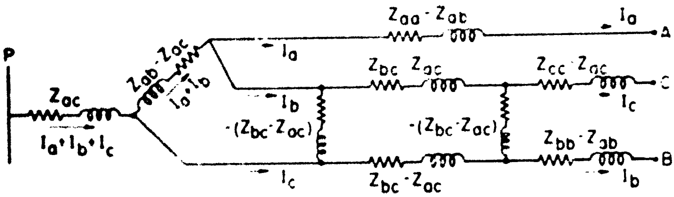
It is interesting to note the effect of zero-sequence resistance from Figs. 4(*a*) and (*b*). With $R_1/X_1 = 0.2$, $R_0/X_1 = 5.0$, and $X_0/X_1 = -2.45$, although in the resonance region of the curves, $V_b = 1.5$ and $V_c = 2.0$.

EQUIVALENT CIRCUITS FOR PARALLEL THREE-PHASE TRANSMISSION LINES

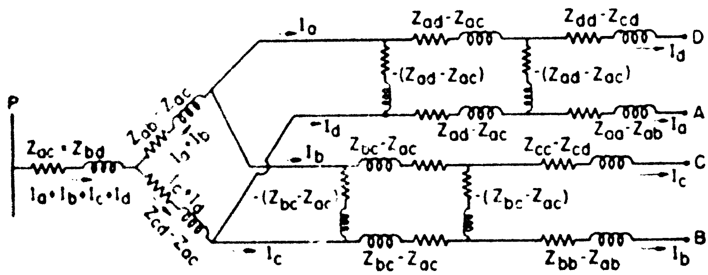
In Chapters XI and XII methods of calculating the self-inductive impedance and capacitive admittance of each line alone and the mutual inductive and capacitive impedances between parallel lines, taken two at a time, are discussed and equations given for determining them. Lines with and without ground wires are considered. Mutual impedances, both inductive and capacitive, between two parallel lines in the positive- or negative-sequence system are small relative to self-impedances and depend upon the arrangements of the phases of the two circuits in their tower positions. Unless a high degree of precision is required, each circuit can be replaced by its equivalent T or Π in the positive- or negative-sequence network without mutual coupling with parallel circuits. If it is desired to include mutual coupling, equivalent circuits similar to those which will be developed for use in the zero-sequence network can be used.

Zero-Sequence Equivalent Circuits. By definition, zero-sequence line currents and voltages to ground in the three phases at any system point are equal in magnitude and phase. Zero-sequence mutual impedances are therefore independent of the arrangements of the phases of the two circuits in their tower positions.

Circuits with Negligible Capacitance. Zero sequence mutual impedances between parallel lines on the same or adjacent towers may be as high as 50% or more of the self-impedance of either circuit alone. Except in special cases, zero-sequence mutual impedances between parallel lines cannot be neglected. Figures 9(*b*) and (*c*), 11(*b*), and 12(*c*) of Chapter I, which show equivalent circuits to replace two mutually coupled circuits, are directly applicable to two parallel transmission lines with negligible capacitance, bussed at both ends, at one end, and at neither end, respectively. For three parallel lines bussed at one end, the equivalent circuits of Fig. 13(*b*) or (*c*) of Chapter I are applicable, the former if two of the mutual impedances are equal and the latter if all three are equal. For three unequal mutual impedances, the four-terminal equivalent circuit, Fig. 5(*a*), may be used. This equivalent circuit is similar to Fig. 13(b) of Chapter I except for the mutual coupling between circuits B and C introduced because $Z_{bc} \neq Z_{ac}$. The mutual impedance $(Z_{bc} - Z_{ac})$ is introduced between circuits B and C by means of the mutual coupling circuit of Fig. 12(c), Chapter I, except that the positions of the terminals at one end are reversed. Figure 5(a) can be checked by



(a)



(b)

FIG. 5. Equivalent circuits for parallel lines of negligible capacitance bussed at one end P for use in analytic calculations. (a) Three lines with unequal mutual impedances between them. (b) Four lines with $Z_{ac} = Z_{bd}$ but other mutual impedances unequal.

following the current in each line with the other lines open to see if it meets an impedance equal to its self-impedance and induces the required voltages in the open lines. With negligible capacitance, a line is opened at any point by opening it at a terminal, without otherwise changing the equivalent circuit. With circuits A and C open at terminals A and C, the current I_b flows through $Z_{ac} + (Z_{ab} - Z_{ac}) + (Z_{bb} - Z_{ab}) = Z_{bb}$. I_b has two paths, both of zero impedance, through the equivalent circuit which couples circuits B and C through the mutual impedance $(Z_{bc} - Z_{ac})$. I_b induces a voltage drop $I_b(Z_{ac} + Z_{ab} - Z_{ac}) = I_bZ_{ab}$ in circuit A in the direction of I_b. In circuit C the induced voltage drop is $I_b(Z_{ac} + Z_{bc} - Z_{ac}) = I_bZ_{bc}$.

The equivalent circuit used in Fig. 5(a) can be extended to four or more parallel lines bussed at one end with unequal mutual impedances between them. For the case of four parallel circuits on the same right-of-way, if $Z_{ac} = Z_{bd}$ but the other mutual impedances are unequal, Fig. 5(*b*) can be used. If $Z_{ac} \neq Z_{bd}$, an additional mutual coupling circuit would be required to insert the mutual impedance $(Z_{bd} - Z_{ac})$ between circuits *B* and *D*.

When parallel lines are not bussed but are supplied through transformers connected Δ–Y, grounded on the line side, the transformers may be included in the zero-sequence self-impedances of the lines, and the lines plus transformers considered to have a ground point in common. The equivalent circuits for two, three, and four parallel lines discussed above can then be used in the zero-sequence network. The only limitation to such equivalent circuits is that zero-sequence voltages at the transformer terminals cannot be obtained directly; but they can be calculated from the transformer impedances and the zero-sequence currents flowing through them.

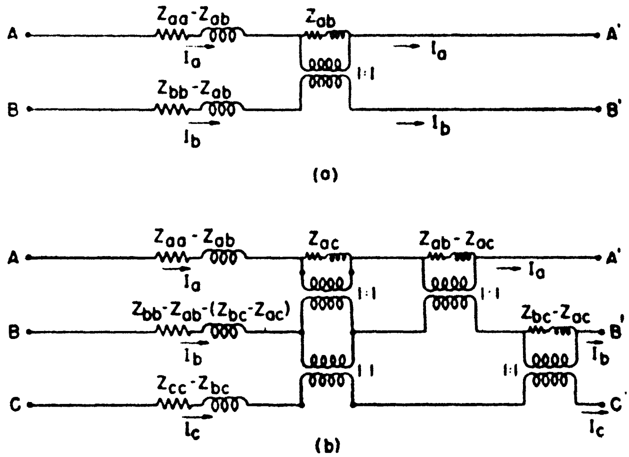


FIG. 6. Equivalent circuits for parallel lines with negligible capacitance for use on an a-c network analyzer. (*a*) Two lines. (*b*) Three lines.

Equivalent Circuits for Use with An A-C *Network Analyzer*. In equivalent circuits used on the network analyzer, mutual coupling transformers are usually employed to secure mutual coupling between circuits. The ideal mutual coupling transformer has infinite exciting impedance, zero resistance, and zero leakage reactance relative to the analyzer impedance units.

Figure 6 and Fig. 8(*d*) with the capacitive shunts to ground omitted show zero sequence equivalent circuits for two, three, and four parallel lines in which capacitance is negligible. If the circuits are so designated that Z_{ac} and Z_{ad}, with three and four parallel lines, respectively, have the lowest resistance components of mutual impedance, there need be no negative resistances in the equivalent circuits. In Fig. 6(a) the mutual impedance Z_{ab} is inserted in one circuit (here circuit A) with the terminals of one winding of a mutual coupling transformer of 1 : 1 turn ratio connected across it, the other transformer winding being connected in series with circuit B. The impedance met by I_a is $Z_{aa} - Z_{ab} + Z_{ab} = Z_{aa}$. The voltage drop induced in circuit B by I_a is $I_a Z_{ab}$ in the direction of I_a.

In Fig. 6(b), two 1 : 1 turn ratio transformers are required to couple the three lines through a common mutual impedance Z_{ac}. An additional transformer is required to obtain the correct mutual impedances between circuits A and B, and another for the correct mutual impedance between circuits B and C. In Fig. 8(d) the method used in Fig. 6(b) has been extended to include four parallel lines. The method can be extended to include any number of parallel circuits, being limited only by the number of coupling transformers available.

Parallel Lines with Appreciable Capacitance. There are two ways in which the zero-sequence capacitances associated with parallel transmission lines may be defined. With each three-phase line represented on a per phase basis, the *capacitive admittance* to ground of any line and its mutual admittances with the other lines may be determined with all other lines *grounded* or with all other lines *open*. By the first method of determining the capacitive admittances of a line, the voltages of all other lines are equated to zero. By the second method, the currents in all other lines are equated to zero. The admittances of a line, determined with all other lines grounded, correspond to driving-point and transfer admittances defined in Chapter I under equation [33]. The admittances of a line, determined with the other lines open, are analogous to self- and mutual admittances between inductively coupled circuits. To avoid confusing capacitive admittances defined in two different ways, self- and mutual *capacitive impedances* instead of admittances will be used in this chapter when capacitances associated with a transmission line are defined as the capacitances determined with all other lines open.

Consider two three-phase circuits A and B, ℓ miles in length, which for the purpose of this discussion will be assumed symmetrical, with all conductors of each circuit equidistant from the conductors of the other circuit. (Unsymmetrical transmission circuits are discussed in Chapters XI and XII.) Three equivalent circuits are shown in Figs. 7(a), (b), and (c), each constructed on a per phase basis. Figure 7(*a*) is a *capacitive admittance equivalent circuit* in which b_{aa} is the self-capacitive susceptance per mile of circuit A with circuit B grounded, b_{bb} the capacitive susceptance per mile of circuit B with circuit A grounded, and b_{ab} is the mutual capacitive susceptance per mile between circuits A and B, determined with either A or B grounded. Figure 7(*b*) is a *capacitive impedance equivalent circuit* in which x_{aa} and x_{bb} are the self-capacitive reactances in ohms-miles of circuits A and B, respectively, each determined with no current in the other circuit; x_{ab} is the mutual capacitive reactance in ohms-miles between circuits A and B, determined with no current in one of the circuits.

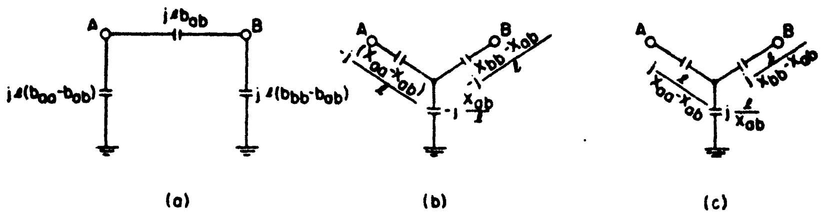


FIG. 7. Nominal equivalent capacitance circuits for two parallel lines of length l. (*a*) Admittance Δ; (*b*) Impedance Y; (*c*) Admittance Y.

Although calculated in different ways, either of these circuits may be obtained from the other, and either may be used in determining equivalent circuits for two parallel transmission lines.

Figure 7(*c*) is a capacitive admittance Y-connected circuit, obtained from Fig. 7(*b*) by replacing its capacitive impedance branches by reciprocal capacitive admittances. In an equivalent circuit composed of static branches, the impedances of all branches may be replaced by their reciprocal admittances to obtain an admittance circuit; likewise, an admittance circuit may be converted to an impedance circuit. Figure 7(*a*) may be obtained by converting the Y of Fig. 7(*c*) into a Δ, or directly from Fig. 7(*b*) by means of the following equations, determined by inverting the fractions on the right-hand sides of equations [40], Chapter I, and simplifying:

$$
\begin{aligned}
b_{aa} &= \frac{x_{bb}}{x_{aa}x_{bb} - x_{ab}^2} \\
b_{bb} &= \frac{x_{aa}}{x_{aa}x_{bb} - x_{ab}^2} \qquad [29] \\
b_{ab} &= \frac{x_{ab}}{x_{aa}x_{bb} - x_{ab}^2}
\end{aligned}
$$

Figure 7(*b*) may be obtained from Fig. 7(*a*) by means of the equations

$$x_{aa} = \frac{b_{bb}}{b_{aa}b_{bb} - b_{ab}^2}$$

$$x_{bb} = \frac{b_{aa}}{b_{aa}b_{bb} - b_{ab}^2} \qquad [30]$$

$$x_{ab} = \frac{b_{ab}}{b_{aa}b_{bb} - b_{ab}^2}$$

With one transmission line only, assumed symmetrical, the capacitive admittance $jb\ell$ in mhos and the capacitive impedance $-j(x/\ell)$ in ohms are reciprocals of each other. With two parallel lines, x_{aa}, x_{bb}, and x_{ab} are *not the reciprocals* of b_{aa}, b_{bb}, and b_{ab} as may be seen from equations [29] and [30]. Also, by definition, b_{aa} is determined with circuit B grounded, while $1/x_{aa}$ is determined with circuit B open.

With ground wires or more than two parallel transmission lines, self- and mutual capacitive impedances are more easily calculated than capacitive admittances. Curves are given in Chapter XII, from which the self-capacitive impedances of each circuit and the mutual capacitive impedances between circuits taken two at a time can be obtained. Capacitive impedances are also more convenient in developing equivalent circuits for three or more parallel lines than capacitive susceptance, and will be used in the work which follows.

Nominal Π *Equivalent Circuits.* Figures 8(*a*) and (*b*) show equivalent circuits for two parallel lines not bussed at either end represented by their *nominal* Π-*lines*. Figure 8(*a*) is for use in analytic calculations; Fig. 8(*b*) may be used on the a-c network analyzer. The architraves of the Π's consist of the inductive self-impedances Z_{aa} and Z_{bb} of the two circuits mutually coupled through the mutual impedance Z_{ab}. Twice the self- and mutual capacitive reactances are used in the capacitive impedance Y's at the terminals of the lines. These impedance Y's can be converted to admittance Y's or Δ's as explained above.

The nominal equivalent circuits of Figs. 8(*a*) and (*b*) for two parallel lines can be extended to three or more parallel lines if the self-capacitive impedances of each line alone and the mutual capacitive impedances between lines, taken two at a time with the other lines open, are given. Figure 8(*c*) shows a nominal equivalent circuit for three parallel lines bussed at one end; in this equivalent circuit, two of the mutual inductive impedances and also two of the mutual capacitive impedances are equal. Figure 8(*d*) shows the nominal equivalent circuit for use on an a-c calculating table for four parallel lines in which the identity of the terminals of the four lines is retained. In this circuit all mutual inductive and capacitive impedances are assumed unequal.

The nominal equivalent circuits of Fig. 8 are combinations of equivalent circuits. The self- and mutual inductive impedances of the system

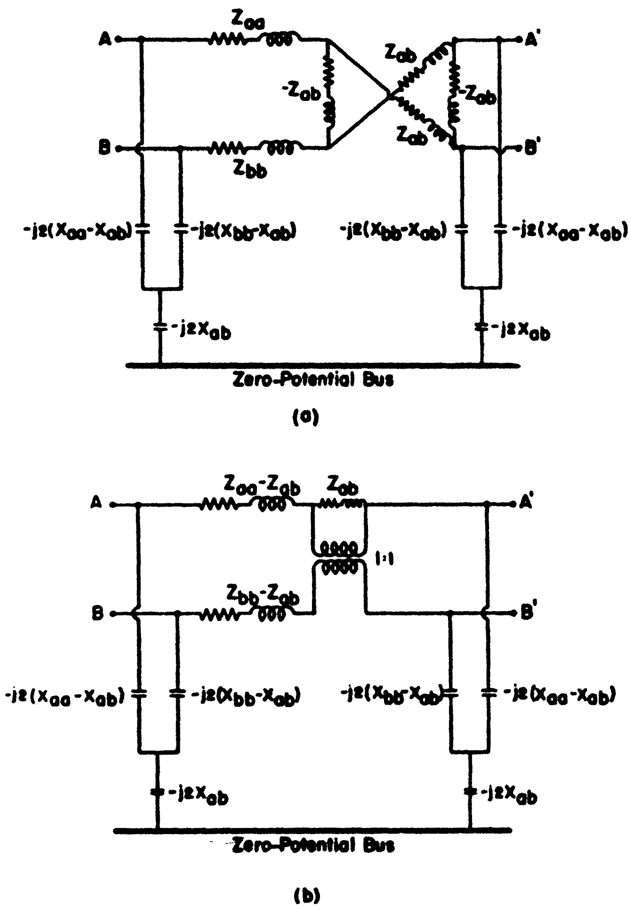


FIG. 8(*a* and *b*). Nominal equivalent circuits for parallel lines in which each line is represented by its nominal Π with inductive and capacitive mutual impedances between them. (*a*) and (*b*) — Two parallel lines.

are represented by an equivalent circuit between the two ends of lines, constructed with capacitance neglected. Self- and mutual capacitive impedances to ground are represented by equivalent circuits at the ends of the lines, constructed with inductance neglected. As half the capacitance is in each shunt circuit, self- and mutual capacitive impedances are multiplied by two. With self-capacitances to ground and mutual capacitances between circuits expressed in terms of capacitive impedances, the construction of the equivalent capacitance circuits between circuit terminals and ground is similar to that of inductively coupled circuits.

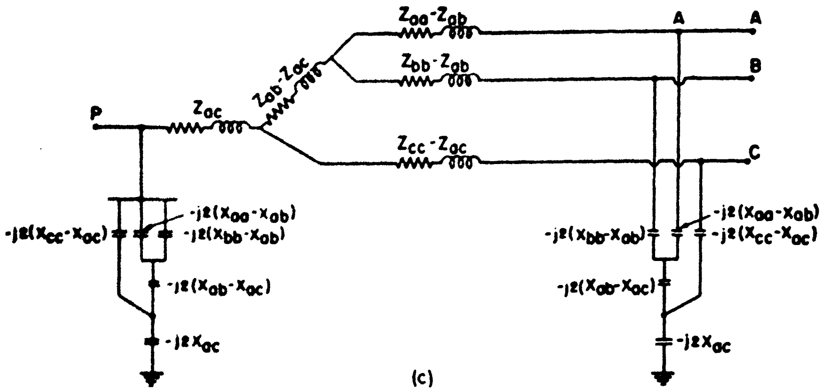


FIG. 8(c). Nominal equivalent circuit for three parallel lines bussed at one end P in which $Z_{ac} = Z_{bc}$ and $-jX_{ac} = -jX_{bc}$.

A fault on one of two or more parallel transmission lines may be considered to divide the parallel lines into two sections, each of which may be replaced by its equivalent circuit, the fault being located on the given circuit at the junction of the equivalent circuits.

NORMAL OPERATION OF A SYMMETRICAL THREE-PHASE TRANSMISSION CIRCUIT

With the power to be transmitted and the distance of transmission given, the *choice of transmission voltage and number of circuits* are influenced by (1) the voltage drop in the line, (2) the power loss under normal operating conditions, and (3) the requirements for power system stability during steady-state operation and under specified transient conditions. The subject of power system stability is not covered in this volume.

At a specified voltage, the choice of conductors is influenced by (1) the diameter of the conductors required to avoid corona under normal operating conditions and (2) the allowable temperature of the conductors when carrying maximum load current. Figure 8 of Appendix B gives approximate corona starting voltages versus geometric mean spacing between conductors for copper and A.C.S.R. conductors used in overhead transmission circuits. Figures 9 and 10 of Appendix B give temperature rise above ambient temperature versus current in the conductors for copper and A.C.S.R. conductors, respectively.

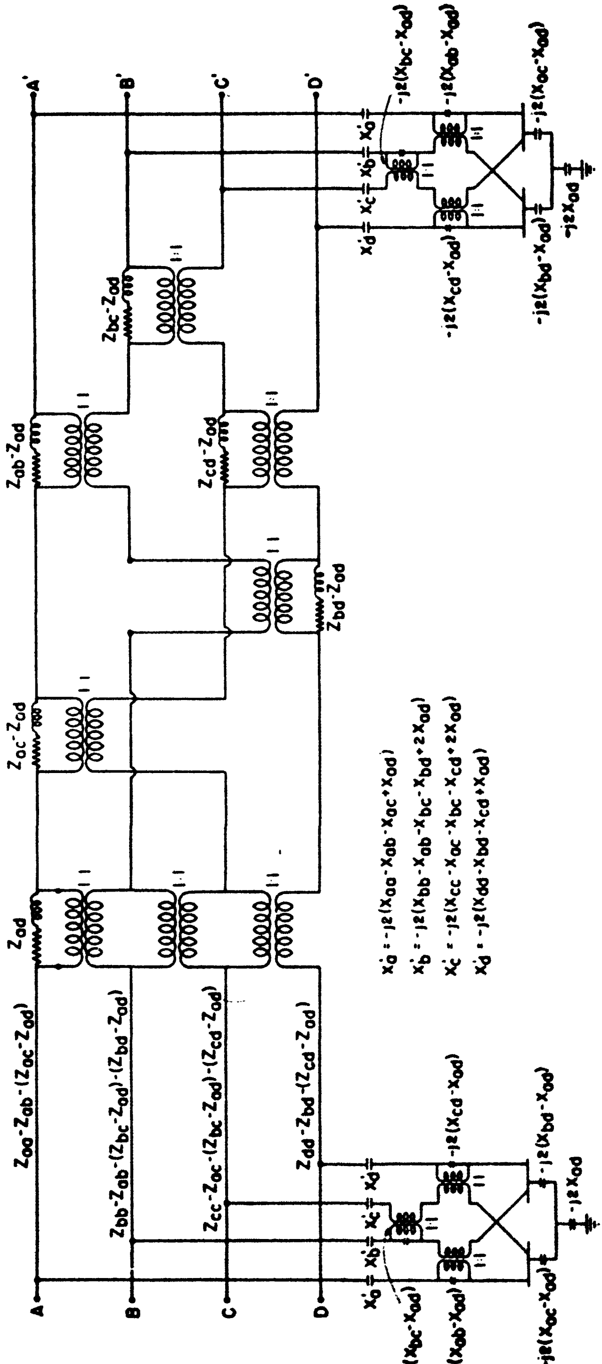


Fig. 8(*d*). Nominal equivalent circuit for four parallel lines in which all inductive and capacitive mutual impedances are unequal, and the identities of all terminals are retained.

During normal operation, the currents and voltages in a symmetrical three-phase power system are positive-sequence currents and voltages. The conditions in the transmission circuit at specified terminal conditions can be calculated from [13]–[16] of this chapter when positive-sequence resistance, reactance, and capacitive susceptance are known. Resistances and internal reactances of commonly used conductors are given in the wire tables of Appendix B; positive-sequence reactances and capacitive susceptances can be obtained from the curves of Appendix B.

Problem 7. Solve Problem 4 with the two Δ-Y transformer banks ungrounded.

Problem 8. Approximately what value of reactance in ohms placed in the neutrals of the transformers in Fig. 3(*a*) would neutralize the zero-sequence capacitance and keep the line from being cut out of service during a line-to-ground fault at *B* caused by lightning? Would it require more or less reactance with reactors in the neutrals of both transformers, or in the neutral of one transformer with the other neutral ungrounded? What would be the kva rating of the ground-fault neutralizer (or neutralizers), based on normal line-to-neutral voltage and the maximum current it would be required to carry during line-to-ground faults?

Problem 9. In Fig. 2, Chapter IV, consider that the per unit impedances in the sequence networks are based on a three-phase kva base of 100,000 kva and base line-to-line voltage of 115 kv in the transmission circuit. The lines *A* and *B* are identical, and are 50 miles long. The frequency is 60 cycles. The positive- and zero-sequence line constants for each line, determined with the other line open, are

$$z_1 = 0.278 + j0.794 \text{ ohm per mile}; \qquad y_1 = j5.2 \times 10^{-6} \text{ mho per mile}$$

$$z_0 = 0.55 + j2.40 \text{ ohms per mile}; \qquad y_0 = j2.85 \times 10^{-6} \text{ mho per mile}$$

The positive-sequence mutual inductive and capacitive impedances are negligible. The zero-sequence mutual impedance $z_{ab} = j1.30$ ohms per mile. The zero-sequence mutual capacitive impedance (determined with one circuit open) is $-jx_{ab} = -j0.125 \times 10^6$ ohm-miles.

Construct a nominal equivalent circuit for the two parallel lines, (*a*) not bussed at either end, (*b*) bussed at the sending end only, (*c*) bussed at both ends. (The capacitive admittance y_0 given for one line alone can be changed to capacitive impedance by taking its reciprocal, which is required in determining a capacitive impedance Y-connected circuit.)

Determine the fault current with a line-to-ground fault on phase *a* at *B*, as in Fig. 2 of Chapter IV, neglecting resistance. Compare this current with that obtained by using the sequence networks given in Figs. 2(*b*), (*c*), and (*d*) of Chapter IV, with capacitance neglected.

Problem 10. Assume that the transformer bank at *C* in Fig. 2(*a*), Chapter IV, is ungrounded and that circuit breakers at *B* have opened, leaving a line-to-ground fault on phase *a* of one of the lines at its sending end terminals. The equivalent excitation on machine *N* is 100% of base voltage referred to the line side of the transformer bank. Using the equivalent circuit for the two parallel lines determined in Problem 9, find the voltages to ground at *B* of the two unfaulted phases, neglecting resistance.

Problem 11. Construct the equivalent Π's for use in the positive- and zero-sequence networks for a transmission line 400 miles long operating at 50 cycles.

$$z_1 = 0.12 + j0.67 \text{ ohm per mile}; \qquad y_1 = j4.40 \times 10^{-6} \text{ mho per mile}$$
$$z_0 = 0.60 + j1.80 \text{ ohms per mile}; \qquad y_0 = j2.40 \times 10^{-6} \text{ mho per mile}$$

Problem 12. Construct an equivalent circuit consisting of two sections, each 200 miles long, using z_1 and y_1 given in Problem 11. By transformations reduce this equivalent circuit to a single Π and compare with the positive-sequence Π of Problem 11.

Problem 13. Derive the equivalent T and Π given in Figs. 1(*d*) and (*e*) for a line with distributed constants from equations [13]–[16]. (Problem for mathematicians only.)

Problem 14. Solve Problem 5 by the second method discussed in Problem 5.

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CHAPTER VII

SIMULTANEOUS FAULTS ON SYMMETRICAL THREE-PHASE SYSTEMS — ANALYSIS BY THE METHOD OF SYMMETRICAL COMPONENTS

The term " fault " is here used to denote an accidental departure from normal operating conditions. A short circuit or an open conductor constitutes a fault. When a fuse opens one end of a short-circuited conductor without clearing the short circuit, the short circuit and open conductor are simultaneous faults on the same conductor. Simultaneous faults may consist of two or more short circuits on the same or on different circuits, open conductors in two or more circuits, or any combinations of short circuits and open conductors. Since each fault affects the voltages and currents resulting from the other, simultaneous faults cannot be treated independently. Simultaneous short circuits and a short circuit and open conductor on the same phase of a symmetrical three-phase system are discussed in this chapter. Simultaneous faults are further discussed in Chapter X.

TWO SIMULTANEOUS SHORT CIRCUITS

As with one short circuit, system currents and voltages during two simultaneous short circuits may be determined analytically or by means of a calculating table. Both methods will be given here. As the faults may occur on the same or on opposite sides of a Δ-Y transformer bank, both cases will be included. Grounded and ungrounded systems will be considered.

Grounded System — No Δ-Y Transformer Bank between Faults

Let the two fault points be C and D, with the conductors at C indicated by a, b, c and those at D by A, B, C, where A and a are of the first phase, B and b of the second phase, and C and c of the third phase in the order of their normal time sequence. Let V_a, V_b, V_c and V_A, V_B, V_C indicate the voltages to ground of conductors a, b, c at C and A, B, C at D, respectively, with the corresponding currents flowing from the conductors into the fault indicated by I_a, I_b, I_c and I_A, I_B, I_C, respectively. Figure 1 shows two faults at different points C and D, with the conductor voltages to ground and currents flowing into the fault indicated by their assigned symbols. Positive direction for currents and their components is taken *into* the faults, as indicated by the arrows.

The symmetrical components of V_a and I_a at C are V_{a1}, V_{a2}, V_{a0} and I_{a1}, I_{a2}, I_{a0}, respectively. The symmetrical components of V_A and I_A at D are V_{A1}, V_{A2}, V_{A0} and I_{A1}, I_{A2}, I_{A0}, respectively. With

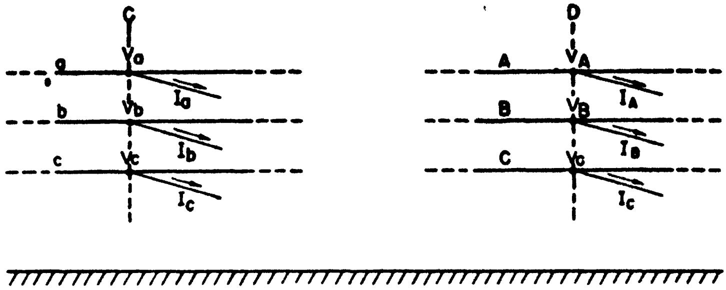


FIG. 1. Simultaneous short circuits at points C and D of a symmetrical three-phase system.

phase a as reference phase, these are the twelve unknowns to be determined. It will be shown that there are twelve independent equations connecting these twelve unknowns: three for each fault point, and two relating components of fault currents and voltages in each of the three sequence networks. These twelve equations are required in an analytic solution. When a calculating table is available, the sequence networks are connected to satisfy the equations which relate symmetrical components of currents and of voltages at both fault points; as explained later, the connections may be direct, through coupling transformers, or through phase shifters. As the six equations relating the components of voltage or current at the two fault points are required in both analytic solutions and in solutions on a calculating table, they will be developed first.

Equations Relating the Components of Fault Voltage or of Fault Current of Different Sequences. Although simultaneous short circuits cannot be treated independently, each may be considered separately to determine the equations relating the symmetrical components of current flowing into the fault, or of voltage to ground at the fault of the reference phase a. Such equations are determined in Chapter III for one short circuit involving various phases. Where there is only one fault on a symmetrical three-phase system, its location with respect to the reference phase may be arbitrarily chosen. With two faults, the location of one of them may be arbitrary, but that of the other will depend upon given fault conditions. For example, let it be stated that a double line-to-ground fault occurs at point C, and a single line-to-ground fault at point D on the normally leading phase of the two phases involved in the fault at point C; then, if the double line-to-ground fault at point C is arbitrarily located on phases b and c, the location of the line-to-ground fault at point D is of necessity on phase b (conductor B) for the assumed phase order abc.

TABLE I

FAULT EQUATIONS EXPRESSING RELATIONS BETWEEN THE SYMMETRICAL COMPONENTS OF I_a AND V_a

Phase a is reference phase. I_a is current flowing into the fault and V_a is voltage to ground at the fault.

Case A. Line-to-Ground Fault

(*a*) *Phase a*

$$I_{a0} = I_{a1} \qquad I_{a2} = I_{a1} \qquad V_{a1} = -(V_{a0} + V_{a2})$$

(*b*) *Phase b*

$$I_{a0} = a^2 I_{a1} \qquad I_{a2} = a I_{a1} \qquad V_{a1} = -(a V_{a0} + a^2 V_{a2})$$

(*c*) *Phase c*

$$I_{a0} = a I_{a1} \qquad I_{a2} = a^2 I_{a1} \qquad V_{a1} = -(a^2 V_{a0} + a V_{a2})$$

Case B. Line-to-Line Fault

(*a*) *Phases b and c*

$$I_{a0} = 0 \qquad I_{a2} = -I_{a1} \qquad V_{a2} = V_{a1}$$

(*b*) *Phases a and c*

$$I_{a0} = 0 \qquad I_{a2} = -a I_{a1} \qquad V_{a2} = a V_{a1}$$

(*c*) *Phases a and b*

$$I_{a0} = 0 \qquad I_{a2} = -a^2 I_{a1} \qquad V_{a2} = a^2 V_{a1}$$

Case C. Double Line-to-Ground Fault

(*a*) *Phases b and c*

$$I_{a1} = -(I_{a0} + I_{a2}) \qquad V_{a0} = V_{a1} \qquad V_{a2} = V_{a1}$$

(*b*) *Phases a and c*

$$I_{a1} = -(a I_{a0} + a^2 I_{a2}) \qquad V_{a0} = a^2 V_{a1} \qquad V_{a2} = a V_{a1}$$

(*c*) *Phases a and b*

$$I_{a1} = -(a^2 I_{a0} + a I_{a2}) \qquad V_{a0} = a V_{a1} \qquad V_{a2} = a^2 V_{a1}$$

Case D. Three-Phase Fault

(*a*) *Phases a, b, and c*

$$I_{a0} = 0 \qquad V_{a1} = 0 \qquad V_{a2} = 0$$

(*b*) *Phases a, b, c, and Ground*

$$V_{a1} = 0 \qquad V_{a2} = 0 \qquad V_{a0} = 0$$

Table I gives three equations connecting the symmetrical components of current flowing into the fault or of voltage to ground at the fault for short circuits involving various phases, phase a being reference phase. The equations given under cases $A(a)$, $B(a)$, $C(a)$, $D(a)$, and $D(b)$ are derived in Chapter III. Derivation of other equations of Table I is given in the following development:

Line-to-Line Fault between Phases a and b. The conditions at the fault are $V_a = V_b$; $I_a = -I_b$; $I_c = 0$. From these equations and those of Chapter II,

$$\begin{aligned} V_a - V_b &= V_{a1} + V_{a2} + V_{a0} - (a^2V_{a1} + aV_{a2} + V_{a0}) \\ &= (1 - a^2)V_{a1} + (1 - a)V_{a2} = 0 \\ I_{a1} &= \tfrac{1}{3}(I_a + aI_b + a^2I_c) = \tfrac{1}{3}I_a(1 - a) \\ I_{a2} &= \tfrac{1}{3}(I_a + a^2I_b + aI_c) = \tfrac{1}{3}I_a(1 - a^2) \\ I_{a0} &= \tfrac{1}{3}(I_a + I_b + I_c) = 0 \end{aligned} \qquad [1]$$

From [1], the relations between the symmetrical components of V_a and of I_a are

$$V_{a2} = -V_{a1}\frac{1 - a^2}{1 - a} = -V_{a1}(1 + a) = a^2V_{a1} \qquad [2]$$

$$I_{a2} = -a^2I_{a1} \qquad [3]$$

Line-to-Ground Fault on Phase b. The conditions at the fault are $V_b = 0$; $I_a = 0$; $I_c = 0$. It therefore follows that

$$\begin{aligned} V_b &= a^2V_{a1} + aV_{a2} + V_{a0} = 0 \\ I_{a1} &= \tfrac{1}{3}(aI_b) \\ I_{a2} &= \tfrac{1}{3}(a^2I_b) \\ I_{a0} &= \tfrac{1}{3}(I_b) \end{aligned} \qquad [4]$$

From [4],

$$V_{a1} = -a^2V_{a2} - aV_{a0} \qquad [5]$$

$$I_{a2} = aI_{a1} \qquad [6]$$

$$I_{a0} = a^2I_{a1} \qquad [7]$$

Double Line-to-Ground Fault on Phases a and c. The conditions at the fault are $V_a = 0$; $V_c = 0$; $I_b = 0$. From the fault equations,

$$\begin{aligned} V_a - V_c &= V_{a1} + V_{a2} + V_{a0} - (aV_{a1} + a^2V_{a2} + V_{a0}) \\ &= (1 - a)V_{a1} + (1 - a^2)V_{a2} = 0 \\ V_a + V_c &= V_{a1} + V_{a2} + V_{a0} + (aV_{a1} + a^2V_{a2} + V_{a0}) \\ &= (1 + a)V_{a1} + (1 + a^2)V_{a2} + 2V_{a0} = 0 \\ I_b &= a^2I_{a1} + aI_{a2} + I_{a0} = 0 \end{aligned} \qquad [8]$$

From [8],

$$V_{a2} = aV_{a1} \tag{9}$$

$$V_{a0} = a^2 V_{a1} \tag{10}$$

$$I_{a1} = -a^2 I_{a2} - aI_{a0} \tag{11}$$

The relations between the symmetrical components of V_a and I_a for a line-to-line fault on phases a and c, a line-to-ground fault on phase c, and a double line-to-ground fault on phases a and b can be determined from [1], [4], and [8], respectively, if a and a^2 are interchanged in these equations.

The three equations connecting the symmetrical components of I_a or of V_a at point C for various types of short circuits can be taken directly from Table I. If a, b, and c in Table I are replaced by A, B, and C, respectively, three equations relating the components of I_A or of V_A at point D are also obtained. Table I furnishes the six equations required when solutions are made on a calculating table. It also provides six of the twelve equations required in an analytic solution. These six equations give relations between the components of fault current or of fault voltage of different sequences and are independent of system impedances.

The other six equations needed in an analytic solution give relations between fault currents and voltages of the same sequence, and therefore depend upon the impedances in the three sequence networks of the symmetrical three-phase system.

Equations Relating Components of Fault Voltage and Fault Current of the Same Sequence. Equations [1]–[3] of Chapter IV express components of fault voltage in terms of components of fault current and the sequence impedances viewed from the fault, when there is but one short circuit. With two short circuits, there are two components of fault voltage and two components of fault current in each of the three sequence networks. If each sequence network is replaced[1] by an equivalent Δ or Y between the two fault points and the zero-potential bus for the network, two equations relating the components of fault currents and voltages of each sequence can be written (as explained below), thereby giving the six additional equations required in an analytic solution.

Zero-Sequence Network. In a symmetrical system, there are no generated zero-sequence voltages. The equivalent Y of the zero-sequence network between fault points C and D and the zero-potential bus for the network, here indicated by S, is shown in Fig. 2(a). The three branches of the equivalent Y are labeled C_0, D_0, and S_0 and are represented as inductive impedances. (This is true also for the equivalent Y's replacing the negative- and positive-sequence networks in

Figs. 2(*b*) and 3, respectively.) This is not intended to exclude capacitive impedances, or negative resistances which may result from the reduction of the sequence networks to equivalent Y's or Δ's.

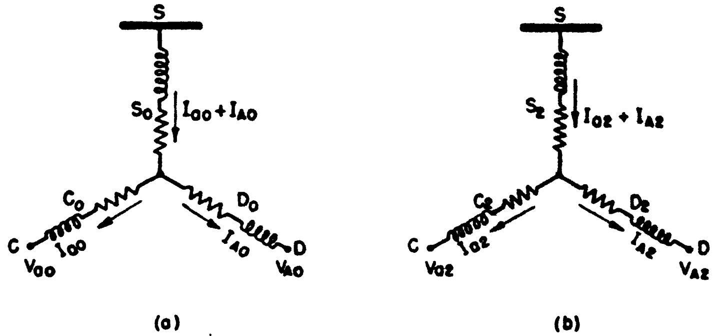


FIG. 2. Equivalent Y's to replace the sequence networks between fault points *C* and *D* and the zero-potential bus for the network, here indicated by *S*. (*a*) Equivalent Y of zero-sequence network. (*b*) Equivalent Y of negative-sequence network.

In Fig. 2(*a*), the zero-sequence impedance viewed from *C*, with no fault at *D*, is $C_0 + S_0$; that from *D*, with no fault at *C*, is $D_0 + S_0$. With positive direction for currents towards the faults, I_{a0} and I_{A0} flow from the zero-potential bus *S* through S_0, then I_{a0} flows through C_0 and I_{A0} through D_0. Before the simultaneous faults occurred there were no currents and no voltages in the zero-sequence network. Superposing the zero-sequence voltage rises from *S* to *C* and from *S* to *D* on the voltages at *C* and *D*, respectively, before the fault (or subtracting the voltage drops from zero), V_{a0} and V_{A0} are

$$V_{a0} = 0 - (I_{a0} + I_{A0})S_0 - I_{a0}C_0 = -I_{a0}(C_0 + S_0) - I_{A0}S_0 \quad [12]$$

$$V_{A0} = 0 - (I_{a0} + I_{A0})S_0 - I_{A0}D_0 = -I_{a0}S_0 - I_{A0}(D_0 + S_0) \quad [13]$$

Equations [12] and [13] express the zero-sequence components of voltage at the two points of fault in terms of the two zero-sequence currents flowing into the faults and the branch impedances of the equivalent Y which replaces the zero-sequence network between *C*, *D*, and *S*.

Solving [12] and [13], the currents I_{a0} and I_{A0} are expressed in terms of V_{a0} and V_{A0}:

$$I_{a0} = -V_{a0}\frac{D_0 + S_0}{\Delta_0} + V_{A0}\frac{S_0}{\Delta_0} \quad [14]$$

$$I_{A0} = V_{a0}\frac{S_0}{\Delta_0} - V_{A0}\frac{C_0 + S_0}{\Delta_0} \quad [15]$$

where $\quad \Delta_0 = C_0D_0 + C_0S_0 + D_0S_0$

Equations [14] and [15] are not independent of [12] and [13]; there are only two independent equations connecting the four unknowns, V_{a0}, V_{A0}, I_{a0}, and I_{A0}, in the zero sequence network.

Negative-Sequence Network. In a symmetrical system, there are no generated negative-sequence voltages. The negative-sequence network, just as the zero-sequence network, can be replaced by an equivalent Y or Δ between the points of fault C and D and the zero-potential bus for the network, here indicated by S. The equivalent Y of the negative-sequence network between points C, D, and S is shown in Fig. 2(b). The branch impedances are indicated by C_2, D_2, and S_2. The negative-sequence impedance viewed from C, with no fault at D, is $C_2 + S_2$; that from D, with no fault at C, is $D_2 + S_2$.

From Fig. 2(b), with positive direction for currents towards the faults, the two negative-sequence voltages in terms of the two negative-sequence currents and the impedances of the Y are

$$V_{a2} = -I_{a2}(C_2 + S_2) - I_{A2}S_2 \quad [16]$$

$$V_{A2} = -I_{a2}S_2 - I_{A2}(D_2 + S_2) \quad [17]$$

Solving [16] and [17], the currents I_{a2} and I_{A2} are expressed in terms of V_{a2} and V_{A2}:

$$I_{a2} = -V_{a2}\frac{D_2 + S_2}{\Delta_2} + V_{A2}\frac{S_2}{\Delta_2} \quad [18]$$

$$I_{A2} = V_{a2}\frac{S_2}{\Delta_2} - V_{A2}\frac{C_2 + S_2}{\Delta_2} \quad [19]$$

where

$$\Delta_2 = C_2D_2 + C_2S_2 + D_2S_2$$

Equations [18] and [19] are not independent of [16] and [17]; there are only two independent equations connecting the four unknowns, V_{a2}, V_{A2}, I_{a2}, and I_{A2} in the negative-sequence network.

Positive-Sequence Network. The positive-sequence impedance network without generated voltages, just as the zero- and negative-sequence impedance networks, can be replaced by an equivalent Y or Δ between the zero-potential bus for the network, here indicated by S, and the two fault points C and D. The equivalent Y is shown in Fig. 3(a) with branch impedances S_1, C_1, and D_1. The positive-sequence impedance viewed from C, with no fault at D, is $C_1 + S_1$; that from D, with no fault at C, is $D_1 + S_1$. The positive-sequence network of a symmetrical system differs from the negative- and zero-sequence networks in that there are generated voltages in all synchronous machines.

If the system is operating at *no load*, neglecting charging currents, the per unit generated voltages and the voltages at all points in the positive-sequence network are equal and in phase. Let the voltage of phase a be E_a. For this case, the internal voltages of all the machines can be replaced by a voltage E_a between the zero-potential bus S for the network and the branch impedance S_1 of the equivalent Y, as in Fig. 3(b). From Fig. 3(b), subtracting the voltage drops resulting from the fault from the voltages at C and D before the fault,

$$V_{a1} = E_a - I_{a1}(C_1 + S_1) - I_{A1}S_1 \qquad [20]$$

$$V_{A1} = E_a - I_{a1}S_1 - I_{A1}(D_1 + S_1) \qquad [21]$$

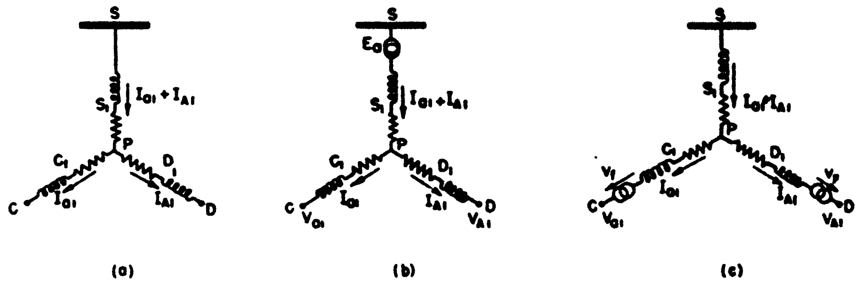


FIG. 3. Positive-sequence equivalent Y's between the zero-potential bus S for the network and the fault points C and D. (a) All generated voltages equated to zero. (b) System operating at no load before the fault. (c) System under load with voltages V_f and V_F at C and D, respectively, before the fault.

If the system is operating *under load*, the voltages and currents throughout the system are determined by the given operating condition. Let the voltages of phase a at C and D before the faults be indicated by V_f and V_F, respectively. When the fault occurs, the positive-sequence voltages at C and D become V_{a1} and V_{A1}, respectively. The positive-sequence currents flowing from the system at C and D before the fault are zero; when the fault occurs, they become I_{a1} and I_{A1}, respectively. The changes in currents are, therefore, I_{a1} and I_{A1}. Subtracting the voltage drops (or superposing the voltage rises) caused by the fault currents flowing from the zero-potential bus through the network to the fault points from the voltages V_f and V_F at C and D, respectively, before the fault occurred, V_{a1} and V_{A1} are

$$V_{a1} = V_f - I_{a1}(C_1 + S_1) - I_{A1}S_1 \qquad [22]$$

$$V_{A1} = V_F - I_{a1}S_1 - I_{A1}(D_1 + S_1) \qquad [23]$$

Equations [22] and [23] are satisfied by the equivalent circuit shown in Fig. 3(c), where *series* voltages V_f and V_F are inserted between the impedances C_1 and D_1, and fault points C and D, respectively. V_f

and V_F represent voltage *rises* in the direction PC and PD, respectively, as indicated by the accompanying arrows. Figure 3(c) is an equivalent circuit for the positive-sequence network to be used for determining positive-sequence voltages at the faults and positive-sequence currents flowing into the faults. In this equivalent circuit, V_{a1}, V_{A1}, I_{a1}, and I_{A1} are unknown quantities which cannot be determined from the positive-sequence network alone; but the voltages V_{a1} and V_{A1} in terms of I_{a1}, I_{A1}, and the known quantities V_f, V_F, S_1, C_1, D_1 are given by [22] and [23]. If $V_F = V_f$, [22] and [23] are similar in form to [20] and [21], respectively, and therefore Fig. 3(b) can be used with V_f replacing E_a.

Analytic Determination of Currents and Voltages

It has been shown that there are twelve equations connecting the twelve unknown components of current and voltage of phase a at the two fault points, three for each fault point and two for each of the three sequence networks. It is proposed to eliminate the eight unknown negative- and zero-sequence components, leaving four equations in terms of the four positive-sequence components. Two of these four equations will be in terms of the negative- and zero-sequence system impedances, and will therefore correspond to [4] of Chapter IV for one short circuit. The other two equations, given by [22] and [23] or by [20] and [21], involve positive-sequence quantities only; they correspond to [1] of Chapter IV for one short circuit. The four positive-sequence equations will be solved for the two positive-sequence currents flowing into the faults and the two positive-sequence voltages at the fault. From the positive-sequence components at the faults and the relations between the symmetrical components at the two fault points, the negative- and zero-sequence components at the faults will be determined. From the symmetrical components of currents and voltages at the faults and the sequence networks, the symmetrical components of current and voltage throughout the system can be obtained.

The two positive-sequence equations in terms of the negative- and zero-sequence system impedances will be considered for the purpose of determining an equivalent circuit to replace the two faults in the positive-sequence network.

Solution of Ten Simultaneous Equations. The six equations for the two fault points from Table I, [12] and [13] from the zero-sequence network, and [16] and [17] from the negative-sequence network give the ten equations needed to eliminate the eight unknowns V_{a0}, V_{A0}, I_{a0}, I_{A0}, V_{a2}, V_{A2}, I_{a2}, and I_{A2}, so that the components of voltage

V_{a1} and V_{A1} may be expressed in terms of the components of current, I_{a1} and I_{A1}, and the known zero- and negative-sequence impedances of the system. When the ten equations are linear, the two resulting equations can be put in the form:

$$V_{a1} = kI_{a1} + mI_{A1} \qquad [24]$$

$$V_{A1} = nI_{a1} + lI_{A1} \qquad [25]$$

where k, l, m, and n depend upon the branch impedances of the equivalent Y's replacing the negative- and zero-sequence networks and the particular combination of conductors involved in the simultaneous faults. They are independent of positive-sequence impedances and of operating conditions, except as operating conditions affect negative- and zero-sequence impedances.

Equations [24] and [25] together with [20] and [21], or [22] and [23], give the four equations required for determining I_{a1}, I_{A1}, V_{a1}, and V_{A1}. Before solving these equations, the constants k, l, m, and n in [24] and [25] will be discussed.

Values of k, l, m, and n in terms of the branch impedances C_2, D_2, S_2, C_0, D_0, S_0 of the equivalent Y's which replace the negative- and zero-sequence networks (determined in reference 1) are tabulated in Table II for various combinations of phases which may be involved in simultaneous short circuits at two points of a grounded three-phase system. For some of the cases[2], equivalent Δ's and their admittances rather than equivalent Y's and their impedances would have given simpler equations and a reduction of work; for consistency, the equivalent Y's with impedance branches are used throughout.

The reduction of ten equations to two equations of the form given by [24] and [25], from which k, m, n, and l can be obtained, is given for the following case to illustrate the procedure.

Line-to-Ground Faults on Conductor A at D and Conductor b at C. From Table I, the three equations with the fault at D are

$$V_{A1} = -V_{A2} - V_{A0} \qquad [26]$$

$$I_{A2} = I_{A1} \qquad [27]$$

$$I_{A0} = I_{A1} \qquad [28]$$

With the fault at C, they are

$$V_{a1} = -(a^2 V_{a2} + a V_{a0}) \qquad [29]$$

$$I_{a2} = aI_{a1} \qquad [30]$$

$$I_{a0} = a^2 I_{a1} \qquad [31]$$

Equations [26]–[31], together with [12], [13], [16], and [17], are the ten equations to be solved. Replacing I_{A2} and I_{A0} by I_{A1}, I_{a2} and I_{a0}

by aI_{a1} and a^2I_{a1}, respectively, in [12], [13], [16], and [17], and then substituting [12] and [16] in [29] and [13] and [17] in [26].

$$\begin{aligned} V_{a1} &= I_{a1}(C_0 + S_0) + aI_{A1}S_0 + I_{a1}(C_2 + S_2) + a^2I_{A1}S_2 \\ &= I_{a1}(C_0 + S_0 + C_2 + S_2) + I_{A1}(aS_0 + a^2S_2) \end{aligned} \qquad [32]$$

$$\begin{aligned} V_{A1} &= a^2I_{a1}S_0 + I_{A1}(D_0 + S_0) + aI_{a1}S_2 + I_{A1}(D_2 + S_2) \\ &= I_{a1}(a^2S_0 + aS_2) + I_{A1}(D_0 + S_0 + D_2 + S_2) \end{aligned} \qquad [33]$$

From [32] and [33] and [24] and [25],

$k = (C_2 + S_2) + (C_0 + S_0) =$ sum of negative- and zero-sequence impedances viewed from C with no fault at D [34]

$l = (D_2 + S_2) + (D_0 + S_0) =$ sum of negative- and zero-sequence impedances viewed from D with no fault at C [35]

$$m = aS_0 + a^2S_2 = -\tfrac{1}{2}(S_0 + S_2) + j\frac{\sqrt{3}}{2}(S_0 - S_2) \qquad [36]$$

$$n = a^2S_0 + aS_2 = -\tfrac{1}{2}(S_0 + S_2) - j\frac{\sqrt{3}}{2}(S_0 - S_2) \qquad [37]$$

TABLE II

VALUES OF k, l, m, AND n TO BE SUBSTITUTED IN EQUATIONS [24] AND [25] FOR SIMULTANEOUS SHORT CIRCUITS AT TWO POINTS ON THE SAME SIDE OF A Δ-Y TRANSFORMER BANK

Let $Z_{cs} = C_0 + S_0 + C_2 + S_2$; $\Delta_0 = C_0D_0 + C_0S_0 + D_0S_0$

$Z_{ds} = D_0 + S_0 + D_2 + S_2$; $\Delta_2 = C_2D_2 + C_2S_2 + D_2S_2$

$Z_{cds} = (C_0 + C_2)(D_0 + D_2) + (C_0 + C_2)(S_0 + S_2) + (D_0 + D_2)(S_0 + S_2)$

Case A. *Single Line-to-Ground Faults at Two Points*

(a) Phases a and A	(b) Phases b and A	(c) Phases c and A
$k = Z_{cs}$	$k = Z_{cs}$	$k = Z_{cs}$
$n = S_0 + S_2$	$n = a^2S_0 + aS_2$	$n = aS_0 + a^2S_2$
$m = S_0 + S_2$	$m = aS_0 + a^2S_2$	$m = a^2S_0 + aS_2$
$l = Z_{ds}$	$l = Z_{ds}$	$l = Z_{ds}$

Case B. *Line-to-Line Faults at Two Points*

(a) Phases b, c and B, C	(b) Phases a, c and B, C	(c) Phases a, b and B, C
$k = C_2 + S_2$	$k = C_2 + S_2$	$k = C_2 + S_2$
$n = S_2$	$n = aS_2$	$n = a^2S_2$
$m = S_2$	$m = a^2S_2$	$m = aS_2$
$l = D_2 + S_2$	$l = D_2 + S_2$	$l = D_2 + S_2$

Case C. *Double Line-to-Ground Faults at Two Points*

(*a*) Phases b, c and B, C

$$k = \frac{\Delta_2(C_0 + S_0) + \Delta_0(C_2 + S_2)}{Z_{cds}}$$
$$n = \frac{\Delta_2 S_0 + \Delta_0 S_2}{Z_{cds}}$$
$$m = \frac{\Delta_2 S_0 + \Delta_0 S_2}{Z_{cds}}$$
$$l = \frac{\Delta_2(D_0 + S_0) + \Delta_0(D_2 + S_2)}{Z_{cds}}$$

(*b*) Phases a, c and B, C

$$k = \frac{\Delta_2(C_0 + S_0) + \Delta_0(C_2 + S_2)}{Z_{cds} + 3S_0S_2}$$
$$n = \frac{a^2 S_0\Delta_2 + aS_2\Delta_0}{Z_{cds} + 3S_0S_2}$$
$$m = \frac{aS_0\Delta_2 + a^2 S_2\Delta_0}{Z_{cds} + 3S_0S_2}$$
$$l = \frac{\Delta_2(D_0 + S_0) + \Delta_0(D_2 + S_2)}{Z_{cds} + 3S_0S_2}$$

(*c*) Phases a, b and B, C: Similar to C (*b*) with a and a^2 interchanged in n and m.

Case D. *Three-Phase Faults at Two Points:* $k = n = m = l = 0$.

Case E. *Line-to-Line Fault at C and Single Line-to-Ground Fault at D*

(*a*) Phases b, c, and A

$$k = C_2 + S_2$$
$$n = -S_2$$
$$m = -S_2$$
$$l = Z_{ds}$$

(*b*) Phases a, c, and A

$$k = C_2 + S_2$$
$$n = -aS_2$$
$$m = -a^2S_2$$
$$l = Z_{ds}$$

(*c*) Phases a, b, and A

$$k = C_2 + S_2$$
$$n = -a^2S_2$$
$$m = -aS_2$$
$$l = Z_{ds}$$

Case F. *Double Line-to-Ground Fault at C. Single Line-to-Ground Fault at D*

(*a*) Phases b, c, and A

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}}$$
$$n = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$
$$m = -\frac{S_0(C_2 + S_2) + S_2(C_0 + S_0)}{Z_{cs}}$$
$$l = Z_{ds} - \frac{(S_0 - S_2)^2}{Z_{cs}}$$

(*b*) Phases a, c, and A

$$k = \frac{(C_0 + S_0)(C_2 + S_2)}{Z_{cs}}$$
$$n = -\frac{a^2S_0(C_2 + S_2) + aS_2(C_0 + S_0)}{Z_{cs}}$$
$$m = -\frac{aS_0(C_2 + S_2) + a^2S_2(C_0 + S_0)}{Z_{cs}}$$
$$l = Z_{ds} - \frac{S_0^2 + S_0S_2 + S_2^2}{Z_{cs}}$$

(*c*) Phases a, b, and A: Similar to F (*b*) with a and a^2 interchanged in n and m.

Case G. *Three-Phase Fault at C. Single Line-to-Ground at D on Phase A*

(*a*) Three-phase fault.

$$k = n = m = 0$$
$$l = D_0 + S_0 + D_2 + \frac{C_2S_2}{C_2 + S_2}$$

(*b*) Three-phase fault involving ground.

$$k = n = m = 0$$
$$l = D_0 + \frac{C_0S_0}{C_0 + S_0} + D_2 + \frac{C_2S_2}{C_2 + S_2}$$

The impedances k and l in [24] and [25] are effective self-impedances met by I_{a1} and I_{A1}, respectively, flowing from the positive-sequence network into the faults; m and n are effective mutual impedances associated with I_{A1} and I_{a1}, respectively. From a study of Table II, it may be seen that, with a line-to-line or a line-to-ground fault at one fault point, k or l at the other fault point is the equivalent circuit which would replace the fault in the positive-sequence network at that point if there were but one fault. (See equations [34] and [35].) With a double line-to-ground or a three-phase fault at one fault point, k or l at the other fault point is not so simply determined. In the solution of ten simultaneous equations, the conditions relating to negative- and zero-sequence currents and voltages imposed by the simultaneous faults are satisfied; therefore, one of the faults is not effectively removed by equating I_{A1} in [24] or I_{a1} in [25] to zero *unless* the corresponding negative- and zero-sequence components of fault current also become zero. This will be illustrated for the case of a three-phase fault not involving ground at C and a line-to-ground fault on conductor A at D. The fault equations at C are

$$V_{a1} = 0; \ V_{a2} = 0; \ I_{a0} = 0 \quad [38]$$

Those at D are

$$I_{A1} = I_{A2} = I_{A0};$$
$$V_{A1} + V_{A2} + V_{A0} = 0 \quad [39]$$

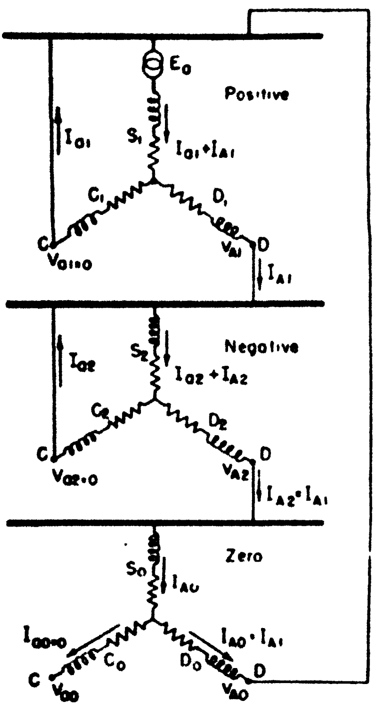


FIG. 4. Connection of the positive-, negative-, and zero-sequence equivalent Y's of a system operated at no load for a three-phase fault at C not involving ground and a line-to-ground fault on conductor A at D.

Equations [38] and [39] are satisfied by the connections indicated in Fig. 4, where the sequence networks are shown as equivalent Y's. The effective self-impedance l met by I_{A1} flowing into the fault, calculated from Fig. 4, is

$$l = \frac{C_2 S_2}{C_2 + S_2} + D_2 + S_0 + D_0 \quad [40]$$

The value of l in [40] is tabulated in Table II, Case $G(a)$. For this case, l is not the equivalent circuit which replaces the line-to-ground fault at D with the three-phase fault removed.

If the three-phase fault involves ground, from Table I, $V_{a0} = 0$ as well as $V_{a2} = 0$. The additional condition $V_{a0} = 0$ requires that point C in Fig. 4 be connected to the zero-potential bus in the zero-sequence network. With $V_{a2} = 0$ and $V_{a0} = 0$, I_{A1} flows through S_0 and C_0 in parallel as well as through S_2 and C_2 in parallel, the value of l met by I_{A1} being

$$l = \frac{C_2S_2}{C_2 + S_2} + D_2 + \frac{C_0S_0}{C_0 + S_0} + D_0$$

This value is tabulated in Table II, Case $G(b)$.

Solution of Four Positive-Sequence Equations and Determination of System Currents and Voltages. Equations [24] and [25] express V_{a1} and V_{A1} in terms of I_{a1} and I_{A1} and k, l, m, and n, which are functions of the negative- and zero-sequence system impedances. Equations [22] and [23] express V_{a1} and V_{A1} in terms of I_{a1} and I_{A1}, the impedances C_1, D_1, and S_1 of the positive-sequence equivalent Y, and V_f and V_F the voltages of phase a at C and D, respectively, before the fault. Eliminating V_{a1} and V_{A1} from these four equations,

$$V_f = I_{a1}(C_1 + S_1 + k) + I_{A1}(S_1 + m) \qquad [41]$$

$$V_F = I_{a1}(S_1 + n) + I_{A1}(D_1 + S_1 + l) \qquad [42]$$

Solving [41] and [42] for I_{a1} and I_{A1},

$$I_{a1} = \frac{V_f}{\Delta_1}(D_1 + S_1 + l) - \frac{V_F}{\Delta_1}(S_1 + m) \qquad [43]$$

$$I_{A1} = -\frac{V_f}{\Delta_1}(S_1 + n) + \frac{V_F}{\Delta_1}(C_1 + S_1 + k) \qquad [44]$$

where

$$\Delta_1 = (C_1 + S_1 + k)(D_1 + S_1 + l) - (S_1 + m)(S_1 + n) \qquad [45]$$

If $V_F = V_f$,

$$I_{a1} = \frac{V_f}{\Delta_1}(D_1 + l - m) \qquad [46]$$

$$I_{A1} = \frac{V_f}{\Delta_1}(C_1 + k - n) \qquad [47]$$

where Δ_1 is defined by [45].

I_{a1} and I_{A1} are given by [43] and [44] or [46] and [47] in terms of known voltages and impedances. Knowing I_{a1} and I_{A1}, V_{a1} and V_{A1} can be obtained from [24] and [25], respectively; and the negative- and zero-sequence components from the fault equations and [12], [13], [16], and [17]. The phase currents and voltages at the fault are calculated by substituting the symmetrical components of I_a and V_a in the equations of Chapter II. The negative- and zero-sequence system currents and voltages are determined by calculation from the negative- and zero-sequence components of fault currents and voltages and the complete negative- and zero-sequence networks, respectively. The *changes* in positive-sequence system currents and voltages resulting from the faults can be determined from the positive-sequence fault currents and the complete positive-sequence network with internal generated voltages equated to zero. The currents due to the faults superimposed upon load currents before the fault give total initial symmetrical rms positive-sequence currents; the voltage drops in the system due to the fault currents subtracted from the voltages at various system points before the fault give initial symmetrical rms positive-sequence system voltages.

When the positive-sequence network contains many loops, analytic determination of positive-sequence currents and voltages is simplified by the use of an equivalent circuit to replace both faults in the positive-sequence network.

Equivalent Circuits to Replace Both Faults in the Positive-Sequence Network. Equations [24] and [25] may be written

$$V_{a1} = (k - n)I_{a1} + \frac{m+n}{2}(I_{a1} + I_{A1}) + \frac{m-n}{2}(I_{A1} - I_{a1}) \quad [48]$$

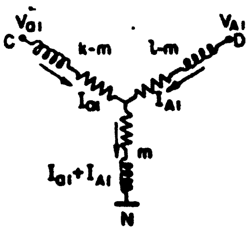
$$V_{A1} = (l - m)I_{A1} + \frac{m+n}{2}(I_{a1} + I_{A1}) + \frac{m-n}{2}(I_{A1} - I_{a1}) \quad [49]$$

m and n Equal. When m and n are equal [48] and [49] become

$$V_{a1} = (k - m)I_{a1} + m(I_{a1} + I_{A1}) \quad [50]$$

$$V_{A1} = (l - m)I_{A1} + m(I_{a1} + I_{A1}) \quad [51]$$

The relations expressed in [50] and [51] are satisfied if the fault is replaced by an equivalent Y in the positive-sequence network with branch impedances $(k - m)$, $(l - m)$, and m, connecting the points C, D, and the zero-potential bus, here indicated by N. The equivalent circuit and the connection of this equivalent circuit at points C and D of the complete positive-sequence network are shown in Fig. 5. For simplicity, the network is represented as a rectangle with one side a heavy line to indicate the zero-potential bus for the network. The internal generated voltages of all synchronous machines of the system are understood to be present. The fault points C and D are shown as points within the rectangle with leads brought out to which the equivalent circuit is connected to satisfy fault conditions.



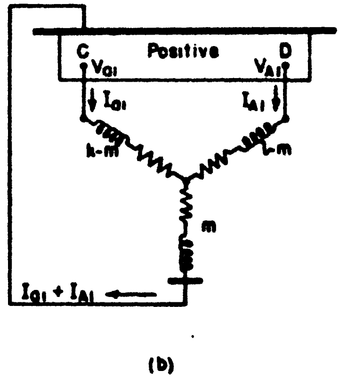


FIG. 5. (*a*) Equivalent Y to replace two short circuits in the positive-sequence network for special case of $m = n$ in [24] and [25]. (*b*) Positive-sequence network with equivalent circuit replacing two simultaneous short circuits when $m = n$ in [24] and [25].

The equivalent circuit in Fig. 5 is a function of the negative- and zero-sequence system impedances only; and, since these impedances are substantially the same for initial, transient, or steady-state operation, the equivalent circuit is a general one suitable for determining positive-sequence currents and voltages during initial, transient, or steady-state conditions.

Solution by Means of a Calculating Table

When a calculating table is used, it is desirable to set up the complete positive-, negative-, and zero-sequence networks and to connect them so that the relations between the symmetrical components of current and voltage of phase a at both fault points are satisfied. These relations are given in Table I for various types of short circuits, involving various phases.

Short Circuits Symmetrical with Respect to the Reference Phase. A line-to-ground fault on phase a and a line-to-line or a double line-to-ground fault on phases b and c are symmetrical with respect to the reference phase a. For those cases in which both faults are symmetrical with respect to phase a, direct connections of the sequence networks to satisfy the fault equations at one fault point (considered alone) can always be made. For the second fault point, direct connections can be made only if the three fault equations for both fault points are simultaneously satisfied. Direct connection of the sequence networks to satisfy the equations for both faults is illustrated in Fig. 4. When one of the faults is a three-phase fault, and also with line-to-line or double line-to-ground faults involving the same phases at both faults, the connections between the sequence networks made for each fault point are those which would be made if there were but one fault. These connections, shown in Fig. 6 where each sequence network is

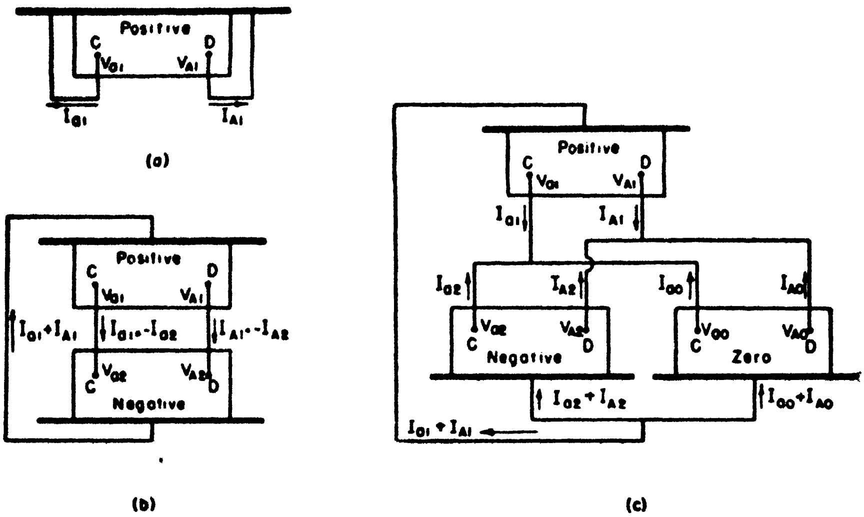


FIG. 6. Direct connections of the sequence networks for simultaneous faults. (*a*) Three-phase faults at *C* and *D*. (*b*) Line-to-line faults at *C* and *D* between phases *b* and *c* at both faults. (*c*) Double line-to-ground faults at *C* and *D* on phases *b* and *c* at both faults.

represented by a rectangle with points *C* and *D* within the rectangle and the zero-potential bus a heavy line, can be made on either an a-c or a d-c calculating table.

With one of the faults a line-to-ground fault on phase *a* and the other a line-to-ground, line-to-line, or double line-to-ground fault symmetrical with respect to phase *a*, direct connections of the sequence networks to satisfy voltage conditions at both faults may not (and in general do not) satisfy current conditions. Expressed in another way, voltage restrictions are introduced by direct connections for both faults which may not be true restrictions. For such cases, Dr. E. W. Kimbark[3] represents one fault by connecting the sequence networks through 1 : 1 turn ratio mutual coupling transformers; for the other fault, direct connection of the sequence networks is made. Figure 7 illustrates the connection of the sequence networks through 1 : 1 turn ratio transformers for a line-to-ground fault on phase a at C, direct connection of the sequence network being made for the line-to-ground fault on phase a at D. If the connections for the fault at C had been made in the same way as for the fault at D, the relations $V_{a1} = V_{A1}$, $V_{a2} = V_{A2}$, and $V_{a0} = V_{A0}$ would have been introduced. Except for fault points symmetrically located in all three sequence networks (for example, faults involving the same phase of two identical parallel lines bussed at both ends) these restricting equations are untrue.

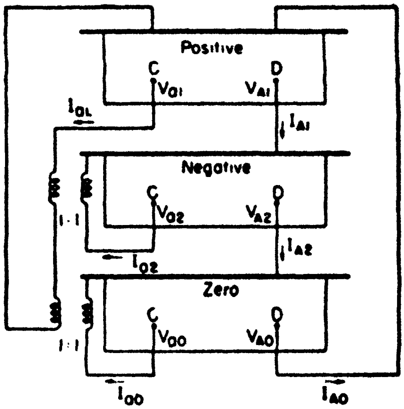


FIG. 7. Connection of the sequence networks through transformers of 1 : 1 turn ratio. Line-to-ground faults at C and D both on phase a.

One of Two Short Circuits Unsymmetrical with Respect to the Reference Phase. With the phases so named that the faulted phase or phases at one fault point are symmetrical with respect to phase a, while those at the other point are unsymmetrical, direct connections or connections by means of coupling transformers between the sequence networks can be made for the fault symmetrical with respect to phase a. Because of the operators a and a^2 in the fault equations of Table I which must be satisfied, direct connections or connections by means of coupling transformers cannot be made at the other fault point. Phase converters capable of rotating fault voltages and currents through 120° and 240° would be required.[3]

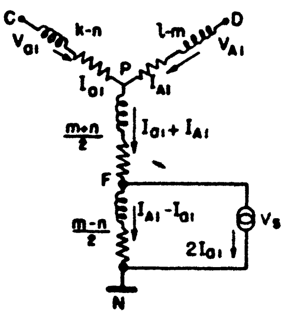


FIG. 8. A-c network analyzer equivalent circuit to replace two faults in the positive-sequence network when $m \neq n$ in [24] and [25].

An alternate method of solution[1] is by means of an equivalent circuit which replaces both faults in the positive-sequence network. By this method, the equivalent Y's to replace the negative- and zero-sequence networks are determined analytically or on the calculating table, and k, l, m, and n in terms of C_2, D_2, S_2, C_0, D_0, and S_0 read from Table II corresponding to the given fault conditions. Figure 8 shows a general equivalent circuit to replace the two short circuits in the positive-sequence network when an a-c calculating table is used. This equivalent circuit consists of a Y having branch impedances $k - n$, $l - m$, and $(m + n)/2$ connected between points C, D, and F; and between F and the zero-potential bus N, an impedance $(m - n)/2$ [or $(n - m)/2$] paralleled by an adjustable voltage V_e. Equations [48] and [49] will be satisfied if current $(I_{A1} - I_{a1})$ is made to flow through the impedance $(m - n)/2$, or if current $(I_{a1} - I_{A1})$ flows through the impedance $(n - m)/2$. If the voltage, V_e, is adjusted in phase and magnitude until the current through it is double I_{a1}, the current entering the fault at C, then the current $(I_{A1} - I_{a1})$ will flow in the impedance $(m - n)/2$. If the impedance $(n - m)/2$ is used, V_e must be adjusted until the current through it is double I_{A1}, the current entering the fault at D. (See reference 4.)

When a *d-c calculating table* is used and a direct connection between the sequence networks cannot be made for both faults, the equivalent circuit shown in Fig. 5 can replace both faults in the positive sequence network for those cases where $m = n$.

A study of Table II shows that *with resistance and capacitance neglected*, k and l have no resistance components, but are positive reactive impedances larger in magnitude than m and n. When m and n are equal, they also have no real components, but are positive or negative reactive impedances: $m = n = \pm jx$. When m and n are unequal, they have real components which are equal in magnitude and opposite in sign, while their reactive components are equal in magnitude and of the same sign: $m = \pm r \pm jx$, $n = \mp r \pm jx$. The error made by neglecting the real components of m and n will ordinarily be no greater than the error made by neglecting line resistances and capacitances. When the real components of m and n are neglected, $(m - n)/2 = 0$, and the equivalent circuit in Fig. 8 becomes an impedance Y connecting C, D, and the zero-potential bus N with branch impedances $(k - n)$, $(l - m)$ and $(m + n)/2$, as in Fig. 9. The impedances $(k - n)$, and $(l - m)$ will be positive reactive impedances and therefore can be represented on the d-c calculating table, while $(m + n)/2$ may be either positive or negative; if positive, it can also be represented on the d-c table. If $(m + n)/2$ is negative, the following procedure is suggested: The branch impedance $(m + n)/2$ which is connected to the zero-potential bus for the positive-sequence network (as are the neutrals of the generators) is in series with the genera-

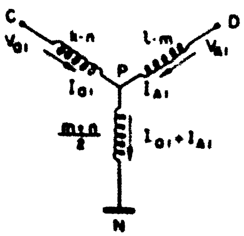


FIG. 9. Approximate equivalent circuit to replace two faults in the positive-sequence network when $m \neq n$ in [24] and [25].

tor reactances. If there is but one generating source, $(m + n)/2$ may be combined with its reactance. With several synchronous machines, or groups of machines, $(m + n)/2$ between P and N in Fig. 9 can be set to zero and the distribution of currents obtained, these currents to be increased by the ratio $X_p/\left(X_p + \frac{m+n}{2}\right)$, where X_p is the equivalent impedance between generator neutrals and P, and is found by dividing generator voltage by total current when P is shorted to N.

Simultaneous Faults on Opposite[3] Sides of a Δ-Y Transformer Bank

A Δ-Y transformer bank usually divides the zero-sequence system into two parts which have no connection with each other in the zero-sequence network. The positive- and negative-sequence equivalent circuits for three-phase power systems, used in analytic calculations or on a calculating table, are based on equivalent Y-Y transformer banks. With any circuit selected as reference circuit and any voltage or current vector in that circuit (or referred to that circuit) as reference vector, the positive-sequence currents and voltages in a circuit separated from the reference circuit by a Δ-Y transformer bank are correctly determined in magnitude and in phase relative to each other, but their phases relative to the system reference vector are given for an equivalent Y-Y transformer bank and not for the actual Δ-Y bank. To refer them to the system reference vector, a phase correction must be applied. This is true also for negative-sequence currents and voltages. The phase correction to be applied to negative-sequence currents and voltages to refer them to the system reference vector is not the same as that for positive-sequence currents and voltages. As explained in Chapter III, and illustrated in Figs. 19(a) and (b), Chapter III, there are two possible connections of a Δ-Y transformer bank. With either connection, the positive-sequence phase correction can be determined from the angular displacement at no load with magnetizing current neglected between the voltages to neutral of the reference phases on the two sides of the bank. Consider the transformer connection diagram given in Fig. 10(a). This diagram may also be used as a positive-sequence no-load voltage vector diagram. Let the circuit at the transformer terminals D with phases labeled ABC be arbitrarily chosen as the reference circuit, and phase A the reference phase. In the circuit at transformer terminals C, the phase to be selected as reference phase (that is the phase which determines the positive-sequence phase correction for the Δ-Y bank) is arbitrary; any one of the three may be chosen. Following the convention adopted in Chapter III, the reference phase is designated a and so chosen that the line-to-neutral voltage V_{a1} at no load and no faults is 90° out of phase with the line-to-neutral voltage V_{A1}. Whether V_{a1} leads or lags V_{A1} is determined from the transformer connection diagram. In Fig. 10(*a*), V_{a1} lags V_{A1} by 90°; and, as shown in Chapter III, V_{a2} leads V_{A2} by 90° for this connection. Positive-sequence

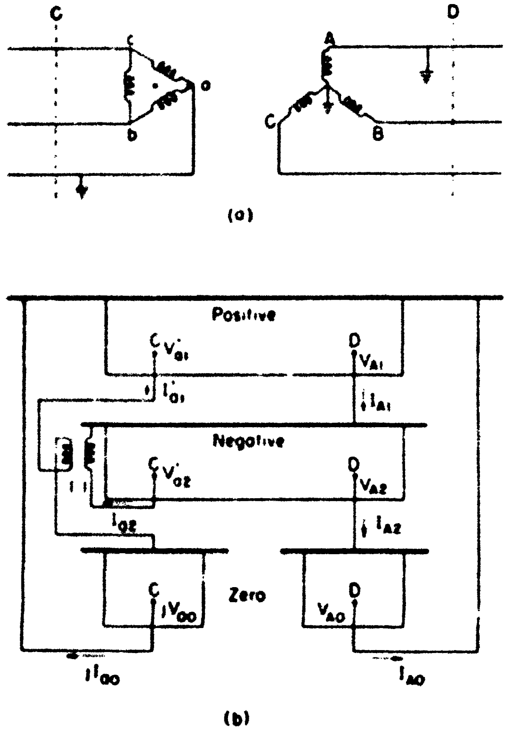


FIG. 10. (*a*) Connection diagram of a Δ-Y transformer bank with line-to-ground faults on conductors *A* and *a* as indicated. (*b*) Connections of the sequence networks for solution on an a-c network analyzer. Line-to-ground faults as indicated in (*a*). Circuit *D* is reference circuit. There is no connection between the zero-sequence impedances viewed from *C* and *D*.

currents and voltages in circuit *C*, determined from the positive-sequence network based on equivalent Y-Y transformers with the circuit at *D* as reference circuit, are therefore to be turned backward through 90° or multiplied by $-j$ to be referred to the system reference vector. Negative-sequence currents and voltages in circuit *C* determined from the negative-sequence network are to be turned forward through 90° or multiplied by j. In calculating system currents and voltages by means of symmetrical components when there is a single fault, the circuit in which the fault occurs is selected as the reference circuit and the sequence networks connected to satisfy fault equations. Positive- and negative-sequence currents and voltages throughout the system are determined in magnitude and phase based on equivalent Y-Y transformer banks. Before combining the components of current and voltage in circuits separated from the reference circuit by a Δ-Y transformer bank, it is necessary to apply the phase corrections resulting from the presence of Δ-Y transformer banks. This has been explained in Chapter III and illustrated in Problem 6 of that chapter.

With simultaneous faults at points C and D in circuits separated by a Δ-Y transformer bank, the positive- and negative-sequence equivalent circuits based on equivalent Y-Y transformer banks may be used, with either the circuit at D or that at C as reference circuit. With the circuit at D arbitrarily chosen, the equations at the fault point D can be taken from Table I, with subscripts ABC replacing abc, respectively, in these equations. Assuming the connection diagram of Fig. 10(a), let V_{a1}, V_{a2}, V_{a0} be the components of V_a the voltage to ground of conductor a at the fault point C and I_{a1}, I_{a2}, and I_{a0} the components of current I_a flowing into the fault, all referred to the *reference vector for the system*. Let these same symbols primed be the components of fault current and voltage at C referred to the circuit at D; i.e., the primed components are components which appear in the positive- and negative-sequence networks based on equivalent Y-Y transformer banks. With no connection between the zero sequence impedances viewed from C and D,

$$\begin{aligned} V_{a1} &= -jV'_{a1}; \quad I_{a1} = -jI'_{a1} \\ V_{a2} &= jV'_{a2}; \quad I_{a2} = jI'_{a2} \\ V_{a0} &= V'_{a0}; \quad I_{a0} = I'_{a0} \end{aligned} \qquad [52]$$

Line-to-Ground Faults on Conductor a at C and Conductor A at D. The fault equations are given by Case A (a) of Table I. At D,

$$I_{A1} = I_{A2} = I_{A0} \qquad [53]$$

$$V_{A1} + V_{A2} + V_{A0} = 0 \qquad [54]$$

The equations at C are the same with the subscript A replaced by a. Referring the components at C to the circuit at D by means of [52], the fault equations at C become

$$-jI'_{a1} = jI'_{a2} = I'_{a0}$$

$$-jV'_{a1} + jV'_{a2} + V'_{a0} = 0$$

Multiplying by j,

$$I'_{a1} = -I'_{a2} = jI'_{a0} = jI_{a0} \quad [55]$$

$$V'_{a1} - V'_{a2} + jV'_{a0} = V'_{a1} - V'_{a2} + jV_{a0} = 0 \quad [56]$$

Equations [53]–[56] are satisfied on an a-c network analyzer by the connections given in Fig. 10(b). Coupling transformers are used to satisfy the fault conditions at C, and direct connections of the networks are made at D. A comparison of Fig. 10(b) with Fig. 7 shows that the connections to the negative-sequence network in Fig. 10(b) made through a mutual coupling transformer are the reverse of those in Fig. 7. As the circuit at D is the reference circuit, the components of current and voltage at D are given directly. Components at C referred to the system reference vector are obtained by substituting values obtained from Fig. 10(b) in equations [52]. The fault current in the zero-sequence network at C in Fig. 10(b) is jI_{a0} and the fault voltage is jV_{a0}. Zero-sequence currents and voltages at C are obtained by multiplying the values obtained from Fig. 10(b) by $-j$.

With a *line-to-ground fault on conductor A at D and a line-to-ground fault on conductor b at C*, the fault equations at D are given in [53] and [54]. Those at C from Table I, case A (b), are

$$I_{a1} = a^2I_{a2} = aI_{a0} \quad [57]$$

$$V_{a1} + a^2V_{a2} + aV_{a0} = 0 \quad [58]$$

Replacing the components at C in [57] and [58] by their values from [52], and multiplying the equations by j,

$$I'_{a1} = -a^2I'_{a2} = jaI'_{a0} \quad [59]$$

$$V'_{a1} - a^2V'_{a2} + jaV'_{a0} = 0 \quad [60]$$

Equations [53] and [54] are satisfied by direct connections of the sequence networks. Equations [59] and [60] require phase converters to turn negative-sequence currents and voltages through $-a^2 = \underline{/60°}$, and zero-sequence currents and voltages through $ja = \overline{/150°}$. When there is no connection between the zero-sequence networks at C and D, the phase converter for the zero-sequence network may be omitted. Zero-sequence currents and voltages at C read on the a-c calculating table are then multiplied by $-ja^2$ to obtain V_{a0} and I_{a0} at C.

In an *analytic solution*, the components of fault currents and voltages referred to circuit D are calculated as for faults on the same side of a Δ-Y transformer bank, except that equations [59] and [60] instead of [57] and [58] are used. With I'_{a2}, V'_{a2}, I'_{a0}, and V'_{a0} replacing I_{a2}, V_{a2}, I_{a0}, and V_{a0}, respectively, equations [12] and [13] apply to the zero-sequence network, and [16] and [17] to the negative-sequence network. If I_{A2}, I_{A0}, I'_{a2}, and I'_{a0} in these equations are replaced by their values in terms of I_{A1} and I'_{a1} given in [53] and [59], and V_{A2} and V_{A0} then substituted in [54] and V'_{a2} and V'_{a0} in [60], the following equations will be obtained:

$$V'_{a1} = I'_{a1}(C_0 + S_0 + C_2 + S_2) + I_{A1}(jaS_0 - a^2S_2) \quad [61]$$

$$V_{A1} = I'_{a1}(-ja^2S_0 - aS_2) + I_{A1}(D_0 + S_0 + D_2 + S_2) \quad [62]$$

Comparing these equations with [24] and [25], with V_{a1} and I_{a1} replaced by V'_{a1} and I'_{a1}:

$$\begin{aligned} k &= C_0 + S_0 + C_2 + S_2 \\ l &= D_0 + S_0 + D_2 + S_2 \\ m &= jaS_0 - a^2S_2 \\ n &= -ja^2S_0 - aS_2 \end{aligned} \quad [63]$$

When there is no connection in the zero-sequence network between the zero-sequence impedances viewed from C and D, S_0 in the above equations disappears.

I'_{a1} and I_{A1} can be calculated from [43] and [44], or [46] and [47], if E_a is referred to circuit D and I'_{a1} replaces I_{a1} in these equations. Knowing I'_{a1} and I_{A1}, the other components at the faults and in the system referred to circuit D are obtained. The components of currents and voltages at C referred to the reference vector for the system are obtained by substituting the components referred to circuit D in [52].

The procedure outlined above can be applied to faults of any type involving any phases. When the equivalent Y's representing the sequence networks can be obtained by inspection, an analytic solution is not difficult. Consider the system in Fig. 11(a). The equivalent Y's of the sequence networks are shown in Figs. 11(b), (c), and (d). For this case, no calculations are required to obtain the equivalent Y's. Each equivalent Y has one branch of zero impedance: $C_1 = C_2 = S_0 = 0$. In Figs. 11(b), (c), and (d), the components of fault currents and voltages at C are primed, indicating that they are referred to the circuit at D, i.e., based on an equivalent Y–Y transformer bank.

Ungrounded System — Simultaneous Ground Faults

In ungrounded systems of appreciable capacitance, currents and voltages resulting from two simultaneous faults can be determined just as in grounded systems. In the zero-sequence network the impedance to ground viewed from either fault point is a capacitive impedance, usually high relative to the inductive impedances of the system — but finite.

In ungrounded systems of negligible capacitance, there is little or no fault current during a single ground fault. In the equivalent Y representing the zero-sequence network, $S_0 = \infty$. With two simultaneous

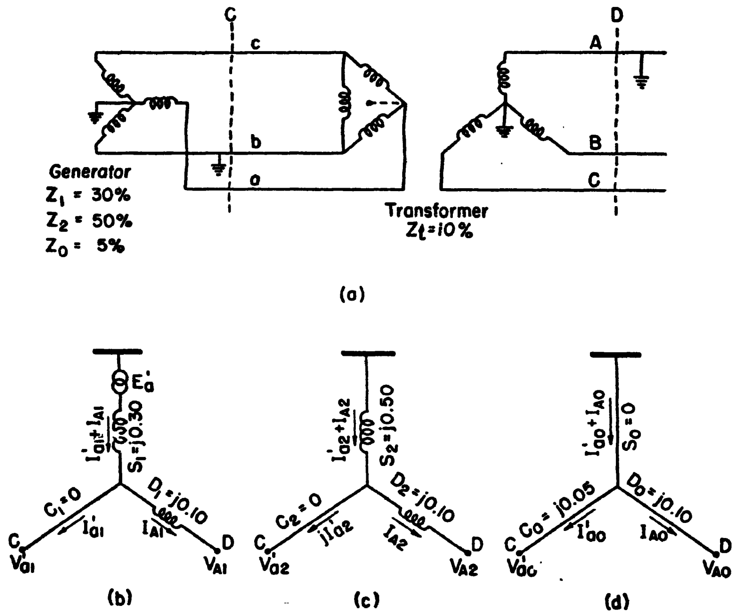


FIG. 11. (*a*) Three-phase system with line-to-ground faults on conductor *A* at *D* and on conductor *b* at *C*, *D* and *C* on opposite sides of a Δ–Y transformer bank. (*b*), (*c*), and (*d*) Per unit equivalent Y's representing the positive-, negative-, and zero-sequence networks, respectively, based on an equivalent Y–Y transformer bank, with circuit *D* the reference circuit.

ground faults, there is a path for zero-sequence currents, out of the system into the ground at one fault point and from the ground back to the system at the other fault. The zero-sequence impedance, in terms of the impedances of the equivalent Y of the zero-sequence network, is $C_0 + D_0$. In an ungrounded loop, the impedance met by zero-sequence currents is the zero-sequence impedance of the two paths between C and D in parallel. Representing the zero-sequence impedance between C and D by z_0, the following equations replace [12] and [13] in the zero-sequence network:

$$I_{a0} = -I_{A0} \tag{64}$$

$$V_{a0} = V_{A0} - I_{a0}z_0 = V_{A0} + I_{A0}z_0 \tag{65}$$

where $z_0 = C_0 + D_0$ and $S_0 = \infty$.

The equations given in Table I are applicable to ungrounded as well as grounded systems. The equations in Table II are also applicable, but are indeterminate with $S_0 = \infty$. If k, l, m, and n from Table II are substituted in [43] and [44], or [46] and [47], these equations can be evaluated for $S_0 = \infty$. The procedure is to divide both numerator and denominator of fractions containing powers of S_0 by S_0 to the highest power appearing in the denominator after simplification. Then, letting $S_0 = \infty$, simple expressions for the two positive-sequence components of fault current are obtained. Components of fault current and voltage, and positive- and negative-sequence currents and voltages in the system are obtained as for grounded systems. In the zero-sequence system, equations [64] and [65] are used instead of [12] and [13].

Solution by Means of an A-C Network Analyzer. The diagrams showing connections between the sequence networks for simultaneous ground faults on grounded systems are also applicable to ungrounded systems of negligible capacitance, provided the zero-sequence impedance network in these diagrams has no direct connections to the zero-potential bus for the network. For example, in Fig. 6(c), which represents double line-to-ground faults on phases b and c at both faults, the rectangle enclosing C and D in the zero-sequence network is to be disconnected from the zero-potential bus to represent an ungrounded system. As in ground systems, zero-sequence voltages are referred to the zero-potential bus for the network, and positive direction for currents is from the network into the fault. In diagrams in which coupling transformers of 1 : 1 turn ratio are used, these transformers may be retained. To make Fig. 7 applicable to an ungrounded system, the rectangle enclosing C and D in the zero-sequence network must be disconnected from the zero-potential bus, but the coupling transformers of 1 : 1 turn ratio remain connected as for the grounded system. An alternate method which eliminates one transformer is to reverse connections to the fault point and the zero-potential bus in the zero-sequence network and replace zero-sequence currents and voltages by their negative values. Representing zero-sequence components of negative signs by primed symbols, let

$$I_{a0} = -I'_{a0}; \quad V_{a0} = -V'_{a0}; \quad I_{A0} = -I'_{A0}; \quad V_{A0} = -V'_{A0}$$

With *line-to-ground faults at C and D on the same phase* (phase a) of an ungrounded loop,

$$I_{a1} = I_{a2} = -I'_{a0}; \quad I_{A1} = I_{A2} = -I'_{A0}$$

$$V_{a1} + V_{a2} - V'_{a0} = 0; \quad V_{A1} + V_{A2} - V'_{A0} = 0$$

These equations are satisfied by the connections shown in Fig. 12. The connections to the zero-potential bus and fault points in the zero-sequence network made in Fig. 7 for the grounded system are reversed in Fig. 12. The zero-potential busses for the positive- and zero-sequence networks in Fig. 12 are directly connected, providing a reference for V'_{A0} and V'_{a0}. Actual zero-sequence currents and voltages are those obtained from the calculating table turned through 180°.

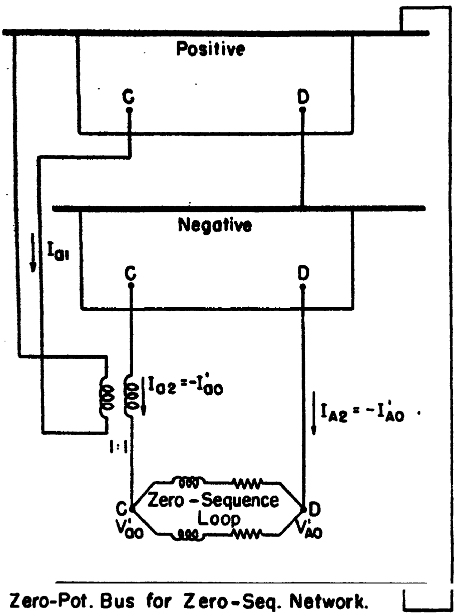


FIG. 12. Connections of the sequence networks for line-to-ground faults at C and D both on phase a of an ungrounded system, showing reversal of the connections to the zero-sequence network for use on an a-c calculating table.

Simultaneous faults in ungrounded systems are further discussed in Chapter X, where solutions for other cases are given.

Fault and Open Conductor Involving the Same Phase

Assume a line-to-ground fault at C and an open conductor at D, both involving phase a as in Fig. 13(a). The fault conditions at C from Table I are

$$I_{a1} = I_{a2} = I_{a0} \quad [66]$$

$$V_{a1} + V_{a2} + V_{a0} = 0 \quad [67]$$

The fault conditions at D from [40] and [41] of Chapter IV are

$$v_{A1} = v_{A2} = v_{A0} \quad [68]$$

$$I'_{A1} + I'_{A2} + I'_{A0} = 0 \quad [69]$$

Here v denotes series voltage drop across the opening and I' line current flowing across the opening.

With no Δ–Y transformer bank between C and D, the connections shown in Fig. 13(b) for use on an a-c calculating table satisfy equations [66]–[69]. The direct connections of the sequence networks at D is the same as that in Chapter IV, Fig. 14(c). The mutual coupling transformers used at C are connected just as in Fig. 7, although this may not be apparent at first glance. In Fig. 13(b), impedance is indicated between points C and D; the portions of the system to the left of C and to the right of D are represented as equivalent synchronous machines.

If C and D are on opposite sides of a Δ–Y transformer bank and there is no connection between the zero-sequence impedance viewed from C and D, the method of representing the open conductor in Fig. 13(*b*) may be retained. For the fault at C on phase a, referred to the circuit at D, the connections given in Fig. 10(*b*) can be used.

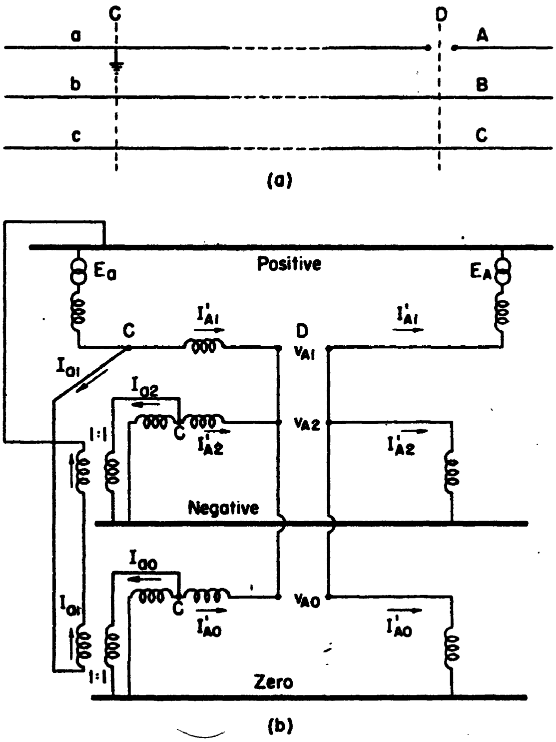


FIG. 13. (*a*) Line-to-ground fault and open conductor on the same phase. (*b*) Connections of the sequence networks to represent a fault and open conductor on the same phase for solution on an a-c network analyzer.

For all cases of simultaneous faults, where the positive- and negative-sequence impedances of rotating machines can be assumed equal, simpler solutions can be obtained by the use of α, β, and 0 components. These components are discussed in Chapter X and applied to the solution of problems involving simultaneous dissymmetries.

Problem 1. Check the equations of Tables I and II.

Problem 2. Draw a three-line diagram showing per unit phase currents and voltages to ground in the system shown in Fig. 11(*a*) with faults as indicated

Problem 3. A conductor breaks and one end falls to ground, the other end is isolated. Develop an equivalent circuit to replace[5] the fault and open conductor in the positive-sequence network. *Suggestion:* Set up an assumed equivalent Y between **points *C*, *D*, and the zero-potential bus, where *C* and *D* are system points at the terminals of the double fault.**

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CHAPTER VIII

UNSYMMETRICAL THREE-PHASE CIRCUITS — ANALYSIS BY THE METHOD OF SYMMETRICAL COMPONENTS

In a symmetrical three-phase system with balanced generated voltages, the currents and voltages under normal operation are balanced. During faults, symmetrical components of current flowing in a symmetrical circuit produce voltage drop of like sequence only. The impedances offered to currents of a given sequence are the same in the three phases and therefore each of the sequence systems can be represented by an equivalent circuit which has no mutual coupling with the equivalent circuits of the other two sequences. In an unsymmetrical three-phase circuit, the voltages and currents are unbalanced under normal operation. If the unbalance is small, it may be relatively unimportant. On the other hand, since it exists during normal operation, its effects may be serious. An example is the heating in a rotating machine resulting from double-frequency currents in the rotor induced by negative-sequence armature currents. Overheating is most likely to occur in the solid rotor of a turbine generator when supplying an unbalanced three-phase or a single-phase load. It is important that the negative-sequence current in a rotating machine does not exceed its allowable safe limit. (See Problem 2.)

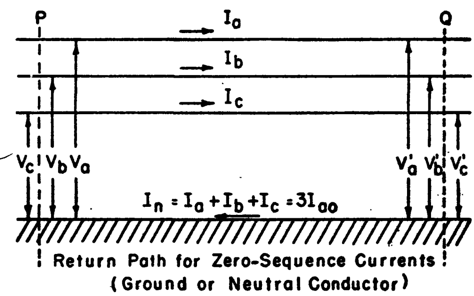


FIG. 1. Three-phase series circuit between P and Q with negligible capacitance and no internal voltages.

Unsymmetrical circuits between two points in an otherwise symmetrical system, and unsymmetrical circuits connected at one point only of the system, will be considered. The simplest type of the unsymmetrical three-phase series circuit is one which provides a direct metallic connection between the two symmetrical parts of the system. In this type of circuit, phase currents flowing between the two symmetrical parts of the system enter the unsymmetrical circuit at one terminal P

and leave at the other terminal Q without change in magnitude or phase, as illustrated in Fig. 1. Examples of this type of circuit are an unsymmetrical three-phase transmission line, with capacitance neglected, and any three series impedances, such as series reactors, which happen to be unequal. Other types of unsymmetrical series circuits include unsymmetrical transformer banks with exciting currents neglected, where the connection between the two parts of the system is by electromagnetic induction rather than by direct metallic connection.

The simplest type of the unsymmetrical three-phase circuit, connected at one point only of an otherwise symmetrical system, is an unsymmetrical Y-connected circuit. The currents in the three phases of the Y are line currents which flow from the symmetrical part of the system into the Y-connected circuit or from the Y into the system, depending upon whether the unsymmetrical Y-connected circuit is receiving or delivering power. The unsymmetrical Y-connected circuit may be treated as a special case of the unsymmetrical three-phase metallically connected series circuit, obtained by connecting the terminals of the three phases at P or Q in Fig. 1 and disconnecting these terminals from the system. Unsymmetrical Y-connected generators, motors, and impedance loads belong in this class. Other unsymmetrical circuits connected at one point only of the system include unsymmetrical Δ-connected circuits and single-phase loads.

Sequence Impedances of the Three Phases of an Unsymmetrical Three-Phase Circuit. The impedances offered to positive-sequence currents in the three phases of a circuit will be defined as the ratios of the voltage drops in the three phases to the corresponding phase currents, with only positive-sequence currents flowing in the circuit. The impedances to positive-sequence currents in phases a, b, and c will be designated by Z_{a1}, Z_{b1}, and Z_{c1}, respectively. In a given circuit, they may be determined by causing positive-sequence currents only to flow in the circuit, or they may be calculated by assuming that only positive-sequence currents are flowing. Positive-sequence currents only can be made to flow in an unsymmetrical three-phase circuit without internal voltages by suitable adjustments of series impedances in the three phases, with positive-sequence voltages applied. The voltage drops in the three phases of the unsymmetrical circuit do not include the voltage drops through the adjustable impedances. A similar procedure can be used to cause only negative- or only zero-sequence currents to flow in an unsymmetrical three-phase circuit. Then, by definition,

$$Z_{a1} = \frac{v_a}{I_{a1}} \qquad Z_{b1} = \frac{v_b}{I_{b1}} \qquad Z_{c1} = \frac{v_c}{I_{c1}}$$

where I_{a1}, I_{b1}, and I_{c1} are positive-sequence currents and v_a, v_b, and v_c the series voltage drops in the three phases resulting from the flow of positive-sequence currents only in the circuit. Similarly, the impedances of the three phases to negative- (or zero-) sequence currents will be defined as the ratios of the voltage drops in the three phases to the corresponding currents with only negative- (or only zero-) sequence currents flowing in the circuit. The negative-sequence impedances of the three phases will be designated by Z_{a2}, Z_{b2}, and Z_{c2}, and the zero-sequence impedances by Z_{a0}, Z_{b0}, and Z_{c0}.

The definitions of the *sequence impedances of the three phases* of an unsymmetrical circuit given here are based on the flow of positive-, negative-, and zero-sequence currents in the circuit and the unbalanced voltage drops resulting from them.

The *sequence admittances of the three phases*, which will be defined in terms of applied positive-, negative-, and zero-sequence voltages and the resulting unbalanced currents are discussed later in this chapter. It is there pointed out that except in a symmetrical three-phase circuit, or in a solidly grounded unsymmetrical three-phase circuit which offers the same impedances to currents of all sequences, the *sequence admittances of the three phases are not the reciprocals of the corresponding sequence impedances*.

Reference Phase and Designation of Conductors. In a symmetrical circuit, the reference phase is conventionally designated a and may be any one of the three phases; when a fault involving one or more phases occurs, it is assumed to be located on the phase or phases which results in the simplest expressions for the sequence currents and voltages. In an unsymmetrical circuit, any phase may be designated the reference phase a; but with phase a specified, the fault or other dissymmetry must be located on the phase or phases relative to phase a which are actually involved. In the work of this and subsequent chapters where circuits are unsymmetrical, the reference phase a will be so selected as to simplify calculations.

Unsymmetrical Three-Phase Series Circuit without Internal Voltages. Let Fig. 1 represent a general three-phase series circuit without internal voltages which provides a direct electrical connection between points P and Q of the system. Currents I_a, I_b, and I_c in phases a, b, and c, respectively, enter the unsymmetrical circuit at P and leave at Q, positive direction for current flow being from P to Q, as indicated by arrows. If the three phases are connected at P, Fig. 1 can represent currents flowing out of an unsymmetrical Y-connected circuit; if at Q, current flowing into such a circuit; if connected at neither P nor Q, current flowing in an unsymmetrical series circuit.

Only the three phases and a return path for zero sequence currents are indicated in Fig. 1; but additional circuit elements can be understood to be present, so that the circuit will represent the desired three-phase circuit.

Phase voltages at any point in a grounded system are referred to ground at that point. If there is a neutral conductor and no ground on the system, phase voltages may be referred to the neutral conductor. Let V_a, V_b, and V_c be the phase voltages at P referred to ground (or some other reference for voltages) at P, and V'_a, V'_b, and V'_c those at Q referred to ground (or some other reference for voltages) at Q. Let the voltage drops between P and Q in the three phases be indicated by v_a, v_b, and v_c; then

$$v_a = V_a - V'_a \qquad v_b = V_b - V'_b \qquad v_c = V_c - V'_c$$

If the resistances and inductances associated with the circuit are constant, superposition can be applied (see Chapter I) and the voltage drop in each phase written as the sum of the voltage drops resulting from the phase current replaced by its symmetrical components. Replacing I_a by $(I_{a1} + I_{a2} + I_{a0})$, I_b by $(a^2I_{a1} + aI_{a2} + I_{a0})$, I_c by $(aI_{a1} + a^2I_{a2} + I_{a0})$, with currents of each sequence in each phase meeting their respective impedances, the voltage drops are

$$\begin{aligned} v_a &= V_a - V'_a = I_{a1}Z_{a1} + I_{a2}Z_{a2} + I_{a0}Z_{a0} \\ v_b &= V_b - V'_b = a^2I_{a1}Z_{b1} + aI_{a2}Z_{b2} + I_{a0}Z_{b0} \\ v_c &= V_c - V'_c = aI_{a1}Z_{c1} + a^2I_{a2}Z_{c2} + I_{a0}Z_{c0} \end{aligned} \qquad [1]$$

where v_a, v_b, and v_c are *series* voltage drops in phases a, b, and c, respectively, in the direction PQ. The lower-case letter v is here used to indicate a series voltage drop as distinguished from a voltage-to-ground, indicated by V. It is important to note that the voltages as defined are *voltage drops*, not voltage rises. (See Chapter I.)

Resolving v_a, v_b, and v_c in [1] into their symmetrical components by [10]–[12], Chapter II,

$$\begin{aligned} v_{a0} &= V_{a0} - V'_{a0} = \tfrac{1}{3}(v_a + v_b + v_c) = I_{a1}\frac{Z_{a1} + a^2Z_{b1} + aZ_{c1}}{3} \\ &\quad + I_{a2}\frac{Z_{a2} + aZ_{b2} + a^2Z_{c2}}{3} + I_{a0}\frac{Z_{a0} + Z_{b0} + Z_{c0}}{3} \\ v_{a1} &= V_{a1} - V'_{a1} = \tfrac{1}{3}(v_a + av_b + a^2v_c) = I_{a1}\frac{Z_{a1} + Z_{b1} + Z_{c1}}{3} \\ &\quad + I_{a2}\frac{Z_{a2} + a^2Z_{b2} + aZ_{c2}}{3} + I_{a0}\frac{Z_{a0} + aZ_{b0} + a^2Z_{c0}}{3} \end{aligned} \qquad [2]$$

$$v_{a2} = V_{a2} - V'_{a2} = \tfrac{1}{3}(v_a + a^2 v_b + a v_c) = I_{a1}\frac{Z_{a1} + aZ_{b1} + a^2 Z_{c1}}{3}$$
$$+ I_{a2}\frac{Z_{a2} + Z_{b2} + Z_{c2}}{3} + I_{a0}\frac{Z_{a0} + a^2 Z_{b0} + aZ_{c0}}{3}$$

With positive-sequence currents only flowing in the circuit, I_{a2} and I_{a0} are zero, and equations [2] become

$$\begin{aligned} v_{a0} &= I_{a1}\frac{Z_{a1} + a^2 Z_{b1} + aZ_{c1}}{3} \\ v_{a1} &= I_{a1}\frac{Z_{a1} + Z_{b1} + Z_{c1}}{3} \\ v_{a2} &= I_{a1}\frac{Z_{a1} + aZ_{b1} + a^2 Z_{c1}}{3} \end{aligned} \qquad [3]$$

Equations [3] show that, with positive-sequence currents only flowing in the circuit, voltage drops of all three sequences will occur between P and Q unless the coefficients of I_{a1} are zero. Likewise, with only negative- or only zero-sequence currents flowing in an unsymmetrical circuit, voltage drops of all three sequences may be obtained.

Sequence Self- and Mutual Impedances in Terms of the Sequence Impedances of the Phases. When currents of a given sequence produce voltage drops of unlike as well as like sequence, equations for the sequence voltage drops in the circuit are conveniently expressed in terms of *sequence self- and mutual impedances*. Replacing the coefficients of the currents in [2] by Z's with two subscripts,* the first subscript referring to the sequence of the voltage drop given by the equation and the second to the sequence of the current associated with the coefficient,

$$v_{a1} = V_{a1} - V'_{a1} = I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \qquad [4]$$

$$v_{a2} = V_{a2} - V'_{a2} = I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \qquad [5]$$

$$v_{a0} = V_{a0} - V'_{a0} = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00} \qquad [6]$$

where the coefficients are defined below in [7].

$Z_{11} = \frac{1}{3}(Z_{a1} + Z_{b1} + Z_{c1})$ = self-impedance to positive-sequence currents

$Z_{22} = \frac{1}{3}(Z_{a2} + Z_{b2} + Z_{c2})$ = self-impedance to negative-sequence currents

* Notation and expression "non-reciprocal," suggested by Dr. E. W. Kimbark's letter to Editor, *Electrical Engineering*, October, 1938, p. 431.

$Z_{00} = \frac{1}{3}(Z_{a0} + Z_{b0} + Z_{c0})$ = self-impedance to zero-sequence currents

$Z_{12} = \frac{1}{3}(Z_{a2} + a^2Z_{b2} + aZ_{c2})$ = ratio of the positive-sequence voltage drop produced by I_{a2} to I_{a2}

$Z_{10} = \frac{1}{3}(Z_{a0} + aZ_{b0} + a^2Z_{c0})$ = ratio of the positive-sequence voltage drop produced by I_{a0} to I_{a0} [7]

$Z_{21} = \frac{1}{3}(Z_{a1} + aZ_{b1} + a^2Z_{c1})$ = ratio of the negative-sequence voltage drop produced by I_{a1} to I_{a1}

$Z_{20} = \frac{1}{3}(Z_{a0} + a^2Z_{b0} + aZ_{c0})$ = ratio of the negative-sequence voltage drop produced by I_{a0} to I_{a0}

$Z_{01} = \frac{1}{3}(Z_{a1} + a^2Z_{b1} + aZ_{c1})$ = ratio of the zero-sequence voltage drop produced by I_{a1} to I_{a1}

$Z_{02} = \frac{1}{3}(Z_{a2} + aZ_{b2} + a^2Z_{c2})$ = ratio of the zero-sequence voltage drop produced by I_{a2} to I_{a2}

Equations [4]–[6] express the symmetrical components of voltage drop in an unsymmetrical three-phase series circuit in which there are no internal voltages in terms of the symmetrical components of current flowing through the circuit and the sequence self- and mutual impedances defined by [7]. Self-impedances are indicated by Z with two like subscripts, mutual impedances by Z with two unlike subscripts. Z_{11}, Z_{22}, Z_{00} represent the positive-, negative-, and zero-sequence self-impedances, respectively, of the circuit and are the impedances met by currents of positive, negative, and zero sequence flowing in their respective networks.

In an unsymmetrical circuit, positive-sequence current flowing in the positive-sequence network meets the positive-sequence self-impedance Z_{11} of the circuit and produces negative- and zero-sequence voltage drops in the negative- and zero-sequence networks. The positive-sequence network is therefore coupled with the other two sequence networks. The mutual or coupling impedances in the positive-sequence network are Z_{21} with the negative-sequence network and Z_{01} with the zero-sequence network. The negative-sequence voltage drop produced in the negative-sequence network by I_{a1} is $Z_{21}I_{a1}$ and the zero-sequence voltage drop produced in the zero-sequence network by I_{a1} is $Z_{01}I_{a1}$. Negative-sequence current flowing in the negative-sequence network meets the negative-sequence self impedance Z_{22} of the circuit and produces positive- and zero-sequence voltage drops in the positive and zero-sequence networks equal to $Z_{12}I_{a2}$ and $Z_{02}I_{a2}$, respectively. The mutual or coupling impedances between the negative-sequence network and the positive- and zero-sequence networks are Z_{12} and Z_{02}, respectively. Zero-sequence current flowing in the zero-sequence network meets the zero-sequence self-impedance Z_{00} of the circuit and produces positive- and negative-sequence voltage drops in the positive- and negative-sequence networks equal to $Z_{10}I_{a0}$ and $Z_{20}I_{a0}$, respectively. The mutual impedances between the zero-sequence network and the positive- and negative-sequence networks are Z_{10} and Z_{20}, respectively.

It is of interest to note that Z_{12} and Z_{21}, as defined in [7], are not equal in the general case. This means that, except in special cases, the ratio of the positive-sequence voltage drop to the negative-sequence current producing it is not the same as the ratio of the negative-sequence voltage drop to the positive-sequence current producing it. Also Z_{10} and Z_{01}, Z_{20} and Z_{02}, except in special cases, will be unequal. The mutual impedances between the sequence networks resulting from an unsymmetrical circuit are therefore *non-reciprocal* in the general case.

Unsymmetrical Three-Phase Series Circuit with Unbalanced Generated or Induced Voltages. Let E_a, E_b, and E_c be the internally generated, or externally induced, voltage rises in phases a, b, and c, respectively, of Fig. 1 in the direction PQ. These internal voltages, if unbalanced, can be resolved into their symmetrical components of voltage by [1], Chapter III. If the components of voltage drop in [4]–[6] are subtracted from the symmetrical components of voltage rise E_{a1}, E_{a2}, and E_{a0}, respectively, the resultant components of voltage rise from P to Q are given by the following equations:

$$\begin{aligned} v_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} - I_{a0}Z_{10} \\ v_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} - I_{a0}Z_{20} \\ v_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}Z_{00} \end{aligned} \qquad [8]$$

where the sequence self- and mutual impedances are defined in [7]. If the applied voltages are balanced, E_{a2} and E_{a0} in [8] are zero.

An unsymmetrical Y-connected synchronous machine, with its neutral grounded through an impedance Z_n, is represented by Fig. 1 if the three phases are connected at P and grounded through an impedance Z_n, as in Fig. 2. P then represents the neutral of the machine and Q its terminals. The symmetrical components of the voltage to ground of phase a at the terminals of the machine, with positive direction for line currents from neutral towards the terminals, are given

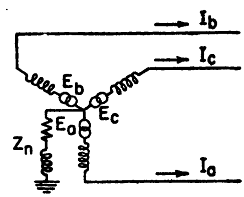


FIG. 2. Y-connected synchronous machine with neutral ground through Z_n.

by [8] if V_{a1}, V_{a2}, and V_{a0} replace v_{a1}, v_{a2}, and v_{a0} in these equations. If Z'_{00} represents the zero-sequence self-impedance between neutral and terminals, then

$$\begin{aligned} V_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} - I_{a0}Z_{10} \\ V_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} - I_{a0}Z_{20} \\ V_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}(Z'_{00} + 3Z_n) \\ &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{02} - I_{a0}Z'_{00} + V_n \end{aligned} \tag{9}$$

where $V_n = -I_{a0}(3Z_n)$ is the voltage above ground of the neutral. If the neutral is solidly grounded, $Z_n = 0$ and $V_n = 0$.

If the *machine neutral is ungrounded,* $Z_n = \infty$ and $I_{a0} = 0$. If the zero-sequence impedances in the three phases between terminals and neutral are finite, as is the case in a Y-connected machine, the zero-sequence impedances Z'_{00}, Z_{10}, and Z_{20} in [9] are finite, and these equations become

$$\begin{aligned} V_{a1} &= E_{a1} - I_{a1}Z_{11} - I_{a2}Z_{12} \\ V_{a2} &= E_{a2} - I_{a1}Z_{21} - I_{a2}Z_{22} \\ V_{a0} &= E_{a0} - I_{a1}Z_{01} - I_{a2}Z_{01} + V_n \end{aligned} \tag{10}$$

where V_{a0} and V_n, the zero-sequence voltages at the machine terminals and neutral, respectively, can be determined for a given problem when the zero-sequence diagram for the rest of the system is known.

Equations [9] and [10] give components of voltage to ground at the terminals of a synchronous machine in terms of the symmetrical components of line current flowing *from* the machine; if I_{a1}, I_{a2}, and I_{a0} are components of line current flowing *into* the machine, the negative signs in [9] and [10] become positive.

In an *unsymmetrical static load* with a path for zero-sequence currents, the symmetrical components of phase voltages to ground at the load terminals are given in terms of the positive-, negative-, and zero-sequence line currents flowing into the circuit and the sequence self- and mutual impedances of the circuit viewed from its terminals by the following equations:

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00} \end{aligned} \tag{11}$$

where the sequence self- and mutual impedances in [11] depend upon the characteristics of the unsymmetrical static load.

If *the load is* Y-*connected* with neutral ground through an impedance

Z_n, equations [11] can be written

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}(Z'_{00} + 3Z_n) \\ &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z'_{00} + V_n \end{aligned} \qquad [11a]$$

where $V_n = I_{a0}(3Z_n)$ is the voltage of the neutral of the load above ground, and Z'_{00} is the zero-sequence self-impedance between terminals and neutral.

If the *load is* Y-*connected with neutral ungrounded*, $I_{a0} = 0$. With finite zero-sequence impedances in the three phases between terminals and neutral, [11a] becomes

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + V_n \end{aligned} \qquad [11b]$$

Δ-*connected loads* are discussed under Δ-connected circuits.

Determination of Sequence Self- and Mutual Impedances from Circuit Impedances

The sequence self- and mutual impedances in [4]–[6] are defined in [7] in terms of the *sequence impedances of the three phases*. The sequence impedances of the phases have been introduced to simplify calculations. It is the sequence self- and mutual impedances that are required, the sequence impedance of the phases being merely helpful in obtaining them. In certain unsymmetrical circuits, the sequence impedances of the phases can be obtained by inspection. This is the case in three-phase circuits with unequal self-impedances in the phases and no mutual impedances between phases or with other circuits. With static circuits having unequal circuit self- and mutual impedances, the introduction of the sequence impedances of the three phases allows the sequence self- and mutual impedances associated with the currents of each sequence to be calculated separately. This is especially useful in the calculation of zero-sequence impedances of overhead transmission lines with ground wires. (See Chapter XI.)

To avoid confusing the two different types of self- and mutual impedances and the two different types of impedances associated with the phases, these impedances will be redefined:

Sequence self- and mutual impedances refer to the self-impedances in the sequence networks (Z_{11}, Z_{22}, and Z_{00}) and the mutual impedances between these networks (Z_{12}, Z_{21}, Z_{10}, Z_{01}, Z_{20}, Z_{02}).

Sequence impedances of the three phases refer to the impedances in the three phases to the sequence currents, when currents of each sequence are applied separately (Z_{a1}, Z_{b1}, Z_{c1}; Z_{a2}, Z_{b2}, Z_{c2}; Z_{a0}, Z_{b0}, Z_{c0}).

Circuit or conductor self- and mutual impedances refer to the self-impedances of the elements or conductors of the actual circuit ($Z_{aa}, Z_{bb}, Z_{cc}, \cdots Z_{nn}$) and the mutual impedances between elements ($Z_{ab}, Z_{bc}, \cdots Z_{cn}$).

Three-Phase Static Circuit with Neutral Conductor — Presence of Earth Neglected. Let Fig. 1 represent an unsymmetrical, reciprocal, static circuit between P and Q in which capacitance is negligible, there are no internal voltages, and the return path for zero-sequence currents is through a neutral conductor with the presence of the earth neglected. Let the self-impedances of phases a, b, c be Z_{aa}, Z_{bb}, Z_{cc}, respectively, and the self-impedance of the neutral conductor n be Z_{nn}. Let the mutual impedances between phases be Z_{ab}, Z_{ac}, Z_{bc}, and between phases and neutral conductor be Z_{an}, Z_{bn}, Z_{cn}. In a reciprocal static circuit $Z_{ab} = Z_{ba}$, $Z_{bn} = Z_{nb}$, etc. (Formulas for the self-impedance of a linear conductor and the mutual impedance between two parallel linear conductors are given in Chapter XI, equations [1] and [2].)

The voltages referred to the neutral conductor of phases a, b, c at P are V_a, V_b, V_c; at Q they are V'_a, V'_b, V'_c. The phase voltage drops in the circuit are $v_a = V_a - V'_a$; $v_b = V_b - V'_b$; $v_c = V_c - V'_c$. If the three phases of the circuit are shorted to the neutral conductor at Q, $V'_a = V'_b = V'_c = 0$ and the voltage drops in the three phases between P and Q are the voltage drops in the three loop circuits, each consisting of a phase conductor with neutral conductor return. Shorting all three conductors at one terminal of a circuit to the reference point for voltages at that terminal is a convenient way of determining its impedances. The neutral conductor return is included because the voltages at P and Q are not referred to the same point of the neutral conductor.

The voltage drops in the three loop circuits, each consisting of a phase conductor and neutral conductor return, are

$$
\begin{aligned}
v_a = V_a - V'_a &= (I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} - I_nZ_{an}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn}) \\
v_b = V_b - V'_b &= (I_aZ_{ab} + I_bZ_{bb} + I_cZ_{bc} - I_nZ_{bn}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn}) \\
v_c = V_c - V'_c &= (I_aZ_{ac} + I_bZ_{bc} + I_cZ_{cc} - I_nZ_{cn}) \\
&\quad + (I_nZ_{nn} - I_aZ_{an} - I_bZ_{bn} - I_cZ_{cn})
\end{aligned}
\qquad [12]
$$

where $I_n = I_a + I_b + I_c$.

There are two ways of determining the sequence self- and mutual impedances of the circuit defined by [12]. The first method, already described, consists of determining the positive-, negative-, and zero-sequence impedances of the three phases of the circuit in accordance with their given definitions, and from them the sequence self- and mutual impedances using [7]. In the second method, after I_n has been replaced by $I_a + I_b + I_c$ in [12], the procedure is to replace the phase currents by their symmetrical components of current ($I_a = I_{a1} + I_{a2} + I_{a0}, I_b = a^2 I_{a1} + aI_{a2} + I_{a0}$, $I_c = aI_{a1} + a^2 I_{a2} + I_{a0}$); then to resolve v_a, v_b, and v_c into their symmetrical components of voltage by [10]–[12], Chapter II; and finally to equate the coefficients of I_{a1}, I_{a2}, and I_{a0} in the resultant equations for v_{a1}, v_{a2}, and v_{a0} to the corresponding coefficients in [4]–[6], obtaining the sequence self- and mutual impedances in terms of the self- and mutual impedances of the conductors.

By the first method, the self- and mutual impedances of each sequence are determined separately. This tends to simplify the equations, but increases their number because of the introduction of the sequence impedances of the three phases. When the second method is used, the sequence impedances of the three phases are not required as the self- and mutual impedances of the sequence networks are expressed directly in terms of conductor self- and mutual impedances. The notation is therefore simpler and the number of equations less; the equations, however, are longer. The first method is employed in the following development. The second is reserved for an exercise (see Problem 7).

With positive-sequence currents only flowing in the three phases, $I_a = I_{a1}$, $I_b = a^2 I_{a1}$, $I_c = aI_{a1}$, and $I_n = 0$. Substituting these values in [12],

$$\begin{aligned} v_a &= V_a - V_a' \\ &= I_{a1}(Z_{aa} + a^2 Z_{ab} + aZ_{ac}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\ v_b &= V_b - V_b' \\ &= I_{a1}(Z_{ab} + a^2 Z_{bb} + aZ_{bc}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \\ v_c &= V_c - V_c' \\ &= I_{a1}(Z_{ac} + a^2 Z_{bc} + aZ_{cc}) - I_{a1}(Z_{an} + a^2 Z_{bn} + aZ_{cn}) \end{aligned}$$

The positive-sequence impedances of the three phases by definition, using the above equations, are

$$Z_{a1} = \frac{v_a}{I_{a1}} = (Z_{aa} + a^2 Z_{ab} + aZ_{ac}) - (Z_{an} + a^2 Z_{bn} + aZ_{cn})$$

$$Z_{b1} = \frac{v_b}{a^2 I_{a1}} = (Z_{bb} + a^2 Z_{bc} + a Z_{ab}) - a(Z_{an} + a^2 Z_{bn} + a Z_{cn})$$

$$Z_{c1} = \frac{v_c}{a I_{a1}} = (Z_{cc} + a^2 Z_{ac} + a Z_{bc}) - a^2(Z_{an} + a^2 Z_{bn} + a Z_{cn})$$

Similar equations can be written for the voltage drops with only negative- and only zero-sequence currents flowing in the three phases and the negative- and zero-sequence impedances of the three phases determined.

The nine sequence impedances of the three phases of a static circuit with return for zero-sequence currents through a neutral conductor with the presence of the earth neglected are given in the following equations in terms of the self- and mutual impedances of the conductors:

$$\begin{aligned}
Z_{a1} &= (Z_{aa} + a^2 Z_{ab} + a Z_{ac}) - (Z_{an} + a^2 Z_{bn} + a Z_{cn})\\
Z_{b1} &= (Z_{bb} + a^2 Z_{bc} + a Z_{ab}) - a(Z_{an} + a^2 Z_{bn} + a Z_{cn})\\
Z_{c1} &= (Z_{cc} + a^2 Z_{ac} + a Z_{bc}) - a^2(Z_{an} + a^2 Z_{bn} + a Z_{cn})\\[6pt]
Z_{a2} &= (Z_{aa} + a Z_{ab} + a^2 Z_{ac}) - (Z_{an} + a Z_{bn} + a^2 Z_{cn})\\
Z_{b2} &= (Z_{bb} + a Z_{bc} + a^2 Z_{ab}) - a^2(Z_{an} + a Z_{bn} + a^2 Z_{cn}) \qquad [12a]\\
Z_{c2} &= (Z_{cc} + a Z_{ac} + a^2 Z_{bc}) - a(Z_{an} + a Z_{bn} + a^2 Z_{cn})\\[6pt]
Z_{a0} &= Z_{aa} + Z_{ab} + Z_{ac} - 3Z_{an} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn})\\
Z_{b0} &= Z_{bb} + Z_{ab} + Z_{bc} - 3Z_{bn} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn})\\
Z_{c0} &= Z_{cc} + Z_{ac} + Z_{bc} - 3Z_{cn} + 3Z_{nn} - (Z_{an} + Z_{bn} + Z_{cn})
\end{aligned}$$

The sequence self- and mutual impedances of a reciprocal three-phase static circuit with no internal voltages, having a return path for zero-sequence currents through a neutral conductor with the presence of the earth neglected, obtained by substituting [12a] in [7], are

$$\begin{aligned}
Z_{11} &= Z_{22} = \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) - \tfrac{1}{3}(Z_{ab} + Z_{ac} + Z_{bc})\\
Z_{00} &= \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) + \tfrac{2}{3}(Z_{ab} + Z_{ac} + Z_{bc})\\
&\qquad + 3Z_{nn} - 2(Z_{an} + Z_{bn} + Z_{cn})\\
Z_{12} &= \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + a Z_{cc}) + \tfrac{2}{3}(a Z_{ab} + a^2 Z_{ac} + Z_{bc})\\
Z_{21} &= \tfrac{1}{3}(Z_{aa} + a Z_{bb} + a^2 Z_{cc}) + \tfrac{2}{3}(a^2 Z_{ab} + a Z_{ac} + Z_{bc}) \qquad [13]\\
Z_{10} &= Z_{02} = \tfrac{1}{3}(Z_{aa} + a Z_{bb} + a^2 Z_{cc}) - \tfrac{1}{3}(a^2 Z_{ab} + a Z_{ac} + Z_{bc})\\
&\qquad - (Z_{an} + a Z_{bn} + a^2 Z_{cn})\\
Z_{20} &= Z_{01} = \tfrac{1}{3}(Z_{aa} + a^2 Z_{bb} + a Z_{cc}) - \tfrac{1}{3}(a Z_{ab} + a^2 Z_{ac} + Z_{bc})\\
&\qquad - (Z_{an} + a^2 Z_{bn} + a Z_{cn})
\end{aligned}$$

Equations [13] are evaluated in terms of circuit dimensions in Chapter XI, equations [12]–[14] and [7]–[9].

Unsymmetrical Static Y-Connected Circuit with Mutual Impedances between Phases. If the three-phase circuit with mutual impedances between phases has a return for zero-sequence currents through a neutral grounding self-impedance Z_n and the length of the ground path is insignificant, the mutual impedances Z_{an}, Z_{bn}, and Z_{cn} in [12] and [13] disappear and Z_{nn} becomes Z_n in these equations.

Symmetrical Three-Phase Static Circuit. In a symmetrical three-phase static circuit,

$$Z_{aa} = Z_{bb} = Z_{cc}, \qquad Z_{ab} = Z_{ac} = Z_{bc}, \qquad Z_{an} = Z_{bn} = Z_{cn}$$

The sequence impedances of the three phases from [12] are

$$\begin{aligned} Z_{a1} &= Z_{b1} = Z_{c1} = (Z_{aa} - Z_{ab}) = Z_1 \\ Z_{a2} &= Z_{b2} = Z_{c2} = (Z_{aa} - Z_{ab}) = Z_2 \\ Z_{a0} &= Z_{b0} = Z_{c0} = Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = Z_0 \end{aligned} \qquad [14]$$

The sequence self- and mutual impedances from [13] are

$$\begin{aligned} Z_{11} &= Z_{22} = (Z_{aa} - Z_{ab}) = Z_1 = Z_2 \\ Z_{00} &= Z_{aa} + 2Z_{ab} + 3Z_{nn} - 6Z_{an} = Z_0 \\ Z_{12} &= Z_{21} = Z_{10} = Z_{02} = Z_{20} = Z_{01} = 0 \end{aligned} \qquad [15]$$

Substituting these impedances in [4]–[6],

$$\begin{aligned} v_{a1}' &= I_{a1}Z_1 \\ v_{a2} &= I_{a2}Z_2 \\ v_{a0} &= I_{a0}Z_0 \end{aligned} \qquad [16]$$

From [14], in a symmetrical static circuit, the impedances to currents of a given sequence are the same for the three phases, and the impedances to positive- and negative-sequence currents are equal. From [15] and [16], there is no mutual coupling between the sequence networks of a symmetrical circuit and currents of a given sequence produce voltage drops of like sequence only. If voltages of a given sequence are applied to the circuit, currents of like sequence only will flow. This follows because the voltage drops must equal the applied voltages, and voltage drops of a given sequence in a symmetrical circuit can be produced only by currents of that sequence.

Equal Self-Impedances in the Three Phases and Mutual Impedances of Two Phases Equal. If $Z_{aa} = Z_{bb} = Z_{cc}$, $Z_{ab} = Z_{ac}$, and $Z_{bn} = Z_{cn}$, the sequence self- and mutual impedances from [13] are

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{aa} - \tfrac{1}{3}(Z_{bc} + 2Z_{ab}) \\ Z_{00} &= Z_{aa} + \tfrac{2}{3}(Z_{bc} + 2Z_{ab}) + 3Z_{nn} - 2(Z_{an} + 2Z_{bn}) \\ Z_{12} &= Z_{21} = \tfrac{2}{3}(Z_{bc} - Z_{ab}) \\ Z_{10} &= Z_{01} = Z_{20} = Z_{02} = -\tfrac{1}{3}(Z_{bc} - Z_{ab}) - (Z_{an} - Z_{bn}) \end{aligned} \qquad [17]$$

In [17] the mutual impedances between the sequence networks are reciprocal.

Self-Impedance Circuits. When the impedances of the three phases of the circuit are unequal self-impedances without mutual coupling between phases or with other circuits, the impedance of any phase to positive-, negative-, and zero-sequence currents is the same. Let the phase impedances be Z_a, Z_b, and Z_c; then $Z_{a1} = Z_{a2} = Z_{a0} = Z_a$; $Z_{b1} = Z_{b2} = Z_{b0} = Z_b$; $Z_{c1} = Z_{c2} = Z_{c0} = Z_c$. The self- and mutual impedances defined in [7] for use in [4]–[6] become

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{00} = \tfrac{1}{3}(Z_a + Z_b + Z_c) \\ Z_{12} &= Z_{20} = Z_{01} = \tfrac{1}{3}(Z_a + a^2 Z_b + aZ_c) \\ &= \tfrac{1}{3}\left[Z_a - \frac{Z_b + Z_c}{2} - j\frac{\sqrt{3}}{2}(Z_b - Z_c)\right] \\ Z_{10} &= Z_{21} = Z_{02} = \tfrac{1}{3}(Z_a + aZ_b + a^2 Z_c) \\ &= \tfrac{1}{3}\left[Z_a - \frac{Z_b + Z_c}{2} + j\frac{\sqrt{3}}{2}(Z_b - Z_c)\right] \end{aligned} \qquad [18]$$

In the general self-impedance circuit, $Z_{12} \neq Z_{21}$, $Z_{10} \neq Z_{01}$, $Z_{20} \neq Z_{02}$, and therefore the mutual impedances between the sequence networks are non-reciprocal.

Self-Impedances Equal in Two Phases. If the self-impedances of two of the phases are equal, let these phases be b and c. Then $Z_b = Z_c$ and [18] become

$$\begin{aligned} Z_{11} &= Z_{22} = Z_{00} = \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{12} &= Z_{21} = Z_{10} = Z_{01} = Z_{20} = Z_{02} = \tfrac{1}{3}(Z_a - Z_b) \end{aligned} \qquad [19]$$

In the special case of two equal circuit self-impedances, the sequence mutual impedances are reciprocal.

Unsymmetrical Δ-Connected Circuits with Unbalanced Generated Voltages

Figure 3 shows two unsymmetrical Δ-connected circuits with unbalanced generated voltages. In part (a), line current flows from the Δ; in part (*b*), towards the Δ. In each case it is required to express the positive- and negative-sequence voltages to neutral at the circuit terminals in terms of the positive- and negative-sequence line currents and the sequence impedances of the Δ-connected windings. There are no zero-sequence components of current flowing from the Δ into

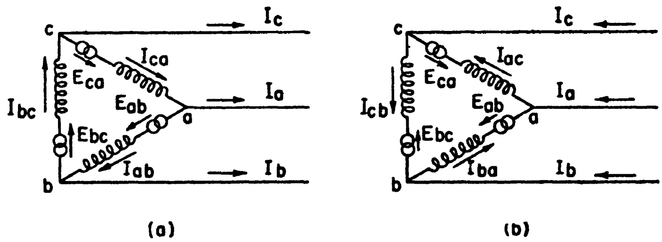


FIG. 3. General unsymmetrical Δ-connected circuits with unbalanced generated voltages. (*a*) Line currents flowing from the Δ. (*b*) Line currents flowing towards the Δ.

the line or from the line into the Δ, but there may be zero-sequence currents circulating in the Δ. There are no zero-sequence components of voltage in the line-to-line voltages (since their sum is zero) but there may be zero-sequence voltages in the voltages to ground at the circuit terminals.

Δ currents and voltages will be indicated by V and I, respectively, with two subscripts, the voltages by $V_{bc} = V_c - V_b$, $V_{ca} = V_a - V_c$, and $V_{ab} = V_b - V_a$, the currents by I_{bc}, I_{ca}, and I_{ab} or I_{cb}, I_{bc}, and I_{ac}, where positive direction of current flow is from the point indicated by the first subscript towards the point indicated by the second.

E_{bc}, E_{ca}, and E_{ab} are the generated voltage rises in the three phases of the Δ in the directions *bc*, *ca*, and *ab*, respectively. The positive-sequence impedances of phases *bc*, *ca*, and *ab* are Z_{bc1}, Z_{ca1}, and Z_{ab1}, respectively; the negative-sequence impedances are Z_{bc2}, Z_{ca2}, and Z_{ab2}, respectively; the zero-sequence impedances are Z_{bc0}, Z_{ca0}, and Z_{ab0}, respectively. The positive-sequence impedances are defined as the ratios of voltage drops in the impedances in the three phases to the corresponding phase currents with only positive-sequence currents flowing. The negative- and zero-sequence impedances are similarly defined. With $V_{bc} = V_c - V_b$ as reference phase,

V_{bc1}, $V_{ca1} = a^2 V_{bc1}$, $V_{ab1} = a V_{bc1}$ are the positive-sequence line-to-line voltages of the Δ

V_{bc2}, $V_{ca2} = a V_{bc2}$, $V_{ab2} = a^2 V_{bc2}$ are the negative-sequence line-to-line voltages of the Δ

The notation for the positive- and negative-sequence currents corresponds to that for the voltages. The zero-sequence circulating current in the Δ will be indicated by I_{bc0}.

If superposition can be applied, the phase voltages of the Δ which are line-to-line voltages will be the generated Δ voltages minus the voltage drops caused by the symmetrical components of current flowing through their respective impedances. From Fig. 3(a) or 3(b),

$$\begin{aligned} V_{bc} &= E_{bc} - I_{bc1}Z_{bc1} - I_{bc2}Z_{bc2} - I_{bc0}Z_{bc0} \\ V_{ca} &= E_{ca} - a^2I_{bc1}Z_{ca1} - aI_{bc2}Z_{ca2} - I_{bc0}Z_{ca0} \\ V_{ab} &= E_{ab} - aI_{bc1}Z_{ab1} - a^2I_{bc2}Z_{ab2} - I_{bc0}Z_{ab0} \end{aligned} \tag{20}$$

Resolving the line-to-line voltages into their symmetrical components of voltage, by equations analogous to [10]–[12] of Chapter II, the resultant equations are

$$\begin{aligned} V_{bc1} &= \tfrac{1}{3}(V_{bc} + aV_{ca} + a^2V_{ab}) = E_{bc1} - I_{bc1}Z_{\Delta 11} - I_{bc2}Z_{\Delta 12} \\ &\qquad - I_{bc0}Z_{\Delta 10} \\ V_{bc2} &= \tfrac{1}{3}(V_{bc} + a^2V_{ca} + aV_{ab}) = E_{bc2} - I_{bc1}Z_{\Delta 21} - I_{bc2}Z_{\Delta 22} \\ &\qquad - I_{bc0}Z_{\Delta 20} \\ V_{bc0} &= \tfrac{1}{3}(V_{bc} + V_{ca} + V_{ab}) = 0 = E_{bc0} - I_{bc1}Z_{\Delta 01} - I_{bc2}Z_{\Delta 02} \\ &\qquad - I_{bc0}Z_{\Delta 00} \end{aligned} \tag{21}$$

where

$$\begin{aligned} E_{bc1} &= \tfrac{1}{3}(E_{bc} + aE_{ca} + a^2E_{ab}) \\ E_{bc2} &= \tfrac{1}{3}(E_{bc} + a^2E_{ca} + aE_{ab}) \\ E_{bc0} &= \tfrac{1}{3}(E_{bc} + E_{ca} + E_{ab}) \end{aligned} \tag{22}$$

and

$$\begin{aligned} Z_{\Delta 11} &= \tfrac{1}{3}(Z_{bc1} + Z_{ca1} + Z_{ab1}) \\ Z_{\Delta 22} &= \tfrac{1}{3}(Z_{bc2} + Z_{ca2} + Z_{ab2}) \\ Z_{\Delta 00} &= \tfrac{1}{3}(Z_{bc0} + Z_{ca0} + Z_{ab0}) \\ Z_{\Delta 12} &= \tfrac{1}{3}(Z_{bc2} + a^2Z_{ca2} + aZ_{ab2}) \\ Z_{\Delta 10} &= \tfrac{1}{3}(Z_{bc0} + aZ_{ca0} + a^2Z_{ab0}) \\ Z_{\Delta 21} &= \tfrac{1}{3}(Z_{bc1} + aZ_{ca1} + a^2Z_{ab1}) \\ Z_{\Delta 20} &= \tfrac{1}{3}(Z_{bc0} + a^2Z_{ca0} + aZ_{ab0}) \\ Z_{\Delta 01} &= \tfrac{1}{3}(Z_{bc1} + a^2Z_{ca1} + aZ_{ab1}) \\ Z_{\Delta 02} &= \tfrac{1}{3}(Z_{bc2} + aZ_{ca2} + a^2Z_{ab2}) \end{aligned} \tag{23}$$

Equations [21] express the positive-, negative-, and zero-sequence components of the phase voltages of the Δ, or the line-to-line voltages across the Δ, in terms of the sequence components of voltages generated in the Δ, the sequence components of currents flowing in the Δ, and the sequence self- and mutual impedances of the Δ-connected circuit. Equations [22], which define the sequence generated Δ voltages in terms of the voltages generated in the three phases of the Δ, are similar to [1], Chapter III. Equations [23] are similar to [7], the sequence impedances in the three phases of the Δ replacing the sequence impedances in the three phases of the series circuit. The Δ impedances in [21], defined in [23], have an additional subscript Δ to indicate that they are the impedances of a Δ-connected circuit.

Solving the last equation of [21] for I_{bc0},

$$I_{bc0} = \frac{1}{Z_{\Delta 00}}(E_{bc0} - I_{bc1}Z_{\Delta 01} - I_{bc2}Z_{\Delta 02}) \qquad [24]$$

Substituting I_{bc0} from [24] in the first two equations of [21],

$$\begin{aligned} V_{bc1} &= \left(E_{bc1} - E_{bc0}\frac{Z_{\Delta 10}}{Z_{\Delta 00}}\right) - I_{bc1}\left(Z_{\Delta 11} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{Z_{\Delta 00}}\right) \\ &\quad - I_{bc2}\left(Z_{\Delta 12} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{Z_{\Delta 00}}\right) \\ V_{bc2} &= \left(E_{bc2} - E_{bc0}\frac{Z_{\Delta 20}}{Z_{\Delta 00}}\right) - I_{bc1}\left(Z_{\Delta 21} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{Z_{\Delta 00}}\right) \\ &\quad - I_{bc2}\left(Z_{\Delta 22} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{Z_{\Delta 00}}\right) \end{aligned} \qquad [25]$$

If currents, voltages, and impedances are expressed in amperes, voltages, and ohms, respectively, or in per unit on common kva and voltage bases, from [53], Chapter III,

$$\begin{aligned} V_{bc1} &= j\sqrt{3}V_{a1} \\ V_{bc2} &= -j\sqrt{3}V_{a2} \end{aligned} \qquad [26]$$

With direction of line currents away from the Δ as in Fig. 3(a), from [60], Chapter III,

$$\begin{aligned} I_{bc1} &= \frac{jI_{a1}}{\sqrt{3}} \\ I_{bc2} &= -\frac{jI_{a2}}{\sqrt{3}} \end{aligned} \qquad [27]$$

Substituting [26] and [27] in [25] and [27] in [24] and simplifying, V_{a1} and V_{a2}, the positive- and negative-sequence components of phase voltage to ground at the Δ terminals, and I_{bc0}, the zero-sequence current in the Δ in the direction *bca*, are expressed in terms of I_{a1} and I_{a2}, the positive- and negative-sequence components of line current flowing *from* the Δ terminals, the Δ-generated voltages, and sequence self- and mutual impedances of the Δ by the following equations:

$$V_{a1} = \frac{-j}{\sqrt{3}}\left(E_{bc1} - E_{bc0}\frac{Z_{\Delta 10}}{Z_{\Delta 00}}\right) - I_{a1}\left(\frac{Z_{\Delta 11}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) + I_{a2}\left(\frac{Z_{\Delta 12}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [28]$$

$$V_{a2} = \frac{j}{\sqrt{3}}\left(E_{bc2} - E_{bc0}\frac{Z_{\Delta 20}}{Z_{\Delta 00}}\right) + I_{a1}\left(\frac{Z_{\Delta 21}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) - I_{a2}\left(\frac{Z_{\Delta 22}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [29]$$

$$I_{bc0} = \frac{1}{Z_{\Delta 00}}\left(E_{bc0} - j\frac{I_{a1}}{\sqrt{3}}Z_{\Delta 01} + j\frac{I_{a2}}{\sqrt{3}}Z_{\Delta 02}\right) \qquad [30]$$

V_{a0} is indeterminate but can be evaluated in any given problem when the zero-sequence network for the system is known. If the generated line-to-line voltages are balanced, $E_{bc2} = E_{bc0} = 0$. With per unit quantities based on a common kva base, but base voltages in the Δ and line in the ratio of $\sqrt{3}$ to 1, the factors $1/\sqrt{3}$ and $\frac{1}{3}$, which change line-to-line generated voltages and Δ impedances to line-to-neutral voltages and impedances, respectively, disappear in [28]–[30].

Unsymmetrical Δ-Connected Synchronous Machine with Unbalanced Generated Voltages. Equations [28]–[30] give the positive- and negative-sequence components of line-to-neutral voltage at the machine terminals in terms of the positive- and negative-sequence components of line currents flowing *from* the Δ, the sequence self- and mutual impedances of the Δ-connected circuit, and the symmetrical components of generated Δ voltages. (See Fig. 3(*a*).) If I_{a1} and I_{a2} are components of line current flowing *towards* the Δ, as in Fig. 3(*b*), the signs of the coefficients of I_{a1} and I_{a2} in [28]–[30] are reversed.

Unsymmetrical Δ-Connected Static Circuit. If the generated voltages in [28]–[30] are equated to zero and the signs of I_{a1} and I_{a2} reversed, the following equations apply to the unsymmetrical Δ-connected static load:

$$V_{a1} = I_{a1}\left(\frac{Z_{\Delta 11}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) - I_{a2}\left(\frac{Z_{\Delta 12}}{3} - \frac{Z_{\Delta 10}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [31]$$

$$V_{a2} = -I_{a1}\left(\frac{Z_{\Delta 21}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 01}}{3Z_{\Delta 00}}\right) + I_{a2}\left(\frac{Z_{\Delta 22}}{3} - \frac{Z_{\Delta 20}Z_{\Delta 02}}{3Z_{\Delta 00}}\right) \qquad [32]$$

$$I_{bc0} = \frac{j}{\sqrt{3}}\left(I_{a1}\frac{Z_{\Delta 01}}{Z_{\Delta 00}} - I_{a2}\frac{Z_{\Delta 02}}{Z_{\Delta 00}}\right) \qquad [33]$$

where I_{a1} and I_{a2} are the symmetrical components of line current flowing towards the Δ, and I_{bc0} is the zero sequence current circulating in the Δ, in the direction *bca*.

Equations [31]–[33], as well as [28]–[30], apply where currents, voltages, and impedances are expressed in amperes, volts, and ohms, respectively, or in per unit on common kva and voltage bases. With all quantities in per unit based on a common kva, but base voltages in the Δ and line in the ratio of $\sqrt{3}$: 1, the factors $\frac{1}{3}$ in [31] and [32] and $1/\sqrt{3}$ in [33] disappear.

Δ-Connected Self-Impedance Circuit. For a self-impedance Δ-connected circuit without mutual impedances between phases, $Z_{bc1} = Z_{bc2} = Z_{cb0} = Z_{bc}$; $Z_{ca1} = Z_{ca2} = Z_{ca0} = Z_{ca}$; $Z_{ab1} = Z_{ab2} = Z_{ab0} = Z_{ab}$ From [23],

$$\begin{aligned} Z_{\Delta 11} &= Z_{\Delta 22} = Z_{\Delta 00} = \tfrac{1}{3}(Z_{bc} + Z_{ca} + Z_{ab}) \\ Z_{\Delta 12} &= Z_{\Delta 20} = Z_{\Delta 01} = \tfrac{1}{3}(Z_{bc} + a^2 Z_{ca} + aZ_{ab}) \qquad [34] \\ Z_{\Delta 21} &= Z_{\Delta 10} = Z_{\Delta 02} = \tfrac{1}{3}(Z_{bc} + aZ_{ca} + a^2 Z_{ab}) \end{aligned}$$

For the Δ-connected self-impedance circuit, [34] may be substituted in [31]–[33], or an equivalent Y can replace the Δ-connected self-impedance circuit for calculating current and voltage conditions at its terminals. (See [39], Chapter I.) The first two equations of [11*b*] then apply, where the sequence self- and mutual impedances Z_{11}, Z_{22}, Z_{12}, and Z_{21} are defined in [18] in terms of the self-impedances of the equivalent Y. It should be noted that it is the self-impedance Δ only which can be replaced by a single equivalent Y. The zero-sequence current circulating in the unsymmetrical Δ cannot be determined from the equivalent Y, but the total currents in the three branches of the Δ can be obtained by dividing the calculated line-to-line voltages at the terminals of the equivalent Y by the corresponding phase impedances of the Δ. One-third the sum of these Δ currents is the zero-sequence current which circulates in the Δ.

DETERMINATION OF CURRENTS AND VOLTAGES IN A SYSTEM CONTAINING AN UNSYMMETRICAL CIRCUIT BY MEANS OF EQUATIONS

As pointed out, the mutual impedances between the sequence networks are non-reciprocal for the general unsymmetrical circuit. This is true for the general unsymmetrical circuit with mutual impedances between phases (see [13]) and also for the general unsymmetrical circuit composed of self-impedances only (see [18]). With non-reciprocal mutual impedances between the sequence networks, there is no equivalent static circuit to replace the actual circuit in these networks. Solution, however, can be obtained by means of equations. On an a-c network analyzer, a phase shifter is required. Equations will be developed from which the currents and voltages can be determined in a system, symmetrical except for (1) a general unsymmetrical static series circuit, (2) a general unsymmetrical circuit connected at one point only of the system. When the mutual impedances are reciprocal, equivalent circuits can be developed to replace the actual circuit in the sequence networks. Such circuits will be developed for special unsymmetrical series and shunt circuits.

General Unsymmetrical Series Circuit. When the system except for the unsymmetrical series circuit is symmetrical and the parts of the system on the two sides of the unsymmetrical circuit are connected only through that circuit, the system can be represented as in

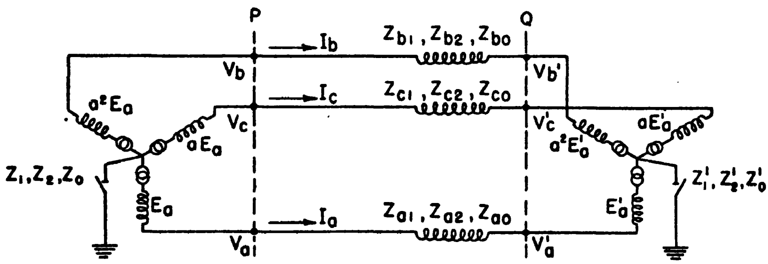


FIG. 4. Unsymmetrical series circuit between points P and Q in an otherwise symmetrical system in which symmetrical parts of the system are represented as equivalent synchronous machines.

Fig. 4, where the points P and Q divide the system into three parts. The parts of the system to the left of P and to the right of Q are symmetrical, the three phases offering the same impedances to currents of a given sequence, while the part between P and Q is unsymmetrical. The symmetrical parts of the system can be replaced by equivalent machines with sequence impedances Z_1, Z_2, Z_0 and Z_1', Z_2', Z_0', which are the sequence impedances of the symmetrical parts of the system viewed from P and Q, respectively, with the unsymmetrical series circuit open. E_a, a^2E_a, aE_a and E'_a, $a^2E'_a$, aE'_a are the balanced internal voltages of the two equivalent machines replacing the symmetrical parts of the system. They are the balanced voltages which would exist at P and Q, respectively, if the three phases were opened at P and Q, all rotors on the system retaining their relative angular positions. E_a and E'_a depend upon operating conditions. For example, if the unsymmetrical circuit is supplied at P through a transformer bank with kva rating so small relative to the kva capacity of the system to the left of P in Fig. 4 that the voltages on the primary side of the bank remain balanced, Z_1, Z_2, and Z_0 will be the positive-, negative-, and zero-sequence impedances of the transformer bank, and the internal voltages of the equivalent machine will be the balanced normal operating voltages on the primary side of the transformer bank. When the current taken at P by the unsymmetrical circuit is appreciable relative to the current in the system to the left of P, the balanced internal voltages of the equivalent machine at P must be high enough to allow for the voltage drop in the symmetrical part of the system caused by the unsymmetrical load. In many problems involving unsymmetrical circuits, the given conditions are such that E_a and E'_a are not required. For example, if the positive-sequence current flowing through an unsymmetrical series circuit in an otherwise symmetrical system is given and the problem is to find the negative-sequence current when the impedance to zero-sequence currents is infinite, it is only necessary to determine the ratio of the negative- to the positive-sequence current. (See [43].) In cases where E_a and E'_a are required, they can usually be satisfactorily estimated. For the present it will be assumed that they can be estimated or will be eliminated in the solution of a given problem. When the symmetrical part of the system to the right of Q is a symmetrical three-phase static circuit, $E'_a = 0$.

In Fig. 4, the line-to-ground voltages of the three phases at P will be designated by V_a, V_b, and V_c, and at Q by V'_a, V'_b, and V'_c and the corresponding line currents which flow in all three circuits by I_a, I_b, and I_c, positive direction for current flow being from P to Q, as indicated by arrows. If the symmetrical components of voltages to ground at P and Q, V_{a1}, V_{a2}, V_{a0}, and V'_{a1}, V'_{a2}, V'_{a0}, respectively, are expressed in terms of the symmetrical components of current flowing from P to Q, there will be three sets of equations, two sets in terms of the impedances of the symmetrical parts of the system and one set in terms of the impedances of the unsymmetrical part. By equating symmetrical components of voltages at P and at Q in these equations, the symmetrical components of current can be determined, and from them the currents and voltages of the system.

From the symmetrical part of the system to the left of P, the symmetrical components of voltage at P from [8]–[10], Chapter III, are

$$\begin{aligned} V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \\ V_{a0} &= -I_{a0}Z_0 \end{aligned} \qquad [35]$$

From the symmetrical part of the system to the right of Q, the symmetrical components of voltage at Q are

$$\begin{aligned} V'_{a1} &= I_{a1}Z'_1 + E'_a \\ V'_{a2} &= I_{a2}Z'_2 \\ V'_{a0} &= I_{a0}Z'_0 \end{aligned} \qquad [36]$$

The sequence voltage drops between P and Q, $(V_{a1} - V'_{a1})$, $(V_{a2} - V'_{a2})$, $(V_{a0} - V'_{a0})$, are given by [4]–[6].

Substituting [35] and [36] in [4]–[6], and simplifying,

$$E_a - E'_a = I_{a1}(Z_{11} + Z_1 + Z'_1) + I_{a2}Z_{12} + I_{a0}Z_{10} \qquad [37]$$

$$0 = I_{a1}Z_{21} + I_{a2}(Z_{22} + Z_2 + Z'_2) + I_{a0}Z_{20} \qquad [38]$$

$$0 = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}(Z_{00} + Z_0 + Z'_0) \qquad [39]$$

Using determinants (see Appendix A), the solution of the simultaneous equations [37]–[39] gives

$$\begin{aligned} I_{a1} &= (E_a - E'_a)\frac{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}}{\Delta} \\ I_{a2} &= -(E_a - E'_a)\frac{Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}}{\Delta} \\ &= -I_{a1}\frac{Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}}{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}} \\ I_{a0} &= -(E_a - E'_a)\frac{Z_{01}(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{02}}{\Delta} \\ &= -I_{a1}\frac{Z_{01}(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{02}}{(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}} \end{aligned} \qquad [40]$$

where

$$\begin{aligned} \Delta = (Z_{11} + Z_1 + Z'_1)&[(Z_{22} + Z_2 + Z'_2)(Z_{00} + Z_0 + Z'_0) - Z_{02}Z_{20}] \\ &- Z_{12}[Z_{21}(Z_{00} + Z_0 + Z'_0) - Z_{01}Z_{20}] \\ &+ Z_{10}[Z_{21}Z_{02} - Z_{01}(Z_{22} + Z_2 + Z'_2)] \end{aligned}$$

Equations [40] express I_{a1}, I_{a2}, and I_{a0} in terms of $(E_a - E_a')$; I_{a2} and I_{a0} are also expressed in terms of I_{a1}. When I_{a1} is known or can be assumed, I_{a2} and I_{a0} are most simply calculated from [40] in terms of I_{a1}.

In circuits where the dissymmetry is but slight and therefore the negative- and zero-sequence currents are known to be small, the negative-sequence voltage drop $I_{a0}Z_{20}$ in [38] produced by I_{a0} and the zero-sequence voltage drop $I_{a2}Z_{02}$ in [39] produced by I_{a2} can be neglected with but slight error. Neglecting these terms in [38] and [39], and solving for I_{a2} and I_{a0} in terms of I_{a1},

$$I_{a2} = -I_{a1}\frac{Z_{21}}{Z_{22} + Z_2 + Z_2'} = -I_{a1}\frac{Z_{21}}{z_{22}}$$

$$I_{a0} = -I_{a1}\frac{Z_{01}}{Z_{00} + Z_0 + Z_0'} = -I_{a1}\frac{Z_{01}}{z_{00}} \qquad [40a]$$

where z_{22} and z_{00} are the negative- and zero-sequence self-impedances met by I_{a2} and I_{a0}, respectively. Equations [40a] could have been obtained from [40] by neglecting the products of two mutual impedances; when the dissymmetry is slight, these products are small relative to the product of one self- and one mutual impedance or two self-impedances. Equations [40a] are used in Problems 6 and 7 of Chapter XI to determine the effect of unsymmetrical transmission circuits upon system currents and voltages during normal operation.

Finite Zero-Sequence Impedances. When Z_0, Z_0', and Z_{00} are all finite, zero sequence currents will flow, and the symmetrical components of current are given by [40] or [40a]. The symmetrical components of voltage at P and Q can be obtained from [35] and [36], respectively, when I_{a1}, I_{a2}, and I_{a0} are known. Substituting the symmetrical components of voltage and current in equations [7]–[9] and [19]–[21], Chapter II, respectively, line currents and voltages to ground at P and Q can be obtained.

Infinite Zero-Sequence Impedances. If any one of the zero-sequence impedances Z_0, Z_0', or Z_{00} is infinite, there will be no zero-sequence currents; and $I_{a0}Z_{10}$ and $I_{a0}Z_{20}$, which represent positive- and negative-sequence voltage drops produced by zero-sequence currents, will be zero. With $I_{a0} = 0$, [37] and [38] become

$$E_a - E_a' = I_{a1}(Z_{11} + Z_1 + Z_1') + I_{a2}Z_{12}$$

$$0 = I_{a1}Z_{21} + I_{a2}(Z_{22} + Z_2 + Z_2') \qquad [41]$$

Solving these equations for I_{a1} and I_{a2},

$$I_{a1} = (E_a - E'_a)\frac{(Z_{22} + Z_2 + Z'_2)}{(Z_{11} + Z_1 + Z'_1)(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{12}} \quad [42]$$

$$I_{a2} = -(E_a - E'_a)\frac{Z_{21}}{(Z_{11} + Z_1 + Z'_1)(Z_{22} + Z_2 + Z'_2) - Z_{21}Z_{12}}$$

$$= -I_{a1}\frac{Z_{21}}{Z_{22} + Z_2 + Z'_2} \quad [43]$$

Knowing I_{a1} and I_{a2}, the positive- and negative-sequence components of the voltages at P and Q can be obtained from [35] and [36], respectively.

With $I_{a0} = 0$ and $(Z_{00} + Z_0 + Z'_0) = \infty$, $V_{a0} - V'_{a0}$ is indeterminate. Zero-sequence voltages at P and Q, however, can be determined from the zero-sequence diagram of the system. For example:

If Z_{00} is infinite and Z_0 and Z'_0 both finite, the zero-sequence voltages at both P and Q, determined from [35] and [36] with $I_{a0} = 0$, are zero.

If Z_{00} is finite, and Z_0 or Z'_0 is infinite, $I_{a0}Z_{00} = 0 \cdot Z_{00} = 0$, and [6] becomes

$$v_{a0} = V_{a0} - V'_{a0} = I_{a1}Z_{01} + I_{a2}Z_{02} \quad [44]$$

where v_{a0} is the zero-sequence voltage drop between P and Q produced by positive- and negative-sequence currents flowing in the unsymmetrical circuit of finite zero-sequence self-impedance. If there is a ground in the part of the system connected at P, but none in the part connected at Q, Z_0 is finite and Z'_0 infinite. With $I_{a0} = 0$ and Z_0 finite, the zero-sequence voltage V_{a0} at P from [35] will be zero. By definition, v_{a0} is the zero-sequence voltage drop between P and Q. The zero-sequence voltage V'_{a0} at Q will therefore be V_{a0}, the zero-sequence voltage at P, minus the zero-sequence voltage drop given by [44]; with $V_{a0} = 0$,

$$V'_{a0}\ (\text{at } Q) = 0 - v_{a0} = -I_{a1}Z_{01} - I_{a2}Z_{02} \quad [45]$$

With no ground on the part of the system connected at Q, the zero-sequence voltage at Q will also be the zero-sequence voltage at all points in the section of the system where the connection to Q is a direct metallic one; or stated more comprehensively, at all points where the connection to Q is through finite zero-sequence impedance.

With Z_{00} finite, and the part of the system connected at Q grounded and the part connected at P ungrounded, Z'_0 is finite and Z_0 infinite. From [36], the zero-sequence voltage at Q will be zero and that at P will be given by [44]. The zero-sequence voltage at P will also be the zero-sequence voltage at all points in the section of the system connected at P where the connection to P is through finite zero-sequence impedance.

Unsymmetrical Circuit Connected at One Point Only of the System. The procedure for this case is similar to that used for the unsymmetrical series circuit. If the unsymmetrical circuit is a generator supplying power to the symmetrical part of the system, the system exclusive of the unsymmetrical generator is replaced by an equivalent synchronous motor with balanced sequence impedances and internal voltages. If the unsymmetrical circuit is receiving power from the symmetrical part of the system, the system exclusive of the unsymmetrical circuit is replaced by an equivalent synchronous generator with balanced sequence impedances and internal voltages. In each case, there are two sets of equations giving the symmetrical components of phase voltages to ground at the junction point of the symmetrical and unsymmetrical parts of the system in terms of the symmetrical components of line current flowing in both parts of the system.

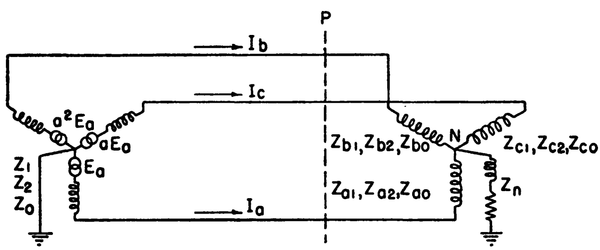


FIG. 5. Unsymmetrical Y-connected load with neutral grounded through Z_n, supplied at P from an otherwise symmetrical system, represented as an equivalent synchronous generator.

Y-Connected Static Circuit. A Y-connected static circuit grounded through a neutral impedance Z_n and supplied from an otherwise symmetrical system is shown in Fig. 5. Equating V_{a1}, V_{a2}, and V_{a0} in [11*a*] and [35] and solving for I_{a1}, I_{a2}, and I_{a0},

$$I_{a1} = E_a \frac{(Z_2 + Z_{22})(Z'_{00} + Z_0 + 3Z_n) - Z_{02}Z_{20}}{\Delta}$$

$$I_{a2} = -E_a \frac{Z_{21}(Z'_{00} + Z_0 + 3Z_n) - Z_{01}Z_{20}}{\Delta} \qquad [46]$$

$$I_{a0} = -E_a \frac{Z_{01}(Z_{22} + Z_2) - Z_{21}Z_{02}}{\Delta}$$

where

$$\begin{aligned}\Delta = (Z_1 + Z_{11})[(Z_2 + Z_{22})(Z'_{00} + Z_0 + 3Z_n) - Z_{02}Z_{20}] \\ - Z_{12}[Z_{21}(Z'_{00} + Z_0 + 3Z_n) - Z_{01}Z_{20}] \\ + Z_{10}[Z_{21}Z_{02} - Z_{01}(Z_{22} + Z_2)]\end{aligned}$$

When the symmetrical components of current have been determined, the symmetrical components of voltage to ground at P can be obtained from either [35] or [11*a*], the former being simpler.

Neutral of Load Ungrounded. With the load ungrounded, $V_n = \infty$ and $I_{a0} = 0$. With the symmetrical part of the system grounded, the zero-sequence voltage at P in Fig. 5 from [35] is zero. Equating V_{a1}, V_{a2}, V_{a0} in [35] and [11*b*] and, simplifying,

$$\begin{aligned}E_a &= I_{a1}(Z_1 + Z_{11}) + I_{a2}Z_{12} \\ 0 &= I_{a1}Z_{21} + I_{a2}(Z_2 + Z_{22}) \\ 0 &= I_{a1}Z_{01} + I_{a2}Z_{02} + V_n\end{aligned}$$

Solving the above equations for I_{a1}, I_{a2}, and V_n,

$$\begin{aligned}I_{a1} &= \frac{E_a(Z_2 + Z_{22})}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} \\ I_{a2} &= \frac{-E_aZ_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} = -I_{a1}\frac{Z_{21}}{Z_2 + Z_{22}}\end{aligned} \qquad [47]$$

and

$$V_n = -E_a\frac{Z_{01}(Z_2 + Z_{22}) - Z_{02}Z_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} \qquad [48]$$

where V_n is the voltage at the neutral of the load. This is the voltage rise from P to N caused by positive- and negative-sequence currents flowing through an unsymmetrical circuit with finite zero-sequence impedance between terminals and neutral.

The positive- and negative-sequence components of voltage at P can be obtained by substituting I_{a1} and I_{a2} from [47] in [35].

If the neutral of the load is grounded but the *system ungrounded*, Z_0 in Fig. 5 is infinite and $I_{a0} = 0$. I_{a1} and I_{a2} are given by [47] and V_{a1} and V_{a2} by substituting these values of I_{a1} and I_{a2} in [35]. The positive- and negative-sequence currents and voltages at P are the same as those for the case of the system grounded and neutral of load ungrounded. With the neutral of the load grounded, $V_n = 0$. The zero-sequence voltage at P, determined from [11b], with V_n replaced by zero, is

$$V_{a0} = E_a \frac{Z_{01}(Z_2 + Z_{22}) - Z_{02}Z_{21}}{(Z_1 + Z_{11})(Z_2 + Z_{22}) - Z_{12}Z_{21}} \tag{48a}$$

This is the zero-sequence voltage drop from P to N caused by positive- and negative-sequence currents flowing in the unsymmetrical circuit of finite zero-sequence impedance. V_{a0} in [48a] is equal in magnitude and opposite in sign to V_n given by [48]. As there is no zero-sequence current in the system supplying the load, the zero-sequence voltage given by [48a] will exist at P and at all other points in the system replaced by the equivalent generator where the connection to P is through finite zero-sequence impedance.

Self-Impedance Y-Connected Circuits. The sequence self- and mutual impedances for use in [46] for the grounded Y, and in [47] for the ungrounded Y (or the Δ replaced by its equivalent Y), are given by [18] in terms of the phase impedances of the Y. For the special case of *equal self-impedances in two phases* of the Y, the sequence self- and mutual impedances in terms of the phase impedances of the Y are given by [19].

Self-Impedances Equal in Two Phases of the Grounded Y. Assuming a grounded system and substituting [19] in [46],

$$\begin{aligned} I_{a1} &= E_a \frac{(Z_2 + Z_{11})(Z_0 + Z_{11} + 3Z_n) - Z_{12}^2}{\Delta} \\ I_{a2} &= -E_a \frac{Z_{12}(Z_0 + Z_{11} + 3Z_n) - Z_{12}^2}{\Delta} \\ I_{a0} &= E_a \frac{Z_{12}^2 - Z_{12}(Z_2 + Z_{11})}{\Delta} \end{aligned} \tag{49}$$

where

$$\begin{aligned} \Delta = {} & (Z_1 + Z_{11})(Z_2 + Z_{11})(Z_0 + Z_{11} + 3Z_n) \\ & - Z_{12}^2(Z_1 + Z_2 + Z_0 + 3Z_{11} + 3Z_n - 2Z_{12}) \\ Z_{11} = {} & \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{12} = {} & \tfrac{1}{3}(Z_a - Z_b) \end{aligned}$$

Symmetrical components of voltage at P can be obtained by substituting I_{a1}, I_{a2}, and I_{a0} from [49] in [35].

Self-Impedances Equal in Two Phases of the Ungrounded Y, *or* Δ *Replaced by Its Equivalent* Y. Assuming a grounded system supplying the ungrounded load, and substituting [19] in [47] and [48],

$$
\begin{aligned}
I_{a1} &= E_a \frac{9Z_2 + 3Z_a + 6Z_b}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2} \\
I_{a2} &= -E_a \frac{3(Z_a - Z_b)}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2} \\
&= -I_{a1} \frac{(Z_a - Z_b)}{3Z_2 + Z_a + 2Z_b} \\
V_n &= -E_a \frac{3(Z_a - Z_b)(Z_2 + Z_b)}{(3Z_1 + Z_a + 2Z_b)(3Z_2 + Z_a + 2Z_b) - (Z_a - Z_b)^2}
\end{aligned} \qquad [50]
$$

V_{a1}, V_{a2}, and V_{a0} at P are obtained from [35] by replacing I_{a1} and I_{a2} by their values from [50], and I_{a0} by zero.

Equal Self-Impedances in Two Phases of the Ungrounded Y *and Infinite Impedance in the Third Phase.* Equations [50] are evaluated for this case by dividing the numerators and denominators of the fractions by $3Z_a$, and then allowing Z_a to approach infinity. The following equations are obtained:

$$
\begin{aligned}
I_{a1} &= \frac{E_a}{Z_1 + Z_2 + 2Z_b} \\
I_{a2} &= \frac{-E_a}{Z_1 + Z_2 + 2Z_b} = -I_{a1} \\
V_n &= -\frac{E_a(Z_2 + Z_b)}{Z_1 + Z_2 + 2Z_b}
\end{aligned} \qquad [51]
$$

where V_n is the voltage at the junction of Z_b and $Z_c = Z_b$; there is no Y and no neutral with $Z_a = \infty$ and phase a therefore open.

The following problem illustrates the procedure for determining the currents in an unsymmetrical load when the phase impedances of the Y are known.

Problem 1. Given a generator with internal voltage $E_a = 1.0$ and impedances $Z_1 = Z_2 = Z_0 = 0$, i.e., infinite bus, supplying an unsymmetrical resistance load connected in Y, $Z_a = 1.0$, $Z_b = 1.5$, $Z_c = 0.5$. The neutral of the load is ungrounded. Find the three line currents and the voltage to ground at the neutral of the load.

Solution. From [18],

$$Z_{11} = Z_{22} = \tfrac{1}{3}(1 + 1.5 + 0.5) = 1$$

$$Z_{12} = Z_{01} = \tfrac{1}{3}(1 + 1.5a^2 + 0.5a) = -j\frac{1}{2\sqrt{3}} = -j0.289$$

$$Z_{21} = Z_{02} = \tfrac{1}{3}(1 + 1.5a + 0.5a^2) = j\frac{1}{2\sqrt{3}} = j0.289$$

Substituting $E_a = 1.0$, $Z_1 = Z_2 = 0$ and the above self- and mutual impedances in [47] and [48],

$$I_{a1} = 1.091$$
$$I_{a2} = -j0.315$$
$$V_n = -0.091 + j0.315$$

$$I_a = 1.091 - j0.315$$
$$I_b = -0.272 - j0.788$$
$$I_c = -0.819 + j1.103$$

For check: $I_a = \dfrac{1 - V_n}{1} = 1.091 - j0.315.$

Faults Considered Unsymmetrical Y-Connected Loads. Line-to-line, line-to-ground, and double line-to-ground faults may all be solved by applying the equations for an unsymmetrical Y-connected load supplied at P by a system represented as an equivalent generator with positive-, negative-, and zero-sequence impedances Z_1, Z_2, and Z_0, respectively, and generated voltage E_a.

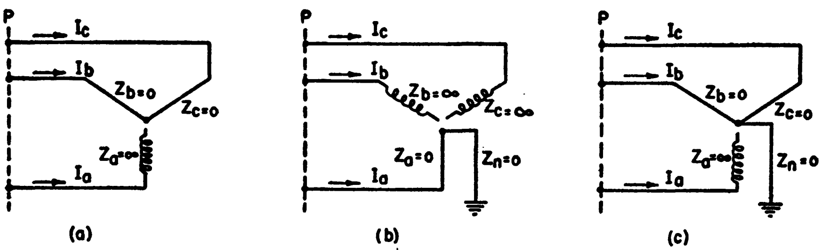


FIG. 6. Faults through zero impedance represented as unsymmetrical shunt loads. (*a*) Line-to-line fault. (*b*) Line-to-ground fault. (*c*) Double line-to-ground fault.

Figure 6(*a*) represents a *line-to-line fault through zero impedance,* if $Z_b = Z_c = 0$ and $Z_a = \infty$. With $Z_b = 0$ in [51],

$$I_{a1} = \frac{E_a}{Z_1 + Z_2} \quad \text{and} \quad I_{a2} = -\frac{E_a}{Z_1 + Z_2} = -I_{a1}$$

$$V_n = -\frac{E_a Z_2}{Z_1 + Z_2} = V_b = V_c$$

A *line-to-ground fault* on phase a is shown in Fig. 6(*b*) as an unbalanced Y-connected load with solidly grounded neutral, and $Z_a = Z_n = 0$, $Z_b = Z_c = \infty$. The current equations can be obtained from [49] by substituting zero for Z_a and Z_n, then dividing numerators and denominators of the fractions by Z_b to the highest power found in the denominator after simplification, and finally allowing Z_b to become infinite. Stated in another way, the fractions are evaluated by neglecting all terms except those in the highest powers of Z_b in numerator and denominator.

Replacing Z_{11} by $\frac{2}{3}Z_b$ and Z_{12} by $-Z_b/3$ in [49],

$$\begin{aligned}\Delta &= Z_1Z_2Z_0 + Z_{11}(Z_1Z_2 + Z_1Z_0 + Z_2Z_0) + Z_{11}^2(Z_1 + Z_2 + Z_0) \\ &\quad + Z_{11}^3 - 3Z_{11}Z_{12}^2 + 2Z_{12}^3 - Z_{12}^2(Z_1 + Z_2 + Z_0) \\ &= Z_1Z_2Z_0 + \tfrac{2}{3}Z_b(Z_1Z_2 + Z_1Z_0 + Z_2Z_0) + \tfrac{1}{3}Z_b^2(Z_1 + Z_2 + Z_0)\end{aligned}$$

With $Z_b = \infty$,

$$I_{a1} = \frac{E_a}{\Delta}[Z_2Z_0 + \tfrac{2}{3}Z_b(Z_2 + Z_0) + \tfrac{1}{3}Z_b^2] = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

$$I_{a2} = \frac{E_a}{\Delta}\left[\tfrac{1}{3}Z_b^2 + \frac{Z_b}{3}Z_0\right] = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

$$I_{a0} = \frac{E_a}{\Delta}\left[\tfrac{1}{3}Z_b^2 + \frac{Z_b}{3}Z_2\right] = \frac{E_a}{Z_1 + Z_2 + Z_0}$$

The sequence currents in a *double line-to-ground* fault can be obtained in a similar manner from [49] if zero is substituted for Z_n, Z_b, and Z_c, and Z_a is allowed to approach ∞. (See Fig. 6(c).)

Open Conductor Considered an Unsymmetrical Series Circuit. With conductor a open between P and Q, the part of the system between P and Q can be treated as an unsymmetrical series circuit in which $Z_a = \infty$, $Z_b = Z_c = 0$. From [19], with $Z_b = Z_c = 0$,

$$Z_{11} = Z_{22} = Z_{00} = Z_{12} = Z_{21} = Z_{10} = Z_{01} = Z_{20} = Z_{02} = \frac{Z_a}{3}$$

Replacing the self- and mutual sequence impedances in [4]–[6] by $Z_a/3$,

$$v_{a1} = V_{a1} - V'_{a1} = \frac{(I_{a1} + I_{a2} + I_{a0})Z_a}{3}$$

$$v_{a2} = V_{a2} - V'_{a2} = \frac{(I_{a1} + I_{a2} + I_{a0})Z_a}{3}$$

$$v_{a0} = V_{a0} - V'_{a0} = \frac{(I_{a1} + I_{a2} + I_{a0})Z_a}{3}$$

Therefore

$$v_{a1} = v_{a2} = v_{a0}$$

With $Z_a = \infty$,

$$I_{a1} + I_{a2} + I_{a0} = 0$$

Therefore

$$I_{a1} = -(I_{a2} + I_{a0})$$

The above equations check [40] and [41] of Chapter IV.

The case of two conductors open can be treated as an unsymmetrical series circuit between P and Q in which $Z_b = Z_c = \infty$, $Z_a = 0$. The procedure is analogous to that for one conductor open, and the results obtained check those given in Chapter IV.

Impedance of a Single-Phase Line-to-Line Load or Piece of Apparatus Designed for Single-Phase Operation. When the impedance Z of a line-to-line load or a piece of apparatus is given in per unit based on its rated line-to-line voltage and a given kva, and this impedance is to be used in calculations in per unit on a line-to-neutral voltage base, it is necessary to convert the given impedance Z to a line-to-neutral voltage base. Let Z' = impedance on a line-to-neutral voltage base and the given kva. Then in per unit of base line-to-neutral voltage and the given kva per phase,

$$Z' = Z\left(\frac{\text{Rated line-to-line voltage}}{\text{Base line-to-neutral voltage}}\right)^2$$

The three-phase kva base will be three times the given kva of the load or piece of apparatus.

Frequently load is given in terms of kva and power factor or kilowatts and power factor. For example, a line-to-line load of 20,000 kw at 0.9 power factor lagging is supplied from a 13.8-kv bus. The kva of the load is therefore 22,200 kva. On a *line-to-line voltage base* of 13.8 kv and 22,200 kva, the impedance is unity, and the impedance angle is the power factor angle, positive for lagging currents, negative for leading currents. Thus

$$Z = 1\underline{/25.8^\circ}$$

where $\underline{/25.8^\circ} = \cos^{-1} 0.9$ lagging. Expressed on a *line-to-neutral voltage base* of $13.8/\sqrt{3}$ and 22,200 kva per phase (three-phase kva base of 66,700 kva),

$$Z' = 1\underline{/25.8^\circ} \times \left(\frac{13.8}{\frac{13.8}{\sqrt{3}}}\right)^2 = 3\underline{/25.8^\circ}$$

On a three-phase kva base of 100,000 kva, the impedance Z' becomes

$$Z'' = 3\underline{/25.8^\circ} \times \frac{100,000}{66,700} = 4.5\underline{/25.8^\circ}$$

A *single-phase load* between phases in an otherwise symmetrical system may be treated as a fault between phases b and c through the load impedance Z_L as in Fig. 7(a). This is represented in Fig. 7(b) as an unsymmetrical Y-connected load with $Z_b = Z_c = \frac{Z_L}{2}$ and $Z_a = \infty$. Applying [51], with Z_b replaced by $Z_L/2$ and the system exclusive of

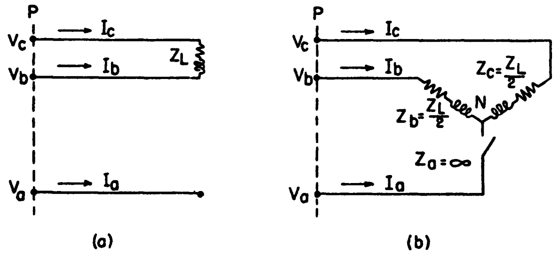


FIG. 7. (a) Single-phase line-to-line load of impedance Z_L between phases b and c. (b) An unsymmetrical Y-connected load, with $Z_b = Z_c = Z_L/2$ and $Z_a = \infty$.

the single-phase load replaced by an equivalent generator of internal voltage E_a and impedances Z_1, Z_2, and Z_0,

$$
\begin{aligned}
I_{a1} &= \frac{E_a}{Z_1 + Z_2 + Z_L} \\
I_{a2} &= \frac{-E_a}{Z_1 + Z_2 + Z_L} = -I_{a1}
\end{aligned}
\qquad [52]
$$

The above equations for I_{a1} and I_{a2} agree with those of Table I, Chapter IV, for the line-to-line fault through fault impedance, if Z_f (the fault impedance) is replaced by Z_L (the load impedance) and V_f by E_a. Symmetrical components of voltage at the fault, line currents flowing into the fault, voltages to ground, and line-to-line voltages at the fault can also be obtained from this table if Z_f and V_f are replaced by Z_L and E_a, respectively. It should be noted that V_f is defined as the voltage of phase a at the fault point F before the fault occurred; V_f could have been measured by a voltmeter. E_a, however, is the fictitious internal voltage of an equivalent machine which replaces the system exclusive of the single-phase load and is not a measurable voltage. From [1], Chapter IV, E_a replacing V_f depends upon the magnitude and phase of the positive-sequence current I_{a1} taken by the load, the positive-sequence system impedance Z_1 viewed from the load, and the positive-sequence voltage V_{a1} at the load. The voltages and currents at the load can be calculated from the equations of Table I, Chapter IV, by assuming a value for E_a which replaces V_f in these equations. As currents and voltages vary directly with E_a, if the calculated voltages at the load are higher or lower than normal allowable operating voltages, they can be decreased or increased in direct proportion as E_a is decreased or increased.

I_{a1} and I_{a2} given by [52] will be in amperes if E_a is in volts and Z_1, Z_2, and Z_L in ohms. If per unit quantities are used, Z_L must be expressed in per unit based on kva per phase and base line-to-neutral voltage which are base quantities in the one-line diagrams of the sequence networks.

The above discussion assumes that the load impedance Z_L is given directly or can be determined. In the usual case, the kva or power taken by the single-phase load and the load power factor are the given quantities, with the allowable range of operating voltages at the load stated, the problem being to determine the negative-sequence currents in a specified generator or an induction motor and the unbalanced voltages in the system. As stated at the beginning of this chapter, negative-sequence currents cause heating in solid rotors of turbine generators which may injure them if the negative-sequence currents are greater than those which the machine can safely carry. The upper limit of negative-sequence current for a three-phase turbine generator of normal design is not more than 15% of rated current. An appreciable amount of three-phase load can be carried with single-phase load, not exceeding its allowable limit. Turbine generators *designed for single-phase operation* are equipped with amortisseur windings of low resistance and reactance to reduce rotor currents resulting from negative-sequence armature currents.

Negative-Sequence Currents and Unbalanced Voltages Resulting for a Single-Phase Line-to-Line Load in an Otherwise Symmetrical System. When a single-phase line-to-line load is taken from an otherwise symmetrical system, there will be no zero-sequence currents and voltages in the system. The only negative-sequence current will be that resulting from the single-phase load. Let the load be between phases b and c; then this case corresponds to that of a line-to-line fault through impedance between phases b and c. The following equations apply. (See Table I, Chapter IV.)

$$\begin{aligned} I_{a1} &= -I_{a2} \\ I_b &= -j\sqrt{3}I_{a1} = j\sqrt{3}I_{a2} \\ V_{cb} &= V_b - V_c = -j\sqrt{3}(V_{a1} - V_{a2}) \\ V_{a2} &= -I_{a2}Z_2 = I_{a1}Z_2 \end{aligned} \qquad [53]$$

where I_b and V_{cb} are the current in the load and the voltage across it, respectively; I_{a1} and I_{a2} are the positive- and negative-sequence components of line current flowing into the load; V_{a1} and V_{a2} are the positive- and negative-sequence components of voltage to neutral at the load.

The magnitude of the load current in amperes is

$$|I_b| = \frac{\text{kva of load}}{|V_{cb}| \text{ in kv}}$$

$$\text{Base line current} = \frac{\text{Base kva per phase}}{\text{Base line-to-neutral voltage in kv}}$$

Dividing $I_b =$ by base line current,

$$|I_b| \begin{pmatrix} \text{in per unit of} \\ \text{base line current} \end{pmatrix} = \frac{\dfrac{(\text{kva of load})}{\text{base kva per phase}}}{\dfrac{|V_{cb}| \text{ in kv}}{\text{base line-to-neutral voltage in kv}}}$$

Let

P_L = power in single-phase load in per unit of base power per phase

V_{cb} = voltage across load in per unit of base line-to-neutral voltage

I_b = load current in per unit of base line current

$\cos\theta$ = power factor of load. θ is here arbitrarily assumed positive for a load with a leading power factor (for example, an over-excited synchronous motor or a capacitive impedance load) and negative for lagging power factor (an underexcited synchronous motor or reactive impedance load).

With base kw numerically equal to base kva,

$$|I_b| = \frac{P_L}{|V_{cb}| \cos\theta}$$

From [53],

$$|I_{a1}| = |I_{a2}| = \frac{|I_b|}{|\sqrt{3}|} = \frac{P_L}{\sqrt{3}|V_{cb}| \cos\theta} \qquad [54]$$

Determination of Negative-Sequence Current in Any Given Machine or Circuit. When the power and power factor of the load and the voltage across it are known, I_{a2}, the total negative-sequence current flowing from the system into the load, can be obtained from [54]. The amount of negative-sequence current in the various circuits of the system can then be determined from the negative-sequence impedance diagram of the system. It will divide between the paths from the zero-potential bus to the load, inversely as their impedances.

Determination of Voltages at the Load. From [53], the load current I_b lags I_{a1} by 90°. V_{cb} lags I_b by the angle θ if θ is positive and leads I_b by θ if θ is negative. With I_{a1} as reference vector and θ positive (leading power factor),

$$V_{cb} = |V_{cb}|\underline{/-\theta - 90} \qquad [55]$$

From [53]–[55],

$$V_{a2} = -I_{a2}Z_2 = I_{a1}Z_2 = \frac{P_L Z_2}{\sqrt{3}|V_{cb}| \cos \theta} \qquad [56]$$

$$V_{a1} = j\frac{V_{cb}}{\sqrt{3}} + V_{a2} = \frac{|V_{cb}|\underline{/-\theta}}{\sqrt{3}} + V_{a2}$$

$$= \frac{|V_{cb}|}{\sqrt{3}}(\cos \theta - j \sin \theta) + V_{a2} \qquad [57]$$

The line-to-ground voltages can be obtained by substituting $V_{a0} = 0$ and V_{a1} and V_{a2} from [56] and [57] in [7]–[9], Chapter II; the line-to-line voltages, by substituting V_{a1} and V_{a2} in [11], Chapter III.

Problem 2. In a steam station there are two turbo alternators, each rated 60,000 kva, 13.8 kv, and each having a negative-sequence reactance of 18% on its rating. There is a single-phase line-to-line load of 20,000 kw at a power factor estimated to be between 0.9 and 0.95 lagging on the line side of a load transformer at the generator terminals. There is a hydro station on the system at a distance from the load; its negative sequence reactance viewed from the generator terminals is 85% on a three-phase base of 100,000 kva. (See Fig. 8.) Find the negative-sequence current in the turbo alternators and the voltages at the load.

Solution. Three-phase loads are not given. It will be assumed that their negative-sequence impedances are high relative to those of the generators. Calculations will be made for a load power factor of 0.9. The voltage across the load is not given; 13.8 kv will be assumed. Calculations will be in per unit based on a three-phase kva of 100,000 and rated generator voltage. Base voltage is $13.8/\sqrt{3}$ and base kva per phase is 100,000/3.

$$P_L = \frac{20{,}000}{\left(\dfrac{100{,}000}{3}\right)} = 0.6 = \text{power in single-phase load in per unit of base power per phase}$$

$$V_{cb} = \frac{13.8}{\left(\dfrac{13.8}{\sqrt{3}}\right)} = \sqrt{3} = \text{voltage across load in per unit of base line-to-neutral voltage}$$

$$\text{Cos } \theta = 0.9; \quad \theta = -25.8°; \quad \sin \theta = -0.436$$

From [54],

$$|I_{a1}| = |I_{a2}| = \frac{0.6}{\sqrt{3} \times \sqrt{3} \times 0.9} = 0.222 \text{ per unit current}$$

The per unit negative-sequence reactance of the two turbo alternators in parallel, viewed from the load, is

$$0.18 \times \frac{100{,}000}{120{,}000} = 0.15 \text{ per unit reactance}$$

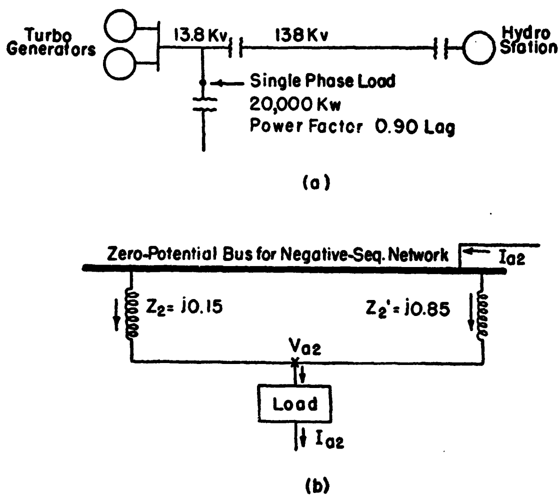


FIG. 8. (*a*) One-line diagram of system described in Problem 2. (*b*) Negative-sequence network for (*a*). Base three-phase kva = 100,000 kva.

The negative-sequence reactance of the hydro station viewed from the load is 0.85 in per unit on the chosen base quantities.

$$Z_2 = j\frac{0.15 \times 0.85}{1.00} = j0.128 \text{ per unit impedance}$$

The negative-sequence current in each turbo alternator (see Fig. 8(*b*)) is

$$\tfrac{1}{2}\left(0.222 \times \frac{85}{100}\right) = 0.0944 \text{ based on } 100{,}000 \text{ kva}$$

or 0.158 on 60,000 kva, the rating of each generator. The calculated negative-sequence line current in each alternator is 15.8% of its rated line current.

From [56] and [57], with I_{a1} as reference vector and $\theta = -25.8°$,

$$V_{a2} = I_{a1}Z_2 = 0.222(j0.128) = j0.028$$

$$V_{a1} = (\cos\theta - j\sin\theta) + V_{a2} = 0.900 + j0.436 + j0.028$$

$$= 0.900 + j0.464$$

The line-to-ground voltages and line-to-line voltages at the load in per unit of $13.8/\sqrt{3}$ kv are

$$V_a = 0.900 + j0.492 = 1.026\underline{/28.7^\circ}$$
$$V_b = -0.072 - j1.026 = 1.027\overline{\backslash 86.0^\circ}$$
$$V_c = -0.828 + j0.534 = 0.985\underline{/147.2^\circ}$$
$$V_{ba} = V_a - V_b = 0.972 + j1.518 = 1.802\underline{/32.6^\circ}$$
$$V_{cb} = V_b - V_c = 0.756 - j1.560 = 1.732\overline{\backslash 64.2^\circ}$$
$$V_{ac} = V_c - V_a = -1.728 + j0.042 = 1.728\underline{/178.6^\circ}$$

The allowable voltages at the load are not given in this problem. If normal operating line-to-line voltage is 13.8 kv, the voltage V_{cb} is normal, the voltage V_{ba} is about 4% above normal, and V_{ac} slightly below normal. Had it been stated that maximum allowable line-to-line voltage is 13.8, the calculated negative-sequence currents and the voltages at the load would be reduced in the ratio 1.732/1.802, so that V_{ba}, the highest line-to-line voltage at the load, would not exceed 13.8 kv. This would give about 4% less negative-sequence current in the alternators and reduce all voltages at the load about 4%. On the other hand, if the maximum allowable voltage is 5% above 13.8 kv, the negative-sequence currents in the alternators and the voltages at the load should be increased by about 1%.

EQUIVALENT CIRCUITS TO REPLACE AN UNSYMMETRICAL CIRCUIT IN THE SEQUENCE NETWORKS

When the sequence mutual impedances of the unsymmetrical circuit are reciprocal, as in [17] and [19], the unsymmetrical circuit can be replaced in the three sequence networks by equivalent circuits which are mutually coupled. These circuits can be used for determining current and voltage distribution during normal operation or during three-phase faults.

If the zero-sequence self-impedance of the unsymmetrical circuit is infinite and the sequence mutual impedances reciprocal, the positive- and negative-sequence networks are coupled through the mutual impedance $Z_{12} = Z_{21}$. The positive- and negative-sequence voltage drops between the terminals P and Q of the unsymmetrical circuit given by [4] and [5] become

$$\begin{aligned} v_{a1} &= V_{a1} - V'_{a1} = I_{a1}Z_{11} + I_{a2}Z_{12} \\ &= I_{a1}(Z_{11} - Z_{12}) + (I_{a1} + I_{a2})Z_{12} \\ v_{a2} &= V_{a2} - V'_{a2} = I_{a1}Z_{12} + I_{a2}Z_{22} \\ &= (I_{a1} + I_{a2})Z_{12} + I_{a2}(Z_{22} - Z_{12}) \end{aligned} \qquad [58]$$

Equations [58] are satisfied by the equivalent circuit for the unsymmetrical circuit between P and Q in the mutually coupled positive- and negative-sequence networks shown in Fig. 9(a), where the positive- and negative-sequence diagrams for the rest of the system are to be connected at points P and Q in each of the sequence networks. Figure 9(a) can be tested for its equivalency by opening the negative-sequence network at P or Q; then, with $I_{a2} = 0$ in both Fig. 9(a) and equations [58], the impedance met by I_{a1} is $Z_{11} - Z_{12} + Z_{12} = Z_{11}$, the positive-sequence self-impedance, and the voltage drop between P and Q in the negative-sequence network is $v_{a2} = I_{a1}Z_{12}$. Likewise, by opening the positive-sequence network at P or Q, the impedance met by

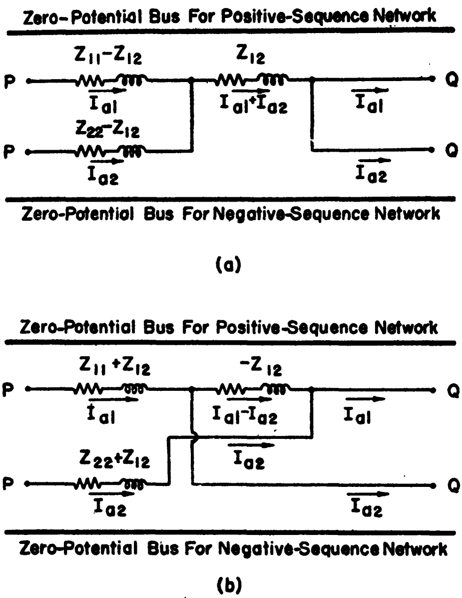


FIG. 9. Equivalent circuits to replace an unsymmetrical series circuit between P and Q in which $Z_{00} = \infty$ and the mutual impedances are reciprocal ($Z_{12} = Z_{21}$). (a) For analytic solutions or for use on a calculating table when Z_{12} is a reactive impedance. (b) Alternate circuit for use on a calculating table when Z_{12} is a capacitive reactance.

I_{a2} is Z_{22}, and the voltage drop between P and Q in the positive-sequence network is $I_{a2}Z_{12}$.

An alternate method of writing [58], suggested by Mr. R. B. Bodine for use on a d-c or an a-c calculating table in cases where Z_{12} is a capacitive reactance ($Z_{12} = -jX$), is

$$\begin{aligned} V_{a1} &= V_{a1} - V'_{a1} = I_{a1}(Z_{11} + Z_{12}) + (I_{a1} - I_{a2})(-Z_{12}) \\ V_{a2} &= V_{a2} - V'_{a2} = I_{a2}(Z_{22} + Z_{12}) - (I_{a1} - I_{a2})(-Z_{12}) \end{aligned} \qquad [58a]$$

Figure 9(b) gives the equivalent circuit corresponding to [58a], convenient for replacing the unsymmetrical circuit if Z_{12} is a capacitive reactance. When an analytic solution is made, it is immaterial which of the two circuits is used. When the d-c calculating table is used, Fig. 9(*a*) is applicable if $Z_{12} = jX_{12}$, and Fig. 9(*b*) if $Z_{12} = -jX_{12}$. Either circuit is applicable when an a-c network analyzer is used, but, because the range of inductances is wider than the range of capacitances, Fig. 9(*b*) is usually preferable if $Z_{12} = -jX_{12}$.

If *zero-sequence current flows in the unsymmetrical circuit* between P and Q, and the mutual impedances are reciprocal ($Z_{12} = Z_{21}$, $Z_{10} = Z_{01} = Z_{20} = Z_{02}$), then [4]–[6] become

$$\begin{aligned} v_{a1} &= V_{a1} - V'_{a1} = I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ v_{a2} &= V_{a2} - V'_{a2} = I_{a1}Z_{12} + I_{a2}Z_{22} + I_{a0}Z_{10} \\ v_{a0} &= V_{a0} - V'_{a0} = I_{a1}Z_{10} + I_{a2}Z_{10} + I_{a0}Z_{00} \end{aligned}$$

The above equations may be written

$$\begin{aligned} v_{a1} &= I_{a1}(Z_{11} - Z_{12}) + (I_{a1} + I_{a2} + I_{a0})Z_{10} \\ &\qquad + (I_{a1} + I_{a2})(Z_{12} - Z_{10}) \\ v_{a2} &= (I_{a1} + I_{a2} + I_{a0})Z_{10} + (I_{a1} + I_{a2})(Z_{12} - Z_{10}) \\ &\qquad + I_{a2}(Z_{22} - Z_{12}) \\ v_{a0} &= (I_{a1} + I_{a2} + I_{a0})Z_{10} + I_{a0}(Z_{00} - Z_{10}) \end{aligned} \qquad [59]$$

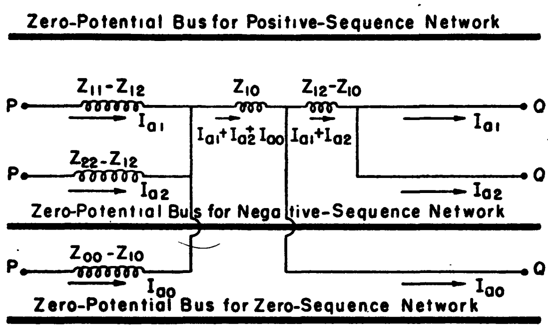


FIG. 10. Equivalent circuit to replace an unsymmetrical series circuit between P and Q in which Z_{00} is finite and the mutual impedances are reciprocal. ($Z_{21} = Z_{12}$; $Z_{01} = Z_{02} = Z_{20} = Z_{10}$.)

Equations [59] are satisfied by the equivalent circuit for the unsymmetrical series circuit between P and Q with reciprocal mutual impedances between the sequence networks given by Fig. 10, where the sequence diagrams for the rest of the system are to be connected at P and Q in each of the sequence networks.

Short Circuits on a System Containing an Unsymmetrical Circuit. *Three-Phase Faults.* When the fault does not involve ground, the conditions at the fault point F are $V_{a1} = 0$, $V_{a2} = 0$, $I_{a0} = 0$. These conditions are satisfied if the point F in both the positive- and negative-sequence networks is shorted to the zero-potential bus for the network. When the three-phase fault involves ground, the additional condition $V_{a0} = 0$ is satisfied by connecting the point F in the zero-sequence network to the zero-potential bus for the network.

Unsymmetrical short circuits on a symmetrical system are satisfied by interconnection of the sequence networks at the fault point F. In a system containing an unsymmetrical circuit, replaced by an equivalent circuit with direct connection between the sequence networks as in Figs. 9 or 10, a second direct connection between the networks cannot be made with any assurance that it will correctly represent conditions.

When an a-c network analyzer is available and the mutual impedances are reciprocal, the sequence networks can be mutually coupled through 1 : 1 turn ratio mutual coupling transformers which have infinite exciting impedances, zero resistances, and zero leakage reactances relative to the analyzer impedances. Figures 11(a) and (b) are equivalent circuits which satisfy [58] and [58a], respectively, when mutual coupling transformers are used. In analytic calculations, the mutual coupling circuit of Chapter I, Fig. 12(c), can be used instead of the directly connected equivalent circuits of Figs. 9 (a) and (b).

In developing the equations upon which the equivalent circuits for the unsymmetrical circuit are based, the reference phase a was arbitrarily chosen to give the simplest equivalent circuit during normal operation. With phase a specified, the fault must be located on the phase or phases relative to phase a which are actually involved. The relations between the symmetrical components of voltage and current of phase a at the fault for various types of faults involving various phases and ground are given in Table I of Chapter VII. For a line-to-ground fault on phase a, a line-to-line or a double line-to-ground fault on phases b and c, the sequence networks can be directly connected when mutual coupling transformers are used in the equivalent circuit for the unsymmetrical circuit. When the relations between the symmetrical components of voltage and current of phase a at the fault depend upon the operators a and a^2, phase shifters capable of turning the sequence currents and voltages at the F through 120° or 240° would be required. It should be remembered that equivalent circuits and connections of the sequence networks to satisfy fault conditions are derived from equations and used for the purpose of simplifying calculations and permitting a more comprehensive view of the problem.

Therefore, when equivalent circuits and connections of the sequence network are not indicated by the equations, solutions can always be made by means of additional equations.

The use of equivalent circuits will be illustrated for the open Δ transformer bank, and the Y–Δ grounded or ungrounded bank in

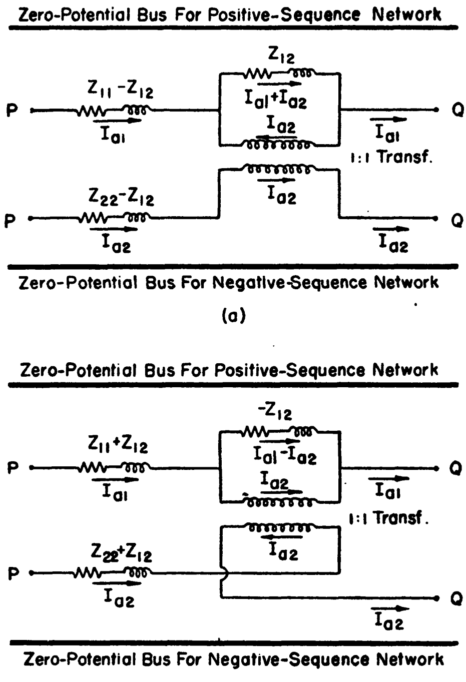


FIG. 11. A-c network analyzer equivalent circuits to replace an unsymmetrical circuit between P and Q in which Z_{00} is infinite, and the mutual impedances are reciprocal ($Z_{12} = Z_{21}$).

which one transformer has a different leakage impedance from that of the other two. With exciting currents neglected and all quantities expressed in per unit, a transformer bank of single-phase units can be treated as a self-impedance series circuit.

Open-Δ Transformer Bank. An open-Δ transformer bank consisting of two single-phase, two-winding transformers connected between phases a and b, and a and c, is shown in Fig. 12(a) between points P and Q of the system. The open-Δ bank may be considered a Y–Y-connected bank with both windings of one phase shorted as shown in Fig. 12(b). The impedance of the bank to zero-sequence currents is infinite. Positive- and negative-sequence currents in phases b and c meet the transformer leakage impedances in these phases; in phase a they meet no impedance.

With exciting currents neglected, let the leakage impedances of the transformers between phases a and b and a and c be Z_{ab} and Z_{ac}, respectively. If Z_{ab} and Z_{ac} are expressed in ohms, Z_b and Z_c, the impedances of the equivalent Y-connected bank of Fig. 12(b) will also be in ohms.

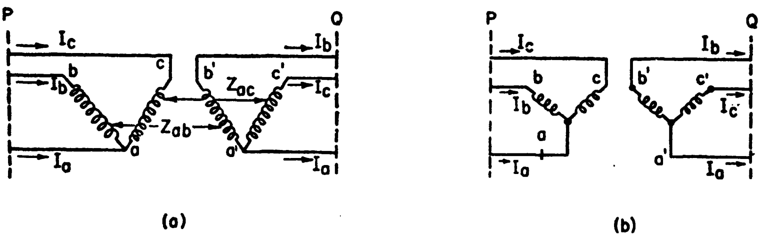


FIG. 12. (a) Open-Δ transformer bank between P and Q. (b) Open-Δ transformer bank considered a Y–Y bank with both windings of one phase shorted.

Z_{ab} and Z_{ac} will ordinarily be given in per cent, or per unit, based on rated kva and voltage of the transformers. Their rated voltage is line-to-line voltage. To express Z_{ab} and Z_{ac} on their rated kva and line-to-neutral voltage they must be multiplied by 3. (See [31], Chapter I.) For two identical single-phase units connected in open Δ, $Z_{ab}(\%) = Z_{ac}(\%) = Z_t(\%)$ and

$$Z_b(\%) = Z_c(\%) = 3Z_t(\%)$$

$$Z_a(\%) = 0$$

where Z_a, Z_b, and Z_c are in per cent based on line-to-neutral voltage and a kva per phase equal to the kva rating of one transformer (three-phase kva is three times the kva rating of one transformer).

The same result would be obtained if $Z_{ab} = Z_{ac} = Z_t$ were first expressed in ohms and then in per cent based on line-to-neutral voltage and a three-phase kva base equal to three times the rating of one transformer. For example: Two single-phase transformers connected in open Δ are each rated 5000 kva, 13,200–2300 volts, and each has a reactance of 6% on its rating. From [24], Chapter I,

$$Z_{ab} \text{ (in ohms referred to 13.2 kv side)} = \frac{6 \times (13.2)^2 \times 10}{5000}$$

$$= 2.09 \text{ ohms}$$

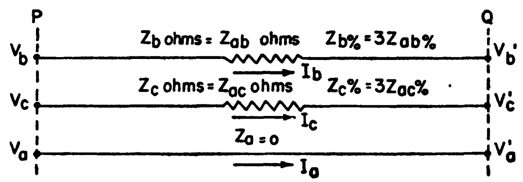
$$Z_b = Z_c = 2.09 \text{ ohms (viewed from 13.2 kv side)}$$

On a kva per phase of 5000 kva (three-phase kva base of 15,000 kva) and a base line-to-neutral voltage of $13.2/\sqrt{3}$ kv, from [26], Chapter I,

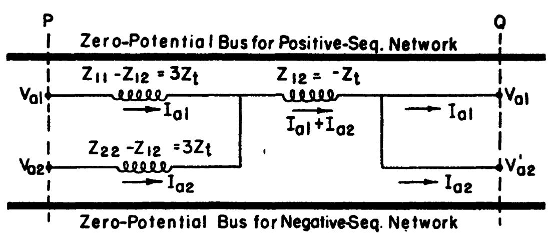
$$Z_b = Z_c = \frac{2.09 \times 15{,}000}{(13.2)^2 \times 10} = 18\% = 3Z_t$$

$$Z_a = 0$$

If the kva ratings of the two transformers are different, their impedances in per cent should be expressed on the same kva per phase before being multiplied by 3. Base three-phase kva is then three times the chosen kva per phase.



(c)



(d)

FIG. 12. (*c*) Three-phase positive- and negative-sequence equivalent circuit for open-Δ transformer bank between P and Q. Z_a, Z_b, and Z_c in per cent are based on line-to-neutral voltage and a kva per phase equal to that upon which Z_{ab} and Z_{ac} are based. (*d*) Equivalent circuit to replace an open-Δ bank of two identical single-phase units in the positive- and negative-sequence networks between P and Q, with three-phase base kva equal to three times the rated kva of one unit. Z_t is the per unit impedance of one transformer based on its rating.

An equivalent three-phase series circuit to replace the open-Δ transformer bank between P and Q in the positive- and negative-sequence three-phase diagrams is shown in Fig. 12(*c*). The per unit sequence self- and mutual impedances for the equivalent Y, replacing the open-Δ transformer bank, from [18] and Fig. 12(*c*) with Z_{ab} and Z_{ac} in per unit, are

$$
\begin{aligned}
Z_{11} = Z_{22} &= \tfrac{1}{3}(Z_b + Z_c) = Z_{ab} + Z_{ac} \\
Z_{12} &= \tfrac{1}{3}(a^2 Z_b + aZ_c) \\
&= -\tfrac{1}{2}(Z_{ab} + Z_{ac}) + j\frac{\sqrt{3}}{2}(Z_{ac} - Z_{ab}) \qquad [60] \\
Z_{21} &= \tfrac{1}{3}(aZ_b + a^2 Z_c) \\
&= -\tfrac{1}{2}(Z_{ab} + Z_{ac}) - j\frac{\sqrt{3}}{2}(Z_{ac} - Z_{ab})
\end{aligned}
$$

For two identical single-phase units, $Z_b = Z_c = 3Z_t$ and [60] becomes

$$
\begin{aligned}
Z_{11} &= Z_{22} = \tfrac{2}{3}Z_b = 2Z_t \\
Z_{12} &= Z_{21} = -\tfrac{1}{3}Z_b = -Z_t
\end{aligned} \qquad [61]
$$

Substituting [61] in [4] and [5] with $I_{a0} = 0$,

$$
\begin{aligned}
v_{a1} &= V_{a1} - V'_{a1} = I_{a1}(2Z_t) - I_{a2}Z_t \\
v_{a2} &= V_{a2} - V'_{a2} = -I_{a1}Z_t + I_{a2}(2Z_t)
\end{aligned} \qquad [62]
$$

Rewriting [62],

$$
\begin{aligned}
v_{a1} &= V_{a1} - V'_{a1} = I_{a1}(3Z_t) + (I_{a1} + I_{a2})(-Z_t) \\
v_{a2} &= V_{a2} - V'_{a2} = (I_{a1} + I_{a2})(-Z_t) + I_{a2}(3Z_t)
\end{aligned} \qquad [63]
$$

The equivalent series circuit determined from [63] to replace the open-Δ transformer bank between P and Q in the positive- and negative-sequence networks is shown in Fig. 12(d). It can be used for determining currents and voltages during normal operation or during three-phase faults. Figure 12(d) is similar to Fig. 9(a), but Fig. 9(b) can be used if preferred. (Unsymmetrical faults on a system containing an open-Δ transformer bank are also discussed in Chapter X.)

Problem 3. With balanced voltages applied to the high side of an open-Δ transformer bank consisting of two identical single-phase units, compare the short-circuit currents with those obtained with a complete Δ–Δ bank for a (1) Three-phase short circuit on the low-voltage side. (2) Line-to-line short circuit on the low-voltage side between (a) phases b and c and (b) phases a and b.

Solution. Let E_a = per unit applied voltage of phase a and Z_t = per unit impedance of each of the transformers of the open-Δ bank, based on its rating.

Complete Bank: The impedance of the equivalent Y–Y bank to replace the complete Δ–Δ bank is the same as that of the Δ–Δ bank when both are expressed in per unit, each on its own voltage base, and the same kva per phase. (See Chapter I.)

(1) *Three-Phase Fault:*

Complete Bank: $I_a = \dfrac{E_a}{Z_t}$; $I_b = \dfrac{E_a/\underline{120°}}{Z_t}$; $I_c = \dfrac{E_a/\underline{120°}}{Z_t}$

Open-Δ Bank: With a three-phase fault on the low-voltage side, the open-Δ bank is equivalent to a Y-connected shunt impedance load with ungrounded neutral in which $Z_b = Z_c = 3Z_t$ and $Z_a = 0$. From [50], with $Z_1 = Z_2 = 0$,

$$I_{a1} = \frac{2E_a}{3Z_t}; \quad I_{a2} = \tfrac{1}{2}I_{a1} = \frac{E_a}{3Z_t}$$

Therefore

$$I_a = \frac{E_a}{Z_t}; \quad I_b = \frac{E_a}{\sqrt{3}Z_t}\overline{/150^\circ}; \quad I_c = \frac{E_a}{\sqrt{3}Z_t}\underline{/150^\circ}$$

(2) *Line-to-Line Fault:*

Complete Bank: (a) $I_b = \dfrac{E_{cb}}{2Z_t} = \dfrac{\sqrt{3}E_a}{2Z_t}\overline{/90^\circ}$

(b) $I_a = \dfrac{E_{ba}}{2Z_t} = \dfrac{\sqrt{3}E_a}{2Z_t}\underline{/30^\circ}$

Open-Δ Bank: The fault currents can be obtained directly from Fig. 12(c), the three-phase equivalent circuit for the positive and negative sequence networks.

$$(a) \quad I_b = \frac{E_{cb}}{6Z_t} = \frac{\sqrt{3}E_a}{6Z_t}\overline{/90^\circ}$$

$$(b) \quad I_a = \frac{E_{ba}}{3Z_t} = \frac{\sqrt{3}E_a}{3Z_t}\underline{/30^\circ}$$

The following problem illustrates the use of the equivalent circuit of Fig. 12(d) for determining negative sequence currents in a system containing an open-Δ transformer bank.

Problem 4. Power is supplied through an open-Δ transformer bank of two identical single-phase units, each having a leakage reactance of 8% on its rating (2000 kva, 66–6.9 kv), to a symmetrical three-phase load on the 6.9-kv side of the bank. (See Fig. 13(a).) The negative-sequence impedance of the load is 16% on 3000 kva and 6.9 kv. The negative-sequence impedance of the symmetrical part of the system on the 66-kv side of the open-Δ transformer bank is 20% on a three-phase kva base of 120,000 kva and 66 kv. With 100% positive-sequence current flowing into the load, what is the negative-sequence current in the load in per cent of the positive-sequence current?

Solution. Calculations will be based on a three-phase kva base of 3000 kva, 1000 kva per phase. The reactance of each transformer is 8% on 2000 kva or 4% on 1000 kva. Z_t in the equivalent circuit of Fig. 12(d) is therefore $j0.04$ on 1000 kva per phase. The negative-sequence system impedance is 20% on a three-phase base of 120,000 kva, and therefore 0.5% on a three-phase base of 3000 kva (1000 kva per phase). The negative-sequence load impedance is 16% on the chosen three-phase kva base of 3000 kva. The positive-sequence equivalent circuit of the open-Δ bank with its mutual connection to the complete negative-sequence network is shown in Fig. 13(b). Because the negative-sequence current in terms of the positive is required, it is unnecessary to consider the complete positive-sequence network.

From Fig. 13(b), I_{a1} has two paths in parallel; one has an impedance of $-j0.04$, the other is through impedances of the negative-sequence network in series and has the impedance $j(0.12 + 0.005 + 0.16) = j0.285$. In the equivalent circuit, the current I_{a1} divides inversely as the impedances in these two paths. With the positive direction of I_{a2} indicated by arrows,

$$I_{a2} = -I_{a1}\frac{-j0.04}{j(0.285 - 0.04)} = 0.163I_{a1} = 16.3\%$$

$$V_{a2} = j0.16I_{a2} = j0.026I_{a1} = 2.6\% \text{ at } Q$$

The direction of the negative-sequence current is that indicated by arrow in Fig. 13 (*b*). Its magnitude is 16.3% of the positive-sequence current flowing into the load. The

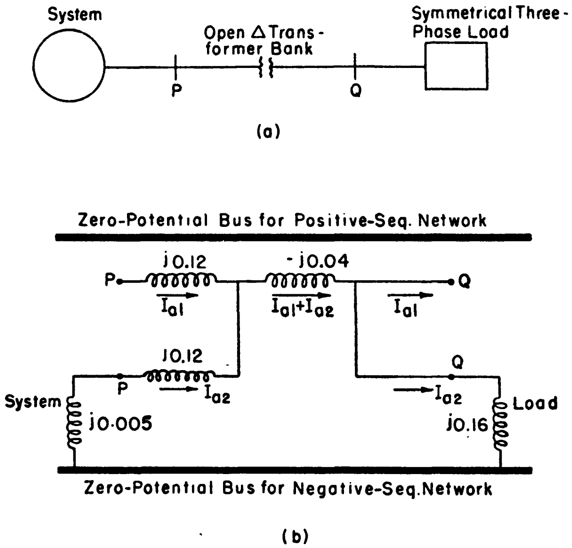


FIG. 13. (*a*) One-line diagram of system described in Problem 3. (*b*) Positive-sequence network between P and Q (the terminals of the open-Δ transformer bank) mutually coupled with the negative-sequence network. Impedances in per unit. Three-phase base kva = 3000 kva.

negative-sequence current resulting from the unsymmetrical series circuit is a circulating current which flows through the system, transformer bank, and load in series. It differs from the negative-sequence current resulting from an unsymmetrical short circuit which flows from the zero-potential bus to the fault through parallel paths. If the negative-sequence impedances of the system or load are lower than the values shown, the negative-sequence current will be higher.

Transformer Banks of Dissimilar Units. Assume the turn ratios of the three units to be equal, but their leakage impedances unequal. If exciting currents can be neglected, the transformer bank may be considered an unsymmetrical self-impedance series circuit. The phase impedances of this unsymmetrical series circuit are conveniently determined by viewing the bank from one set of terminals with the other set shorted to ground. For the Y–Y bank with the neutrals of both Y's solidly grounded, or the solidly grounded Y–Δ bank viewed from the Y side, the leakage impedances of the three units are the phase impedances Z_a, Z_b, and Z_c to be substituted in [18] or [19] to give the sequence self- and mutual impedances. If the Y–Δ bank is grounded through Z_n, $3Z_n$ is added to Z_{00} in [18] or [19]. The Δ–Δ bank may be replaced by an equivalent ungrounded Y–Y bank, or the positive- and negative-sequence self- and mutual impedance can be obtained directly from equations [31] and [32] in terms of the leakage impedances of the Δ-connected units. For the Δ–Y bank there is a shift in phase of positive- and negative-sequence line currents and voltages to neutral in passing through the bank. (See Chapter III, Fig. 19, and Problem 6.) In problems where the magnitude only of negative- or zero-sequence currents is of importance, the shift in phase need not be taken into account. This is illustrated in Problems 5 and 6.

If the leakage impedances of all three units are unequal, the sequence mutual impedances are non-reciprocal. Positive-sequence currents induce negative- and zero-sequence voltage drops $I_{a1}Z_{21}$ and $I_{a1}Z_{01}$, respectively, in the negative- and zero-sequence networks which meet negative- and zero-sequence series impedances. The following discussion applies to unsymmetrical transformer banks in which two units have equal leakage impedances; therefore, from [19] the sequence mutual impedances are reciprocal.

Consider a generator with positive-, negative-, and zero-sequence impedances Z_1, Z_2, and Z_0, respectively, supplying power through an unsymmetrical bank to a load; other generators also supplying power to the load, but not through the unsymmetrical transformer bank.

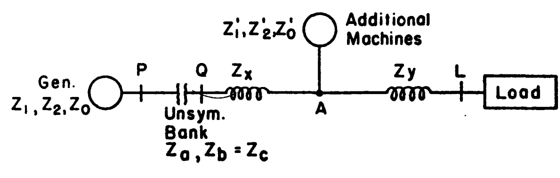


FIG. 14(a). One-line system diagram.

Figure 14(a) shows the one-line system diagram. The unsymmetrical transformer bank is between P and Q. Z_1', Z_2', and Z_0' are the equivalent positive-, negative-, and zero-sequence impedances, respectively, of the other machines on the system viewed from A. The load is at L; Z_x and Z_y indicate impedances between Q and A and between A and L, respectively.

When there are no zero-sequence currents, the positive- and negative-sequence networks and the mutual coupling between them are shown in Fig. 14(*b*), the equivalent circuit for the unsymmetrical transformer bank being that given by Fig. 9(*a*). From this figure it can be seen that positive-sequence currents flowing through the unsymmetrical transformer bank produce a series negative-sequence voltage drop,

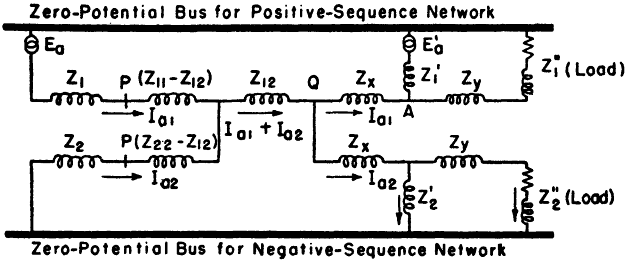


FIG. 14(*b*). Connection of positive- and negative-sequence networks of (*a*) with Z_{00} infinite.

$I_{a1}Z_{12}$, which circulates a negative-sequence current in the negative-sequence network, its value in terms of I_{a1} being

$$I_{a2} = -I_{a1}\frac{Z_{12}}{Z_2 + Z_{22} + Z_x + \dfrac{Z_2'(Z_y + Z_2'')}{Z_2' + Z_y + Z_2''}}$$

Problem 5. In Fig. 14(*b*) the negative-sequence reactances of the generator and the combined additional machines are 16% and 6%, respectively, and the transformer reactances are 7%, 7%, and 13%. All reactances are expressed on the kva rating of the generator. The negative-sequence impedance of the load in series with Z_y is relatively too large to be considered. Z_x offers 2% impedance to positive- and negative-sequence currents. What is the magnitude of the negative-sequence current in the generator with full-load positive-sequence generator current?

Solution. From [19], with $Z_a = j0.13$, $Z_b = Z_c = j0.07$,

$$Z_{11} = Z_{22} = \tfrac{1}{3}(Z_a + 2Z_b) = \tfrac{1}{3}[j0.13 + j2(0.07)] = j0.09$$

$$Z_{12} = Z_{21} = \tfrac{1}{3}(Z_a - Z_b) = \tfrac{1}{3}(j0.13 - j0.07) = j0.02$$

Substituting in the above equation,

$$I_{a2} = -I_{a1}\frac{0.02}{0.16 + 0.09 + 0.02 + 0.06} = -I_{a1}\frac{0.02}{0.33} = -0.06I_{a1}$$

The negative-sequence current in the generator for this case is approximately 6% of full-load generator current.

When there is a path for zero-sequence currents in the transformer, the equivalent circuit of Fig. 10 can be applied. For a self-impedance static circuit with the impedances of two phases equal, all mutual impedances between the sequence networks are equal. (See [19].) Therefore $Z_{12} - Z_{10} = 0$, and the impedance $Z_{12} - Z_{10}$ in Fig. 10 is zero.

Problem 6. Solve Problem 5, assuming the unsymmetrical transformer bank to be connected Δ–Y with the neutral of the Y grounded, and the terminals of the Δ at P and those of the Y at Q. The load is ungrounded but the generators connected at A are grounded and have a zero-sequence impedance of 4% on the kva rating of the generators at P. Z_x offers 3% impedance to zero-sequence currents.

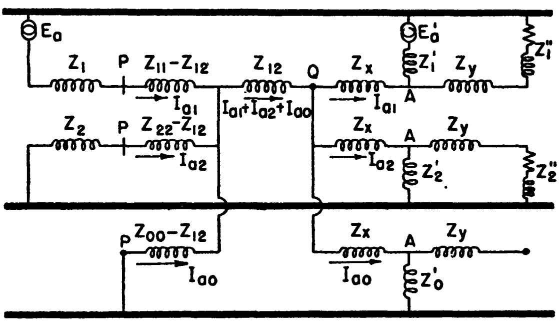


FIG 14(c). Connection of positive-, negative-, and zero-sequence networks of (a) for solution of Problem 6.

Solution. The sequence networks are shown in Fig. 14(c) mutually coupled through the impedance $Z_{12} = Z_{10}$. The positive-sequence current in the equivalent circuit now has three paths in parallel: one path is through the impedance Z_{12}, one through the series impedance of the negative-sequence network exclusive of Z_{12}, and the third through the series impedance of the zero-sequence network exclusive of $Z_{10} = Z_{12}$. If z_2 and z_0 represent the impedances of the paths in the negative- and zero-sequence networks, $Z_y + Z_2''$ being large relative to Z_2',

$$z_2 = Z_2 + (Z_{22} - Z_{12}) + Z_x + Z_2' = j0.16 + j(0.09 - 0.02) + j0.02 + j0.06$$
$$= j0.31$$

$$z_0 = (Z_{00} - Z_{12}) + Z_x + Z_0' = j0.07 + j0.03 + j0.04 = j0.14$$

The parallel impedance of Z_{12} and z_0 is

$$j\frac{0.14 \times 0.02}{0.16} = j0.0175$$

$$I_{a2} = -I_{a1}\frac{0.0175}{0.0175 + 0.31} = -0.053I_{a1}$$

The negative-sequence current in the generator is 5.3% of the positive-sequence current flowing in the generator. By grounding the Δ–Y transformer bank, the negative-sequence current has been decreased from 6% to 5.3%.

The parallel value of Z_{12} and z_2 is $j0.0188$. The zero-sequence current in the transformer is

$$I_{a0} = -I_{a1}\frac{0.0188}{0.0188 + 0.14} = -0.012 I_{a1}$$

$$= 1.2\% \text{ of the positive-sequence current in the transformer}$$

Sequence Admittances of Unsymmetrical Circuits

In an unsymmetrical shunt circuit, as distinguished from a series circuit, it may be of advantage to have equations expressing the symmetrical components of current flowing into the circuit in terms of the symmetrical components of applied voltages to ground at the circuit terminals and the sequence self- and mutual admittances of the circuit. With voltages to ground, V_a, V_b, V_c, applied to the shunt circuit and currents I_a, I_b, I_c flowing into the circuit, the equations will be written

$$\begin{aligned} I_{a1} &= Y_{11}V_{a1} + Y_{12}V_{a2} + Y_{10}V_{a0} \\ I_{a2} &= Y_{21}V_{a1} + Y_{22}V_{a2} + Y_{20}V_{a0} \\ I_{a0} &= Y_{01}V_{a1} + Y_{02}V_{a2} + Y_{00}V_{a0} \end{aligned} \qquad [64]$$

where the Y's with two subscripts represent the sequence self- and mutual admittances of the circuit, the first subscript referring to the sequence of the current given by the equation and the second to the sequence of the voltage associated with the admittance.

Sequence Admittances in Terms of Sequence Impedances. Solving [11] for I_{a1}, I_{a2}, and I_{a0}, then equating coefficients of V_{a1}, V_{a2}, and V_{a0} in the resulting equations to the corresponding coefficients in [64]. Using determinants (see Appendix A), the following equations are written directly:

$$\begin{aligned} Y_{11} &= \frac{(Z_{22}Z_{00} - Z_{02}Z_{20})}{\Delta} \\ Y_{22} &= \frac{(Z_{11}Z_{00} - Z_{01}Z_{10})}{\Delta} \\ Y_{00} &= \frac{(Z_{11}Z_{22} - Z_{21}Z_{12})}{\Delta} \\ Y_{12} &= -\frac{(Z_{12}Z_{00} - Z_{02}Z_{10})}{\Delta} \\ Y_{10} &= \frac{(Z_{12}Z_{20} - Z_{22}Z_{10})}{\Delta} \\ Y_{21} &= -\frac{(Z_{21}Z_{00} - Z_{01}Z_{20})}{\Delta} \end{aligned} \qquad [65]$$

$$Y_{20} = -\frac{(Z_{11}Z_{20} - Z_{21}Z_{10})}{\Delta}$$

$$Y_{01} = \frac{(Z_{21}Z_{02} - Z_{01}Z_{22})}{\Delta}$$

$$Y_{02} = -\frac{(Z_{11}Z_{02} - Z_{01}Z_{12})}{\Delta}$$

where

$$\Delta = Z_{11}(Z_{22}Z_{00} - Z_{02}Z_{20}) - Z_{21}(Z_{12}Z_{00} - Z_{02}Z_{10}) + Z_{01}(Z_{12}Z_{20} - Z_{22}Z_{10})$$

Unsymmetrical, Ungrounded Circuit. In an ungrounded circuit without a neutral conductor there can be no zero-sequence currents, and zero-sequence voltages applied to the circuit will produce no currents of any sequence; [64] for the ungrounded circuit therefore becomes

$$\begin{aligned} I_{a1} &= Y_{11}V_{a1} + Y_{12}V_{a2} \\ I_{a2} &= Y_{21}V_{a1} + Y_{22}V_{a2} \\ I_{a0} &= 0 \end{aligned} \qquad [66]$$

where

$$\begin{aligned} Y_{11} &= \frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \\ Y_{22} &= \frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \\ Y_{12} &= \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \\ Y_{21} &= \frac{-Z_{21}}{Z_{11}Z_{22} - Z_{12}Z_{21}} \\ Y_{00} &= Y_{01} = Y_{02} = Y_{10} = Y_{01} = 0 \end{aligned} \qquad [67]$$

Equations [67] were obtained from [65] by allowing Z_{00} to become infinite with all other sequence self- and mutual impedances remaining finite.

Unsymmetrical Grounded Circuit — System Otherwise Ungrounded. Replacing I_{a0} in [64] by zero, and solving for V_{a0},

$$V_{a0} = -\frac{Y_{01}}{Y_{00}}V_{a1} - \frac{Y_{02}}{Y_{00}}V_{a2} \qquad [68]$$

Substituting [68] in [64],

$$
\begin{aligned}
I_{a1} &= \left(Y_{11} - \frac{Y_{10}Y_{01}}{Y_{00}}\right) V_{a1} + \left(Y_{12} - \frac{Y_{10}Y_{02}}{Y_{00}}\right) V_{a2} \\
I_{a2} &= \left(Y_{21} - \frac{Y_{20}Y_{01}}{Y_{00}}\right) V_{a1} + \left(Y_{22} - \frac{Y_{20}Y_{02}}{Y_{00}}\right) V_{a2} \\
I_{a0} &= 0
\end{aligned}
\qquad [69]
$$

where the sequence self- and mutual admittances in [69] are defined in [65] in terms of the sequence self- and mutual impedances.

In a symmetrical circuit, $Z_{11} = Z_1$; $Z_{22} = Z_2$; $Z_{00} = Z_0$; $Z_{12} = Z_{10} = Z_{21} = Z_{20} = Z_{01} = Z_{02} = 0$. Substituting these values in [65],

$$
\begin{aligned}
\Delta &= Z_{11}Z_{22}Z_{00} \\
Y_{11} &= \frac{Z_{22}Z_{00}}{Z_{11}Z_{22}Z_{00}} = \frac{1}{Z_{11}} = \frac{1}{Z_1} \\
Y_{22} &= \frac{Z_{11}Z_{00}}{Z_{11}Z_{22}Z_{00}} = \frac{1}{Z_{22}} = \frac{1}{Z_2} \\
Y_{00} &= \frac{Z_{11}Z_{22}}{Z_{11}Z_{22}Z_{00}} = \frac{1}{Z_{00}} = \frac{1}{Z_0} \\
Y_{12} &= Y_{10} = Y_{21} = Y_{20} = Y_{01} = Y_{02} = 0
\end{aligned}
\qquad [70]
$$

In a symmetrical circuit, the sequence self-admittances are the reciprocals of the corresponding self-impedances, and there are no mutual admittances between the sequence networks. It is only in a symmetrical circuit that this is true.

Sequence Admittances of the Three Phases. The positive-, negative-, and zero-sequence admittance of the three phases of the circuit were not required in the development of [65] and [67]. As a matter of interest, however, a discussion of them will be given.

By analogy from the corresponding definitions of impedances the positive-sequence admittances of phases a, b, and c will be indicated by Y_{a1}, Y_{b1}, and Y_{c1} and defined as the ratios of the three line currents to the corresponding applied phase voltages with positive-sequence voltages only applied to the circuit. To apply positive-sequence voltages only to an unsymmetrical circuit, it is necessary not only that balanced voltages of positive-sequence phase order be applied through zero impedances but also that the neutral of the applied voltages be grounded. If the neutral is ungrounded, there is no path for zero-sequence currents and, although balanced line-to-line voltages will be applied, the phase voltages may contain zero-sequence components of voltages resulting from the flow of positive- and negative-sequence currents through the unsymmetrical circuit. Likewise to apply negative-sequence voltages to a circuit, the neutral of the applied voltages must be grounded. The negative-sequence admittances of phases a, b, and c will be indicated by Y_{a2}, Y_{b2}, and Y_{c2} and defined as the ratios of the three line currents to the corresponding phase voltages with only negative-sequence voltages applied to the circuit.

The zero-sequence admittances of the three phases a, b, and c will be indicated by Y_{a0}, Y_{b0}, and Y_{c0}, respectively, and defined as the ratios of the three line currents to the corresponding phase voltages with only zero-sequence voltages applied to the circuit. Zero-sequence voltages by definition are equal voltages to ground or to a neutral conductor. The ground in this case is also a neutral point, being common to the three phases, and therefore provides a path to neutral for positive- and negative-sequence currents as well as a path to ground for zero-sequence currents.

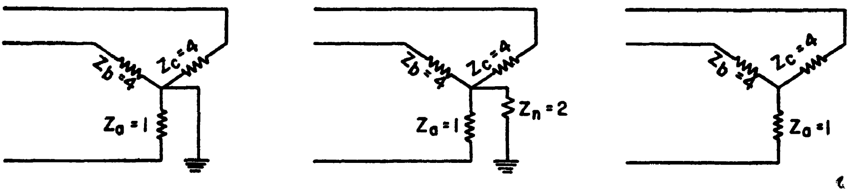
The currents I_a, I_b, and I_c, in terms of the sequence admittances of the three phases and the applied voltages V_a, V_b, and V_c, replaced by their symmetrical components, are given by the following equations:

$$\begin{aligned} I_a &= V_{a1}Y_{a1} + V_{a2}Y_{a2} + V_{a0}Y_{a0} \\ I_b &= a^2V_{a1}Y_{b1} + aV_{a2}Y_{b2} + V_{a0}Y_{b0} \\ I_c &= aV_{a1}Y_{c1} + a^2V_{a2}Y_{c2} + V_{a0}Y_{c0} \end{aligned} \qquad [71]$$

Sequence Self- and Mutual Admittances in Terms of Sequence Admittances of the Phases. Resolving the currents in [71] into their symmetrical components, and equating the coefficients of V_{a1}, V_{a2}, and V_{a0} in the resultant equations to those in [64],

$$\begin{aligned} Y_{11} &= \tfrac{1}{3}(Y_{a1} + Y_{b1} + Y_{c1}) \\ Y_{22} &= \tfrac{1}{3}(Y_{a2} + Y_{b2} + Y_{c2}) \\ Y_{00} &= \tfrac{1}{3}(Y_{a0} + Y_{b0} + Y_{c0}) \\ Y_{12} &= \tfrac{1}{3}(Y_{a2} + a^2Y_{b2} + aY_{c2}) \\ Y_{10} &= \tfrac{1}{3}(Y_{a0} + aY_{b0} + a^2Y_{c0}) \\ Y_{21} &= \tfrac{1}{3}(Y_{a1} + aY_{b1} + a^2Y_{c1}) \\ Y_{20} &= \tfrac{1}{3}(Y_{a0} + a^2Y_{b0} + aY_{c0}) \\ Y_{01} &= \tfrac{1}{3}(Y_{a1} + a^2Y_{b1} + aY_{c1}) \\ Y_{02} &= \tfrac{1}{3}(Y_{a2} + aY_{b2} + a^2Y_{c2}) \end{aligned} \qquad [72]$$

The sequence self- and mutual admittances of [64] are defined in terms of the sequence admittances of the three phases in [72]; in [65] they are defined in terms of the sequence self- and mutual impedances. In the general unsymmetrical circuit, the sequence impedances of the phases, from which the sequence self- and mutual impedances are calculated using [7], are more readily obtained than the sequence admittances. In many unsymmetrical circuits, the sequence impedances of the phases can be written by inspection; this is not usually the case for the sequence admittances of the phases unless the circuit is a symmetrical one or a solidly grounded unsymmetrical circuit in which the admittances of the three phases are independent of the sequence of the applied voltage. In either of these cases, the sequence admittances of the three phases are the reciprocals of the corresponding sequence impedances. In an unsymmetrical solidly grounded circuit in which the admittances of the three phases are independent of the sequence of the applied voltage, although the admittances of the three phases are the reciprocals of the corresponding impedances, the sequence admittances are not the reciprocals of the corresponding sequence impedances. This is illustrated in the following example.



(a) (b) (c)

FIG. 15. Unsymmetrical Y-connected self-impedance circuits. (*a*) Neutral solidly grounded. (*b*) Neutral grounded through impedance. (*c*) Neutral ungrounded.

Figure 15(*a*) shows an unsymmetrical solidly grounded three-phase circuit; $Z_a = 1$, $Z_b = 4$, $Z_c = 4$. There is no mutual impedance between phases. The coefficients of [64] are required.

For this special case the sequence admittances of the three phases are the reciprocals of the corresponding phase impedances: $Y_{a1} = Y_{a2} = Y_{a0} = 1$; $Y_{b1} = Y_{b2} = Y_{c2} = 0.25$; $Y_{c1} = Y_{c2} = Y_{c0} = 0.25$. Substituting these values in [72], the sequence self- and mutual admittances are

$$Y_{11} = Y_{22} = Y_{00} = \tfrac{1}{3}(1 + \tfrac{1}{4} + \tfrac{1}{4}) = 0.5$$

$$Y_{12} = Y_{10} = Y_{21} = Y_{20} = Y_{01} = Y_{10} = \tfrac{1}{3}[Y_{a1} + (a + a^2)Y_{b1}]$$
$$= \tfrac{1}{3}(1 - 0.25) = 0.25$$

The sequence self- and mutual admittances can also be obtained from the sequence self- and mutual impedance, although more calculations are required for this special case. The sequence impedances of the phases are

$$Z_{a1} = Z_{a2} = Z_{a0} = 1$$
$$Z_{b1} = Z_{b2} = Z_{b0} = 4$$
$$Z_{c1} = Z_{c2} = Z_{c0} = 4$$

Substituting the sequence impedances of the phases in [7],

$$Z_{11} = Z_{22} = Z_{00} = \tfrac{1}{3}(1 + 4 + 4) = 3$$
$$Z_{12} = Z_{10} = Z_{21} = Z_{20} = Z_{01} = Z_{02} = \tfrac{1}{3}[1 + (a^2 + a)4]$$
$$= -1$$

Substituting the sequence self- and mutual impedances in [65],

$$\Delta = 3[(3)^2 - 1] + 1(-3 - 1) - 1(1 + 3) = 16$$
$$Y_{11} = Y_{22} = Y_{00} = \frac{9 - 1}{\Delta} = \frac{8}{16} = 0.5$$
$$Y_{12} = Y_{10} = Y_{21} = Y_{20} = Y_{01} = Y_{10} = \frac{3 + 1}{\Delta} = 0.25$$

The sequence self- and mutual admittances obtained by the two methods are the same. It will be noted however that Y_{11} is not the reciprocal of Z_{11} and the sequence mutual admittances are not the reciprocals of the corresponding sequence mutual impedances.

Now assume the neutral of the circuit in Fig. 15(a) to be grounded through an impedance of 2, as in Fig. 15(b); or ungrounded, as in Fig. 15(c). In either case, the only sequence impedance which is affected by the change in the connection of the neutral impedance is Z_{00}, the zero-sequence self-impedance, as shown below. From Fig. 15(b),

$$Z_{a0} = Z_a + 3Z_n = 1 + 6 = 7, \qquad Z_{b0} = Z_b + 3Z_n = 10,$$
$$Z_{c0} = Z_c + 3Z_n = 10$$

From [7],

$$Z_{00} = \tfrac{1}{3}(Z_{a0} + Z_{b0} + Z_{c0}) = \tfrac{1}{3}(7 + 10 + 10) = 9$$
$$Z_{10} = Z_{20} = \tfrac{1}{3}[(1 + 6) + (a^2 + a)(10)] = -1$$

From Fig. 15(c), $Z_n = \infty$.

$$Z_{a0} = Z_a + 3Z_n; \quad Z_{b0} = Z_{c0} = Z_b + 3Z_n$$

From [7], with $Z_n = \infty$

$$Z_{00} = \tfrac{1}{3}(Z_{a0} + Z_{b0} + Z_{c0}) = \infty$$

$$\begin{aligned} Z_{10} = Z_{20} &= \tfrac{1}{3}[Z_a + 3Z_n + (a^2 + a)(Z_b + 3Z_n)] \\ &= \tfrac{1}{3}[(Z_a - Z_b) + (1 + a + a^2)(3Z_n)] \\ &= \tfrac{1}{3}(1 - 4) = -1 \end{aligned}$$

Figure 15 was chosen because of the simplicity of the numerical calculations, but in any unsymmetrical circuit it can be shown that Z_{00} is the only sequence self- or mutual impedance affected by a change in the value of the neutral grounding impedance.

The sequence self- and mutual admittances of the circuits shown in Figs. 15(b) and (c) can be determined from the sequence admittances of the three phases, but the sequence admittances of the phases cannot be determined by inspection, as was the case with the circuit of Fig. 15(a). They can be calculated by applying positive-, negative-, and zero-sequence voltages to the circuit as explained above in defining the sequence admittances of the three phases; however, the simplest procedure is to calculate the sequence self- and mutual impedances and from them the sequence admittances. Sequence admittances of shunt circuits are further discussed in Chapter XII, in connection with the capacitances of overhead transmission lines.

Problem 7. Determine the sequence self-impedances and the mutual impedances between the sequence networks for a three-phase static circuit with a neutral conductor directly from the conductor self- and mutual impedances, neglecting the presence of the earth. Compare with equations [13].

Problem 8. Construct an equivalent circuit with 1 : 1 mutual coupling transformers for use on the a-c network analyzer to replace Fig. 10, when Z_{12} is larger in magnitude than Z_{10} and (a) Z_{12} is a reactive impedance, (b) Z_{12} is a capacitive reactance and Z_{10} has no resistance component.

Problem 9. Tabulate, with their symbols and definitions, the various impedances and admittances used in this chapter.

CHAPTER IX

POLYPHASE SYSTEMS OF MORE THAN THREE PHASES, SINGLE-PHASE AND TWO-PHASE SYSTEMS

The method of symmetrical components provides a general method for the solution of unbalanced polyphase systems of any number of phases. The method, and its application to three-phase systems, has been discussed in the preceding chapters. In this chapter, the method is applied to unbalanced systems of any number of phases, including single-phase systems. For two-phase systems, two sets of components and phase quantities are considered.

POLYPHASE SYSTEMS

Referring to Chapter I, sinusoidal currents or voltages of the same frequency can be represented by vectors revolving at the same angular velocity. As the angular displacements between vectors revolving at the same angular velocity are fixed, sinusoidal currents and voltages of the same frequency can be represented in the same vector diagram, with any voltage or current vector of the same angular velocity as reference vector. If a symmetrical polyphase system of n phases is operating under balanced conditions, the voltages to ground or to neutral of the n phases at any system point form a set of n symmetrical voltage vectors of the phase order and frequency of the generated phase voltages. Likewise, the n line currents at any system point form a set of n symmetrical current vectors of the same phase order and frequency.

When a disturbance occurs, the phase currents and voltages may become unbalanced. By the method of symmetrical components, a set of n unbalanced phase currents or voltages of the same frequency, represented by a group of n unbalanced vectors, can be replaced by n symmetrical systems of vectors consisting of n vectors each. A system of n symmetrical vectors is one in which the n vectors are equal in magnitude and displaced from each other by equal angles. The equal angles between the vectors of the symmetrical systems may be determined from the n independent n^{th} roots of unity. Quoting from Dr. Fortescue's paper:*

* See Bibliography of Chapter II, reference 4.

" The complex roots of unity will be referred to from time to time in the paper. Thus the complete solution of the equation $x^n - 1 = 0$ requires n different values of x, only one of which is real when n is an odd integer. To obtain the other roots we have the relation

$$\begin{aligned} 1 &= \cos 2\pi r + j \sin 2\pi r \\ &= \varepsilon^{j2\pi r} \end{aligned}$$

Where r is any integer. We have therefore

$$1^{\frac{1}{n}} = \varepsilon^{j\frac{2\pi r}{n}}$$

and by giving successive integral values to r from 1 to n, all the n roots of $x^n - 1 = 0$ are obtained namely,

$$\begin{aligned} a_1 &= \varepsilon^{j\frac{2\pi}{n}} = \cos\frac{2\pi}{n} + j\sin\frac{2\pi}{n} \\ a_2 &= \varepsilon^{j\frac{4\pi}{n}} = \cos\frac{4\pi}{n} + j\sin\frac{4\pi}{n} \\ a_3 &= \varepsilon^{j\frac{6\pi}{n}} = \cos\frac{6\pi}{n} + j\sin\frac{6\pi}{n} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ a_n &= \varepsilon^{j2\pi} = 1 \end{aligned} \qquad [1]$$

. . . ."

The n roots of the equation $x^n - 1 = 0$ given in [1] are the operators which determine the angles between the vectors in each of the symmetrical systems of vectors. It will be noted that $a_1 = e^{j\frac{2\pi}{n}}$ is the characteristic operator from which the other operators can be obtained. Thus, $a_2 = a_1^2$, $a_3 = a_1^3$, $\cdots$ $a_{n-1} = a_1^{n-1}$, $a_n = a_1^n$. The operator a_1^n ($= 1\underline{/0^\circ}$) has an angle of zero, indicating that the vectors of one group are equal in phase as well as in magnitude.

In a **three-phase system,** the three operators determined from [1] with $n = 3$ are

$$\begin{aligned} a_1 &= e^{j\frac{2\pi}{3}} = 1\underline{/120^\circ} = a \\ a_{n-1} = a_2 &= e^{j\frac{4\pi}{3}} = 1\underline{/240^\circ} = a^2 \\ a_n = a_3 &= e^{j\frac{6\pi}{3}} = 1\underline{/360^\circ \text{ or } 0^\circ} = a^3 = 1 \end{aligned}$$

It will be noted that the operator for the positive-sequence system of vectors is $a_{n-1} = (a_1)^{n-1} = a^2$. The characteristic operator is the operator of the negative-sequence system.

In an n-phase system let the phases be $a, b, c, \cdots z$, where z is the last or n^{th} phase and the phase order of the generated voltages is $a, b, c, \cdots z$. From [1], operator $a_n = 1$ produces a system of vectors equal in magnitude and in phase. The operator $a_{n-1} = (a_1)^{n-1}$ gives a symmetrical system of vectors of the angular displacement and phase order of balanced generated voltages. The characteristic operator $a_1 = e^{j\frac{2\pi}{n}}$ gives a symmetrical system of vectors of the angular displacement of balanced generated voltages but of phase order the reverse of that of balanced generated voltages. These three systems of vectors, by analogy from the three-phase system, will be called zero-, positive-, and negative-sequence systems, respectively. In an n-phase system of more than three phases, there are $n - 3$ additional systems of symmetrical vectors which have angular displacements determined by the operators $a_1^2, a_1^3, \cdots a_1^{n-2}$. This will be illustrated for the five-phase system.

Five-Phase System. The characteristic operator a_1 of a five-phase system is

$$a_1 = e^{j\frac{2\pi}{5}} = \cos 72° + j \sin 72° = 1\underline{/72°}$$

If the phase order of the generated voltages is *abcde* and phase a is reference phase, the five sets of symmetrical systems of vectors applying to phases a, b, c, d, e, in the order given, are

$$\begin{array}{lccccc}
1^{\text{st}} \text{ set:} & V_{a1}, & a_1^4 V_{a1}, & a_1^3 V_{a1}, & a_1^2 V_{a1}, & a_1 V_{a1} \\
2^{\text{nd}} \text{ set:} & V_{a2}, & a_1 V_{a2}, & a_1^2 V_{a2}, & a_1^3 V_{a2}, & a_1^4 V_{a2} \\
3^{\text{rd}} \text{ set:} & V_{a0}, & V_{a0}, & V_{a0}, & V_{a0}, & V_{a0} \\
4^{\text{th}} \text{ set:} & V_{a3}, & a_1^2 V_{a3}, & a_1^4 V_{a3}, & a_1^6 V_{a3}, & a_1^8 V_{a3} \\
5^{\text{th}} \text{ set:} & V_{a4}, & a_1^3 V_{a4}, & a_1^6 V_{a4}, & a_1^9 V_{a4}, & a_1^{12} V_{a4}
\end{array}$$

In the first three sets of vectors the notation is similar to that used in three-phase systems, and the designations positive-, negative-, and zero-sequence have been retained. Since $a_1^5 = 1$, $a_1^6 = a_1$, $a_1^7 = a_1^2$, $a_1^8 = a_1^3, \cdots a_1^{12} = a_1^2$, the last two systems of vectors can be rewritten

$$\begin{array}{lccccc}
4^{\text{th}} \text{ set:} & V_{a3}, & a_1^2 V_{a3}, & a_1^4 V_{a3}, & a_1 V_{a3}, & a_1^3 V_{a3} \\
5^{\text{th}} \text{ set:} & V_{a4}, & a_1^3 V_{a4}, & a_1 V_{a4}, & a_1^4 V_{a4}, & a_1^2 V_{a4}
\end{array}$$

The five systems of symmetrical vectors are shown in Fig. 1 with the vector of phase a as reference vector for each set. In Fig. 1, the phase relations between the vectors V_{a1}, V_{a2}, V_{a0}, V_{a3}, and V_{a4} are not indicated. In general, the symmetrical component of phase a will be of unequal magnitude and not in phase.

The phase vectors of a five-phase system are expressed in terms of the symmetrical components of phase a by the following equations:

$$
\begin{aligned}
V_a &= V_{a1} + V_{a2} + V_{a0} + V_{a3} + V_{a4} \\
V_b &= a_1^4 V_{a1} + a_1 V_{a2} + V_{a0} + a_1^2 V_{a3} + a_1^3 V_{a4} \\
V_c &= a_1^3 V_{a1} + a_1^2 V_{a2} + V_{a0} + a_1^4 V_{a3} + a_1 V_{a4} \\
V_d &= a_1^2 V_{a1} + a_1^3 V_{a2} + V_{a0} + a_1 V_{a3} + a_1^4 V_{a4} \\
V_e &= a_1 V_{a1} + a_1^4 V_{a2} + V_{a0} + a_1^3 V_{a3} + a_1^2 V_{a4}
\end{aligned}
\qquad [2]
$$

where the operator $a_1 = e^{j\frac{2\pi}{5}} = \cos 72° + j \sin 72° = 1\underline{/72°}$.

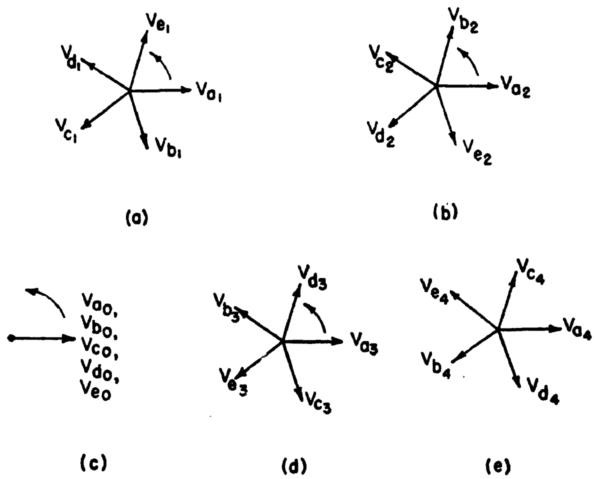


FIG. 1. Symmetrical component systems for a five-phase system. Characteristic operator $= 1\underline{/72°}$. (*a*) Positive-sequence system. (*b*) Negative-sequence system. (*c*) Zero-sequence system. (*d*) and (*e*) Additional systems of symmetrical vectors.

It will be noted from Fig. 1 that the sum of the five vectors of each set, except those of zero sequence, is zero:

$$1 + a_1 + a_1^2 + a_1^3 + a_1^4 = 0 \qquad [3]$$

Making use of [3], the symmetrical components of phase a can be expressed in terms of the phase vectors just as the sequence components of phase a of a three-phase system are expressed in terms of the vectors V_a, V_b, and V_c. To determine V_{a1}, multiply the equations of [2] by 1, a_1, a_1^2, a_1^3, a_1^4, respectively, add, and divide by 5. Since the sums of the coefficients of V_{a2}, V_{a0}, V_{a3}, and V_{a4} in the resultant equation are all zero and the sum of the coefficients of V_{a1} is 5, V_{a1} will be expressed in terms of the phase vectors. Proceeding in a similar manner by multiplying the equations of [2] by operators which make the coefficients of V_{a2}, V_{a0}, V_{a3}, and V_{a4} each in turn unity, the following equations are obtained:

$$\begin{aligned}
V_{a1} &= \tfrac{1}{5}(V_a + a_1 V_b + a_1^2 V_c + a_1^3 V_d + a_1^4 V_e)\\
V_{a2} &= \tfrac{1}{5}(V_a + a_1^4 V_b + a_1^3 V_c + a_1^2 V_d + a_1 V_e)\\
V_{a0} &= \tfrac{1}{5}(V_a + V_b + V_c + V_d + V_e) \qquad [4]\\
V_{a3} &= \tfrac{1}{5}(V_a + a_1^3 V_b + a_1 V_c + a_1^4 V_d + a_1^2 V_e)\\
V_{a4} &= \tfrac{1}{5}(V_a + a_1^2 V_b + a_1^4 V_c + a_1 V_d + a_1^3 V_e)
\end{aligned}$$

The development of the symmetrical component equations of [2] and [4] for the five-phase system illustrates the procedure for any n-phase system where n is prime. When n is not prime, some of the symmetrical systems degenerate into repetitions of systems having numbers of phases corresponding to the factors of n. This is illustrated in Figs. 2 and 3 for four-phase and six-phase systems, respectively.

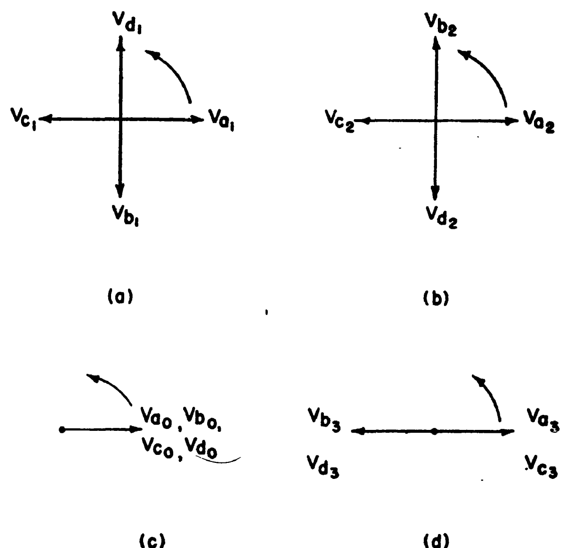


FIG. 2. Symmetrical component system for a four-phase system. Characteristic operator = $1\underline{/90°}$. (*a*) Positive-sequence system. (*b*) Negative-sequence system. (*c*) Zero-sequence system. (*d*) Repeating positive-sequence two-vector system.

The **four-phase system** has the characteristic operator $a_1 = e^{j\frac{2\pi}{4}} = 1\underline{/90°}$. The four sets of symmetrical vectors to replace the phase vectors V_a, V_b, V_c, and V_d of a four-phase system are shown in Fig. 2. In addition to the positive-, negative-, and zero-sequence systems of vectors shown in parts (*a*), (*b*), and (*c*), respectively, of Fig. 2, there is a set of vectors given by part (*d*) which is a repeating positive-sequence two-vector system.

The **six-phase system** has the characteristic operator $a_1 = \epsilon^{j\frac{2\pi}{6}} = 1\underline{/60^\circ}$. Figure 3 shows the six symmetrical systems of vectors. In

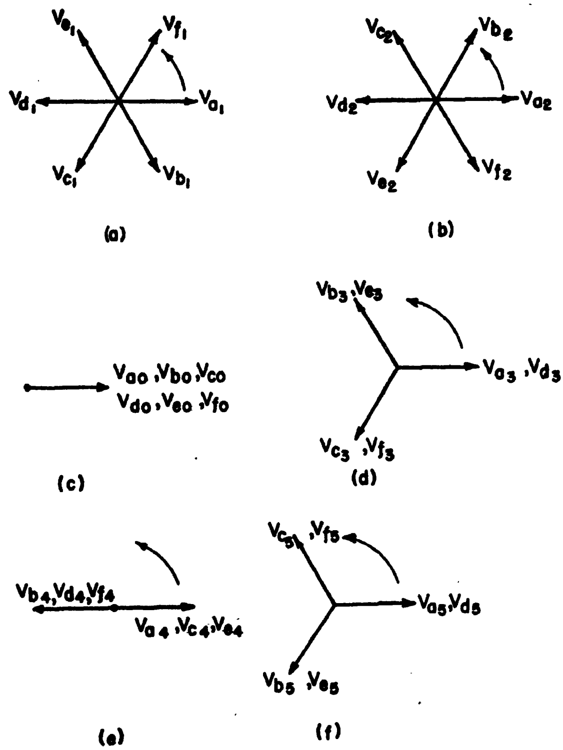


FIG. 3. Symmetrical component systems for a six-phase system. Characteristic operator $= 1\underline{/60^\circ}$. (*a*) Positive-sequence system. (*b*) Negative-sequence system. (*c*) Zero-sequence system. (*d*) and (*f*) Repeating negative- and positive-sequence three-vector systems, respectively. (*e*) Repeating positive-sequence two-vector system.

addition to the positive-, negative-, and zero-sequence systems of vectors, there are two sets of vectors which are repeating three-phase systems and one set which is a repeating two-vector system.

SINGLE-PHASE SYSTEMS

Symmetrical components can be used to determine voltages and currents in a single-phase system during unbalanced conditions if the system is considered a two-vector system. During balanced conditions, V_a and V_b, the voltages to ground or to neutral of conductors *a* and b at any system point, are equal in magnitude and opposite in sign; likewise, the currents I_a and I_b flowing in the same direction in conductors a and b are equal in magnitude and opposite in sign.

Symmetrical Components of a Two-Vector System. Following the method developed for the n-vector system, the characteristic operator of a two-vector system is

$$a_1 = e^{j\frac{2\pi}{2}} = 1\underline{/180^\circ} = -1$$

The only other operator is

$$a_1^n = a_1^2 = (-1)^2 = 1\underline{/0^\circ}$$

For the two-vector system there are two sets of components — the positive-sequence and the zero-sequence.

The phase voltages V_a and V_b in terms of the symmetrical components of voltage of phase a are

$$\begin{aligned} V_a &= V_{a1} + V_{a0} \\ V_b &= -V_{a1} + V_{a0} \end{aligned} \qquad [5]$$

The positive- and zero-sequence components of voltage of phase a in terms of the phase voltages V_a and V_b, obtained by solving [5], are

$$\begin{aligned} V_{a1} &= \tfrac{1}{2}(V_a - V_b) \\ V_{a0} &= \tfrac{1}{2}(V_a + V_b) \end{aligned} \qquad [6]$$

The two symmetrical systems of vectors for a two-vector system are shown in Fig. 4.

The phase currents I_a and I_b, flowing in the same direction, in terms of the symmetrical components of current of phase a are

$$\begin{aligned} I_a &= I_{a1} + I_{a0} \\ I_b &= -I_{a1} + I_{a0} \end{aligned} \qquad [7]$$

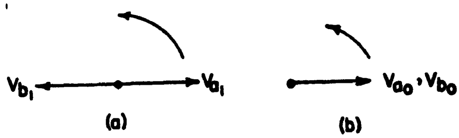


FIG. 4. Symmetrical component systems for a two-vector system. Characteristic operator $= 1\underline{/180^\circ}$. (*a*) Positive-sequence system. (*b*) Zero-sequence system.

The symmetrical components of current of phase a in terms of the phase currents are

$$\begin{aligned} I_{a1} &= \tfrac{1}{2}(I_a - I_b) \\ I_{a0} &= \tfrac{1}{2}(I_a + I_b) \end{aligned} \qquad [8]$$

For the two-wire ungrounded single-phase system in which capacitance to ground is negligible, there will be no zero-sequence currents during faults between conductors or to ground. There is no advantage, therefore, in using positive- and zero-sequence components instead of the single-phase quantities. (See Chapter I for a discussion of the equivalent circuit for a single-phase system.) For the three-wire single-phase system with the third wire grounded or ungrounded and the two-wire single-phase system with neutral grounded or with appreciable capacitance to ground, symmetrical components can be used to advantage in determining voltages and currents during unbalanced conditions.

Three-Wire Single-Phase System Supplied from Three-Phase System through Transformer with Midpoint of Secondary Grounded. Figure 5(*a*) gives a three-line diagram of a three-wire single-phase

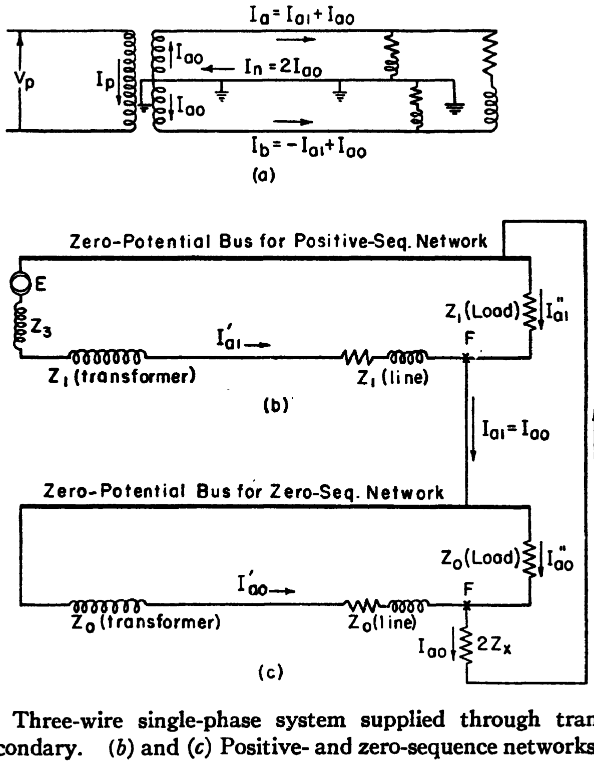


FIG. 5. (*a*) Three-wire single-phase system supplied through transformer with grounded secondary. (*b*) and (*c*) Positive- and zero-sequence networks, respectively, of (*a*), assumed symmetrical, and connected to represent line-to-ground fault on phase *a* through impedance Z_x.

system supplied from a three-phase system through a transformer with the midpoint of the secondary winding grounded and connected to a neutral conductor *n* which may be grounded at various points. If the system is symmetrical and the loads between *a* and *n* and *b* and *n* are equal, no current will flow in the neutral conductor *n* during normal operation. In this case the current in phase *a* is equal and opposite to that in phase *b*, and the system is similar to a two-conductor single-phase system. If the loads are unbalanced, or if there is a fault to ground or to the neutral conductor, zero-sequence currents will flow. Zero-sequence currents are equal in phases *b* and *c* and return in the neutral conductor if the system is ungrounded; if it is multigrounded, zero-sequence currents return in the neutral conductor in parallel with the ground. For determining voltages and currents during unbalanced conditions, the procedure is analogous to that used for three-phase systems. One-line diagrams are drawn for the positive- and zero-sequence systems in which currents and voltages are those of the reference phase *a*. Positive-sequence voltages are referred to neutral. Zero-sequence voltages at any point are referred to ground or to the neutral conductor at that point, depending upon whether the system is grounded or ungrounded. In Fig. 5, parts (*b*) and (*c*), the positive- and zero-sequence networks are drawn for the system shown in part (*a*), assumed symmetrical. These networks are connected to represent a phase-to-ground fault through impedance Z_x, as explained later.

With transformer rated kva and voltage as base quantities in the primary and secondary windings, base kva and base voltage in the positive- and zero-sequence networks of Figs. 5(*b*) and (*c*) are one-half those in the secondary winding; base current is rated secondary current. In per unit, the positive-sequence transformer current I'_{a1} equals I_p, the current in the primary winding. The positive-sequence leakage impedance of the transformer in ohms is one-half the leakage impedance between primary and secondary windings. Expressed in per unit, it is the same as the per unit impedance between the primary and secondary windings based on the transformer rating. The per phase zero-sequence impedance of the transformer is twice the leakage impedance between the two halves of the secondary winding in parallel based on one-half the rated kva and voltage of the secondary winding. Zero-sequence components of current in the single-phase system do not flow in the primary winding. The positive-sequence impedance of the transmission circuit is one-half the impedance of the loop consisting of conductors *a* and *b* in series. In a symmetrical circuit, the neutral conductor *n* is equidistant from conductors *a* and *b*; equal and opposite currents flowing in phases *a* and *b* therefore induce no voltage in *n* and there is no mutual impedance between the positive- and zero-sequence networks. The zero-sequence impedance of the three-wire single-phase, transmission circuit is the impedance per phase offered to equal currents in phases *a* and *b* which return in the neutral conductor *n* if *n* is ungrounded, and in the neutral conductor and ground in parallel if n is multigrounded. The positive- and zero-sequence impedances of three-wire single-phase transmission circuits are determined in Chapter XI. For the present, let it be assumed that the impedance in the sequence networks of Figs. 5(b) and (c) are known, or can be determined.

The equivalent excitation voltage E in Fig. 5(b) is the per unit voltage which would exist across the primary winding and also in phase a of the single-phase circuit with the secondary winding open, but the rotors of all machines in the three-phase system retain the relative angular positions which correspond to the given single-phase load. (See Chapter VIII for a discussion of equivalent excitation.)

Impedance of Three-Phase System Referred to Single-Phase System. Let Z_3 be the per unit impedance met by $I_p (= I'_{a1})$ in the three-phase system, based on rated primary winding kva and voltage, determined as follows: In the three-phase part of the system, the current I_p in the primary winding and the voltage V_p across the primary winding may be treated as the current flowing into an impedance fault and the voltage across that impedance, the fault impedance being the impedance of the single-phase system viewed through the primary winding. Let Z_1, Z_2, and Z_0 be the per unit positive-, negative-, and zero-sequence impedances in the three-phase system viewed from the point where the transformer is connected, based on rated transformer kva per phase and a base line-to-neutral voltage which is rated primary winding voltage if the primary winding is connected between a phase conductor and the neutral conductor, and $1/\sqrt{3}$ times rated primary winding voltage if the primary winding is connected between phases.

With a line-to-neutral fault in the three-phase side on phase a through fault impedance Z_f (see Table I, Chapter IV),

$$I_p = 3I_{a0} = \frac{3E_a}{Z_1 + Z_2 + Z_0 + 3Z_f} = \frac{E_a}{\frac{1}{3}(Z_1 + Z_2 + Z_0) + Z_f} \qquad [9]$$

$$V_p = E_a - I_p \frac{Z_1 + Z_2 + Z_0}{3} \qquad [10]$$

The impedance $\frac{1}{3}(Z_1 + Z_2 + Z_0)$ in [10] is based on rated primary voltage. The equivalent impedance met by $I'_{a1} = I_p$ in Fig. 5(b) is therefore

$$Z_3 = \tfrac{1}{3}(Z_1 + Z_2 + Z_0) \qquad [11]$$

when the primary winding is connected between a phase and a neutral conductor.

For a line-to-line fault on the three-phase side between phases b and c through fault impedance Z_f, where Z_f is based on line-to-line voltage, the current I_p and the voltage V_p of the primary winding in per unit of base line current and line-to-neutral voltage are

$$I_p = I_b = -j\sqrt{3}I_{a1} = \frac{-j\sqrt{3}E_a}{Z_1 + Z_2 + 3Z_f} \quad [12]$$

$$V_p = V_{cb} = -j\sqrt{3}E_a - I_p(Z_1 + Z_2) \quad [13]$$

The impedance $(Z_1 + Z_2)$ in [13] is based on line-to-neutral voltage. Multiplying it by $\frac{1}{3}$ to refer it to rated primary voltage, the equivalent impedance met by $I'_{a1} = I_p$ in Fig. 5(*b*) is

$$Z_3 = \tfrac{1}{3}(Z_1 + Z_2) \quad [14]$$

when the primary winding is connected between phases.

If the kva rating of the three-phase system is large relative to the transformer rated kva, so that voltages on the three-phase side of the transformer are substantially balanced, Z_3 may be neglected and E equated to the per unit normal operating voltage of the primary winding.

Phase-to-Ground Fault in a Three-Wire Single-Phase System. Assume the fault to ground on phase a through impedance Z_x. Let V_a and V_b be voltages to ground of phases a and b, respectively, at the fault and I_a and I_b the line currents flowing from phases a and b into the fault, with positive direction of current flow toward the fault. The conditions at the fault are $I_b = 0$ and $V_a = I_aZ_x$. Substituting $I_b = 0$ in [7],

$$I_b = -I_{a1} + I_{a0} = 0$$

Therefore

$$I_{a1} = I_{a0} \quad [15]$$

Substituting $V_a = I_aZ_x$ in [5],

$$V_a = V_{a1} + V_{a0} = I_aZ_x$$

Therefore

$$V_{a1} = -V_{a0} + (I_{a1} + I_{a0})Z_x \quad [16]$$

To obtain the symmetrical components V_{a1}, V_{a0}, I_{a1}, and I_{a0}, four equations are needed. Two of these equations are given by [15] and [16]. The other two equations which give the positive- and zero-sequence voltages at the fault in terms of the corresponding symmetrical components of current flowing into the fault and impedances viewed from the fault are

$$V_{a1} = V_f - I_{a1}Z_1 \quad [17]$$

$$V_{a0} = -I_{a0}Z_0 \quad [18]$$

where V_f is the voltage to neutral of phase a at the fault point F before the fault occurred and Z_1 and Z_0 are the positive- and zero-sequence impedances, respectively, viewed from the fault. Substituting [18] in [16] and replacing I_{a0} by I_{a1},

$$V_{a1} = I_{a1}(Z_0 + 2Z_x) \tag{19}$$

Solving [17] and [19] for I_{a1},

$$I_{a1} = \frac{V_f}{Z_1 + Z_0 + 2Z_x} \tag{20}$$

When I_{a1} is known, the other symmetrical components and the phase currents and voltages at the fault can be calculated from above equations. From the symmetrical components of current and voltage at the fault and the component networks, the currents and voltages at any system point can be obtained.

From [19], the equivalent circuit to replace the fault in the positive-sequence network is the impedance $(Z_0 + 2Z_x)$.

Connection of Sequence Networks for Phase-to-Ground Fault. If the positive- and zero-sequence networks are connected as in Fig. 5, equations [15]–[20] are satisfied. The impedance Z_x can be used to represent the equivalent impedance of a load taken off between conductor a and neutral conductor n, not balanced by an equal load between b and n. With no impedance in the fault, or with symmetrical loads, $Z_x = 0$.

Equivalent Impedance of Single-Phase System with Fault, Viewed from Three-Phase Side of Transformer. In [9] and [12], Z_f is the per unit impedance of the single-phase system referred to the primary winding. Z_f is therefore the transformer leakage impedance plus the impedance met by the transformer current I'_{a1} in the single-phase system. Neglecting load currents in Fig. 5, Z_f is the sum of the positive- and zero-sequence impedances of the transformer and transmission circuit plus twice the fault impedance. For example, if the fault reactance is 2%, the positive- and zero-sequence leakage reactances of the transformer 6% and 0.3%, and the positive- and zero-sequence reactances of the transmission circuit 10% and 30%, respectively, neglecting resistance,

$$Z_f = j(16 + 30.3 + 4)\% = j50.3\%$$

With a solid fault to ground at the transformer terminals, $Z_f = j6.3\%$.

With a *solid fault between phases* at the transformer terminals, Z_f is the transformer leakage impedance. For the system assumed above, $Z_f = j6\%$.

Positive- and Zero-Sequence Self- and Mutual Impedances. If I_a and I_b are the line currents in the circuit, and V_a and V_b the voltage drops in the circuit between its terminals in the direction of current flow, the positive- and zero-sequence voltage drops V_{a1} and V_{a0} in terms of the positive- and zero-sequence currents, and the self- and mutual sequence impedances will be written

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a0}Z_{10} \\ V_{a0} &= I_{a1}Z_{01} + I_{a0}Z_{00} \end{aligned} \qquad [21]$$

where Z_{11} and Z_{00} are the positive- and zero-sequence self-impedances, and Z_{10} and Z_{01} the mutual impedances between the positive- and zero-sequence networks of the two-vector system.

Three-Wire Circuit with Ungrounded Neutral Conductor. Let the phase conductors be a and b and the neutral conductor n. Neglecting the presence of the earth, the voltage drops V_a and V_b in phases a and b can be expressed in terms of the conductor self- and mutual impedances. With I_a and I_b flowing in conductors a and b and $(I_a + I_b)$ returning in the neutral conductor n, the currents I_a and I_b flow in loops consisting of a phase conductor and the neutral conductor in series.

$$\begin{aligned} V_a &= I_a(Z_{aa} + Z_{nn} - 2Z_{an}) + I_b(Z_{nn} + Z_{ab} - Z_{an} - Z_{bn}) \\ V_b &= I_a(Z_{nn} + Z_{ab} - Z_{an} - Z_{bn}) + I_b(Z_{bb} + Z_{nn} - 2Z_{bn}) \end{aligned} \qquad [22]$$

Replacing I_a and I_b in [22] by their symmetrical component, then substituting V_a and V_b in [6], V_{a1} and V_{a0} will be expressed in terms of I_{a1}, I_{a0}, and the conductor self- and mutual impedances. Equating the coefficients of I_{a1} and I_{a0} in these resulting equations to the corresponding coefficients in [21],

$$\begin{aligned} Z_{11} &= \tfrac{1}{2}(Z_{aa} + Z_{bb} - 2Z_{ab}) \\ Z_{00} &= 2Z_{nn} + \tfrac{1}{2}(Z_{aa} + Z_{bb}) + Z_{ab} - 2Z_{an} - 2Z_{bn} \\ &= Z_{11} + 2(Z_{nn} + Z_{ab} - Z_{an} - Z_{bn}) \\ Z_{10} &= Z_{01} = \tfrac{1}{2}(Z_{aa} - Z_{bb} - 2Z_{an} + 2Z_{bn}) \end{aligned} \qquad [23]$$

If the conductors a and b are identical and equidistant from the neutral conductor, $Z_{aa} = Z_{bb}$ and $Z_{an} = Z_{bn}$. In this case, $Z_{10} = Z_{01} = 0$, and there is no mutual impedance between the component networks; then

$$\begin{aligned} Z_{11} &= Z_{aa} - Z_{ab} \\ Z_{00} &= (Z_{aa} - Z_{ab}) + 2(Z_{nn} + Z_{ab} - 2Z_{an}) \end{aligned} \qquad [24]$$

If the conductors a, b, and n are identical and equidistant from each other, $Z_{nn} = Z_{aa} = Z_{bb}$, and $Z_{an} = Z_{bn} = Z_{ab}$.

$$\begin{aligned} Z_{11} &= Z_{aa} - Z_{ab} \\ Z_{00} &= 3(Z_{aa} - Z_{ab}) = 3Z_{11} \end{aligned} \qquad [25]$$

Equations [23]–[25] are evaluated in Chapter XI in terms of circuit dimensions.

Phase Voltage Drops in Terms of Phase Currents and Phase Self- and Mutual Impedances. In [22], the phase voltage drops V_a and V_b are expressed in terms of the phase current I_a and I_b and the *conductor* self- and mutual impedances. If Z_{AA} and Z_{BB} represent the self-impedances of phases a and b and Z_{AB} and Z_{BA} the mutual impedances between phases, [22] can be written

$$\begin{aligned} V_a &= I_a Z_{AA} + I_b Z_{AB} \\ V_b &= I_a Z_{BA} + I_b Z_{BB} \end{aligned} \qquad [26]$$

where Z_{AA}, Z_{BB}, Z_{AB}, and Z_{BA} in [26] are the coefficients of I_a and I_b in the corresponding equations of [22]. Equations [26] are general equations for phase voltage drops in terms of phase currents, and the *phase* self- and mutual impedances. The subscripts A and B are used here only to distinguish *phase* impedances from *conductor* impedances; lower case letters can be used when there is no danger of confusion. In a reciprocal static circuit, $Z_{BA} = Z_{AB}$.

As $(I_a + I_b)$ returns in the neutral conductor, [26] is conveniently written in terms of I_a, I_b, and $(I_a + I_b)$. For the case of $Z_{BA} = Z_{AB}$,

$$\begin{aligned} V_a &= I_a(Z_{AA} - Z_{AB}) + (I_a + I_b)Z_{AB} \\ V_b &= (I_a + I_b)Z_{AB} + I_b(Z_{BB} - Z_{AB}) \end{aligned} \qquad [27]$$

The equivalent self-impedances in phases a and b and the neutral conductor n from [27] are $(Z_{AA} - Z_{AB})$, $(Z_{BB} - Z_{AB})$, and Z_{AB}, respectively. For the case of identical phase conductors, equidistant from n,

$$\begin{aligned} (Z_{AA} - Z_{AB}) &= (Z_{BB} - Z_{AB}) = Z_{11} = \text{equivalent impedance of phases } a \text{ and } b \\ Z_{AB} &= \tfrac{1}{2}(Z_{00} - Z_{11}) = \text{equivalent impedance of neutral conductor } n \end{aligned} \qquad [28]$$

where Z_{11} and Z_{00} are defined in [24] in terms of conductor self- and mutual impedances and evaluated in Chapter XI in terms of circuit dimensions.

For a symmetrical three-wire single-phase transmission circuit there is no mutual coupling between the positive- and zero-sequence networks. If phase quantities are used, there is a reciprocal mutual coupling Z_{AB} between phases. Three-wire transmission circuits in two-phase systems are discussed in the following section.

TWO–PHASE SYSTEMS

Power may be supplied to a two-phase system from two-phase generators, from a three-phase system through a Scott-connected transformer or autotransformer bank, or from both two-phase generators and a three-phase system. In a two-phase synchronous machine, sometimes called a quarter-phase machine, the generated phase voltages are equal in magnitude and 90° apart in phase. If E_A and E_B are the voltages generated in phases A and B, E_B is equal in magnitude to E_A and lags E_A by 90°, as indicated in Fig. 6. A two-phase transmission circuit may be a four-wire or a three-wire circuit, as shown on Figs. 6(*a*) and (*b*), respectively. The phases of the two-phase system are here indicated by the letters A and B instead of the lower-case letters a and b, which are used for the conductors of a single-phase system, and for the positive- and zero-sequence symmetrical components of a two-vector system.

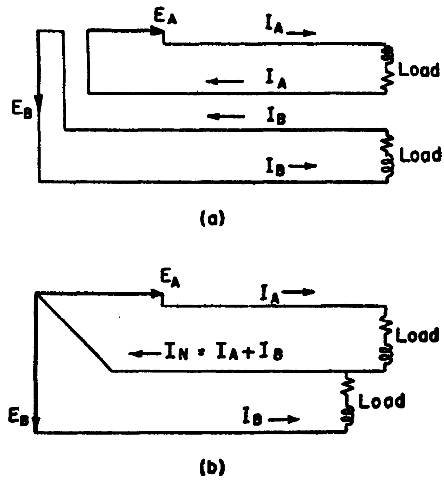


FIG. 6. (*a*) Two-phase four-wire system. (*b*) Two-phase three-wire system.

Two vectors which are equal in magnitude and 90° apart in phase do not form a symmetrical set of vectors because they are not displaced from each other by equal angles. In a symmetrical set of two vectors, the vectors are 180° or 0° apart. A two-phase system consisting of two-phase generators, three-wire transmission circuits, and loads (with or without connection to a three-phase system) is an unsymmetrical system. It can not, therefore, be represented by a one-line diagram, even under balanced operating conditions.

There are several ways in which a two-phase system of vectors can be replaced by two sets of vectors of two vectors each. The most convenient system of vectors to use in calculations will depend on the nature of the problem and the characteristics of the machines, transmission circuits, transformers, loads, etc., of the system.

Two Different Systems of Components and the Phase Quantities

These will be considered under the following headings:

1. Positive- and negative-sequence right-angle components.
2. Positive- and zero-sequence symmetrical components (already described).
3. Phase currents and voltages not replaced by components.

(1) Positive- and Negative-Sequence Right-Angle Components. The two-phase system may be considered a special case of the four-phase system in which the two vectors V_A and V_B are $V_A = (V_a - V_c)$ and $V_B = (V_b - V_d)$. A generator with windings connected for four phases and generated voltages E_a, E_b, E_c, and E_d becomes a two-phase generator if alternate phases are connected to give generated voltages $E_A = E_a - E_c$ and $E_B = E_b - E_d$. Subtracting alternate vectors in the symmetrical four-phase systems of Fig. 2, the sets of vectors in parts (c) and (d) disappear; the positive- and negative-sequence sets of symmetrical vectors in parts (a) and (b) reduce to the two sets of two-phase vectors shown in Fig. 7, parts (a) and (b), respectively. The phase order of the vectors of Fig. 7(a) are of positive sequence; those of Fig. 7(b) are of negative sequence. The vectors of each set are equal in magnitude and displaced from each other by 90°, V_{A1} leading V_{B1} by 90° and V_{A2} lagging V_{B2} by 90°. The components of this system will be called positive- and negative-sequence right-angle components because of the 90° phase displacement between the components of each sequence, and also to distinguish the positive-sequence components of this system from the symmetrical positive-sequence components, discussed under the single-phase two-vector system, which are displaced from each other by 180°, as shown in Fig. 4(a). Upper-case subscripts will be used with V and I to indicate positive- and negative-sequence right-angle components.

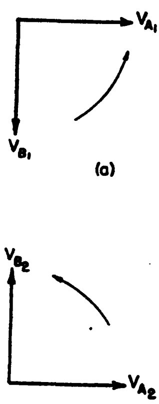


FIG. 7. (a) Positive-sequence right-angle components. (b) Negative-sequence right-angle components.

The phase voltages V_A and V_B are expressed in terms of the positive- and negative-sequence right-angle components of phase A by the following equations:

$$\begin{aligned} V_A &= V_{A1} + V_{A2} \\ V_B &= -jV_{A1} + jV_{A2} \end{aligned} \qquad [29]$$

The corresponding equations for phase currents are

$$\begin{aligned} I_A &= I_{A1} + I_{A2} \\ I_B &= -jI_{A1} + jI_{A2} \end{aligned} \qquad [30]$$

The positive- and negative-sequence right-angle components of voltage of phase A are expressed in terms of the phase voltages V_A and V_B by solving the simultaneous equations of [29]:

$$\begin{aligned} V_{A1} &= \tfrac{1}{2}(V_A + jV_B) \\ V_{A2} &= \tfrac{1}{2}(V_A - jV_B) \end{aligned} \qquad [31]$$

Solving [30] for I_{A1} and I_{A2} in terms of I_A and I_B,

$$\begin{aligned} I_{A1} &= \tfrac{1}{2}(I_A + jI_B) \\ I_{A2} &= \tfrac{1}{2}(I_A - jI_B) \end{aligned} \qquad [32]$$

Right-Angle Self- and Mutual Impedances. The positive- and negative-sequence voltage drops V_{A1} and V_{A2} in a two phase circuit in the terms of the currents I_{A1} and I_{A2} and the right-angle self- and mutual impedances are

$$\begin{aligned} V_{A1} &= I_{A1}Z_{11}(90°) + I_{A2}Z_{12} \\ V_{A2} &= I_{A1}Z_{21} \qquad\quad + I_{A2}Z_{22} \end{aligned} \qquad [33]$$

where $Z_{11}(90°)$ and Z_{22} are the positive- and negative-sequence self-impedances and Z_{12} and Z_{21} are the mutual impedances. The positive-sequence right-angle self-impedance is written $Z_{11}(90°)$ to distinguish it from the self-impedance Z_{11} offered to positive-sequence symmetrical components. The impedances Z_{22}, Z_{12}, and Z_{21} need no additional identification.

(2) Positive- and Zero-Sequence Symmetrical Components. They are shown in Fig. 4 and discussed in connection with the single-phase two-vector system. They can be applied to any two-vector system. Equations [5]–[8] and [21] apply to the two-phase system if V_a, V_b, I_a, and I_b in these equations are replaced by V_A, V_B, I_A, and I_B, respectively.

(3) Phase Currents and Voltages. The voltages in phases A and B are V_A and V_B, and currents are I_A and I_B, respectively. In certain problems, these phase quantities may be more convenient than either of the two sets of components described above. Equations [26] express the phase voltage drops in a two-vector system in terms of the phase self- and mutual impedances. (See also [27] and [28] and Fig. 8.)

For convenience, the phase currents and voltages in terms of positive- and negative-sequence right-angle components and positive- and zero-sequence symmetrical components are summarized in one group of equations:

$$\begin{aligned}
V_A &= V_{A1} + V_{A2} = V_{a1} + V_{a0} \\
V_B &= -jV_{A1} + jV_{A2} = -V_{a1} + V_{a0} \\
I_A &= I_{A1} + I_{A2} = I_{a1} + I_{a0} \\
I_B &= -jI_{A1} + jI_{A2} = -I_{a1} + I_{a0}
\end{aligned} \qquad [34]$$

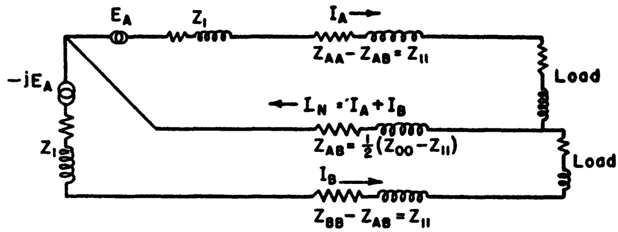


FIG. 8. Phase diagram of two-phase three-wire ungrounded system, with symmetrical three-wire transmission circuit and $Z_1 = Z_2$ in the two-phase generator.

Positive- and Negative-Sequence Right-Angle Components of Voltage and Current in Terms of Phase Voltages and Currents, and in Terms of Positive- and Zero-Sequence Symmetrical Components. Equations [31] and [32] express positive- and negative-sequence right-angle components in terms of phase voltages and currents. Replacing phase voltages and currents in these equations by their values in terms of positive- and zero-sequence symmetrical components from [34], the following equations are obtained:

$$\begin{aligned}
V_{A1} &= \tfrac{1}{2}(V_A + jV_B) = \frac{1-j}{2}V_{a1} + \frac{1+j}{2}V_{a0} \\
V_{A2} &= \tfrac{1}{2}(V_A - jV_B) = \frac{1+j}{2}V_{a1} + \frac{1-j}{2}V_{a0} \\
I_{A1} &= \tfrac{1}{2}(I_A + jI_B) = \frac{1-j}{2}I_{a1} + \frac{1+j}{2}I_{a0} \\
I_{A2} &= \tfrac{1}{2}(I_A - jI_B) = \frac{1+j}{2}I_{a1} + \frac{1-j}{2}I_{a0}
\end{aligned} \qquad [35]$$

Positive- and Zero-Sequence Symmetrical Components of Voltage and Current in Terms of Phase Voltages and Currents and in Terms of Positive- and Negative-Sequence Right-Angle Components. Replacing phase voltages and currents in [6] and [8] by their values in terms of positive- and negative-sequence right-angle component,

$$\begin{aligned}
V_{a1} &= \tfrac{1}{2}(V_A - V_B) = \frac{1+j}{2} V_{A1} + \frac{1-j}{2} V_{A2} \\
V_{a0} &= \tfrac{1}{2}(V_A + V_B) = \frac{1-j}{2} V_{A1} + \frac{1+j}{2} V_{A2} \\
I_{a1} &= \tfrac{1}{2}(I_A - I_B) = \frac{1+j}{2} I_{A1} + \frac{1-j}{2} I_{A2} \\
I_{a0} &= \tfrac{1}{2}(I_A + I_B) = \frac{1-j}{2} I_{A1} + \frac{1+j}{2} I_{A2}
\end{aligned} \qquad [36]$$

By means of equations [35] and [36] components of current and voltage in either of the component systems can be expressed in terms of the phase quantities or the components of the other system. Phase currents and voltages can be expressed in terms of the components of either system by [34].

Generated Voltages of a Two-Phase Synchronous Machine. In a two-phase synchronous machine of balanced design, the generated voltage of phase B is equal in magnitude to the generated voltage of phase A and lags it by 90°. Indicating the generated voltages by E_A and E_B,

$$E_B = -jE_A \qquad [37]$$

From [35] and [37],

$$\begin{aligned}
E_{A1} &= \tfrac{1}{2}(E_A + jE_B) = E_A \\
E_{A2} &= \tfrac{1}{2}(E_A - jE_B) = 0
\end{aligned} \qquad [38]$$

From [36] and [37],

$$\begin{aligned}
E_{a1} &= \tfrac{1}{2}(E_A - E_B) = \frac{1+j}{2} E_A = \frac{E_A}{\sqrt{2}} \angle 45^\circ \\
E_{a0} &= \tfrac{1}{2}(E_A + E_B) = \frac{1-j}{2} E_A = \frac{E_A}{\sqrt{2}} \overline{/45^\circ}
\end{aligned} \qquad [39]$$

Comparing equations [37], [38], and [39], generated voltages in a two-phase synchronous machine are most simply expressed in terms of positive- and negative-sequence right-angle components as there are components of generated voltage in the positive-sequence system only. When symmetrical positive- and zero-sequence components are used, [39] shows that there are generated voltages in both sequence networks. The positive-sequence symmetrical component of generated voltage leads E_A by 45°, while the zero-sequence component of generated voltage lags E_A by 45°. Both voltages are $E_A/\sqrt{2} = 0.707E_A$ in magnitude. When phase quantities are used, there are generated voltages in both phases equal in magnitude but 90° apart in phase.

Relations between the Self- and Mutual Impedances in the Two Component Systems and the Phases. By means of equations [21], [26], [33] and [34]–[36] self- and mutual impedances of the two component systems and of the phases can be expressed in terms of each other. For example, replacing I_{a1} and I_{a0} in [21] by their positive- and negative-sequence right-angle components of current from [36], then substituting V_{a1} and V_{a0} in the equations for V_{A1} and V_{A2} in [35], and finally equating the coefficients of I_{A1} and I_{A2} in the resulting equations for V_{A1} and V_{A2} to the corresponding coefficients in [33], the positive- and negative-sequence right-angle impedances in terms of the symmetrical positive- and zero-sequence self- and mutual impedances are obtained. The following equations express the self- and mutual impedances of the two component systems in terms of each other and in terms of the phase self- and mutual impedances; and also the phase self- and mutual impedances in terms of the self- and mutual impedances of each of the two component systems.

$$\begin{aligned}
Z_{11}(90°) &= \tfrac{1}{2}[Z_{AA} + Z_{BB} - j(Z_{AB} - Z_{BA})] \\
&= \tfrac{1}{2}[Z_{11} + Z_{00} - j(Z_{10} - Z_{01})] \\
Z_{22} &= \tfrac{1}{2}[Z_{AA} + Z_{BB} + j(Z_{AB} - Z_{BA})] \\
&= \tfrac{1}{2}[Z_{11} + Z_{00} + j(Z_{10} - Z_{01})] \\
Z_{12} &= \tfrac{1}{2}[Z_{AA} - Z_{BB} + j(Z_{AB} + Z_{BA})] \\
&= \tfrac{1}{2}[Z_{10} + Z_{01} + j(Z_{00} - Z_{11})] \\
Z_{21} &= \tfrac{1}{2}[Z_{AA} - Z_{BB} - j(Z_{AB} + Z_{BA})] \\
&= \tfrac{1}{2}[Z_{10} + Z_{01} - j(Z_{00} - Z_{11})]
\end{aligned} \tag{40}$$

$$\begin{aligned}
Z_{11} &= \tfrac{1}{2}[Z_{AA} + Z_{BB} - (Z_{AB} + Z_{BA})] \\
&= \tfrac{1}{2}[Z_{11}(90°) + Z_{22} + j(Z_{12} - Z_{21})] \\
Z_{00} &= \tfrac{1}{2}[Z_{AA} + Z_{BB} + (Z_{AB} + Z_{BA})] \\
&= \tfrac{1}{2}[Z_{11}(90°) + Z_{22} - j(Z_{12} - Z_{21})] \\
Z_{10} &= \tfrac{1}{2}[Z_{AA} - Z_{BB} + (Z_{AB} - Z_{BA})] \\
&= \tfrac{1}{2}[Z_{12} + Z_{21} - j\{Z_{22} - Z_{11}(90°)\}] \\
Z_{01} &= \tfrac{1}{2}[Z_{AA} - Z_{BB} - (Z_{AB} - Z_{BA})] \\
&= \tfrac{1}{2}[Z_{12} + Z_{21} + j\{Z_{22} - Z_{11}(90°)\}]
\end{aligned} \tag{41}$$

$$
\begin{aligned}
Z_{AA} &= \tfrac{1}{2}[Z_{11}(90°) + Z_{22} + (Z_{12} + Z_{21})] \\
&= \tfrac{1}{2}[Z_{11} + Z_{00} + (Z_{10} + Z_{01})] \\
Z_{BB} &= \tfrac{1}{2}[Z_{11}(90°) + Z_{22} - (Z_{12} + Z_{21})] \\
&= \tfrac{1}{2}[Z_{11} + Z_{00} - (Z_{10} + Z_{01})] \\
Z_{AB} &= j\tfrac{1}{2}[Z_{11}(90°) - Z_{22} - (Z_{12} - Z_{21})] \\
&= \tfrac{1}{2}[Z_{00} - Z_{11} + (Z_{10} - Z_{01})] \\
Z_{BA} &= j\tfrac{1}{2}[Z_{22} - Z_{11}(90°) - (Z_{12} - Z_{21})] \\
&= \tfrac{1}{2}[Z_{00} - Z_{11} - (Z_{10} - Z_{01})]
\end{aligned}
\qquad [42]
$$

A circuit may have non-reciprocal mutual impedances between the component network when one system of components is used, but reciprocal or zero mutual impedances when phase quantities or the other system of components is used. In two-phase system studies, with a choice of two systems of components and the phase quantities, selection is based on simplicity of calculations, taking into consideration the various circuits which make up the system. The impedances of a given circuit can be obtained in terms of phase impedances, symmetrical component impedances, or positive- and negative-sequence right-angle impedances — whichever is easiest. These impedances can then be converted to the type of impedances selected for system analysis by means of equations [40]–[42].

When a system of components or the phase quantities has been selected, the procedure is similar to that with symmetrical components in a three-phase or in a single-phase system. One-line diagrams are drawn of the component networks (or of the phases), with equivalent circuits replacing transmission circuits, machines, and other equipment. In cases where there is no mutual coupling between the component networks (or phases), the component networks (or phases) can be directly connected to represent fault or other abnormal conditions which may exist in the system.

Impedances of Two-Phase Synchronous Machines. The impedances of a two-phase synchronous machine to positive- and negative-sequence right-angle components of current can be obtained by calculation or test. Let these impedances be Z_1 and Z_2, respectively. The two-phase synchronous machine, just as the three-phase synchronous machine (Chapter I, and Fig. 17, Chapter I), can be represented in the positive-sequence right-angle network by a voltage E_A acting through an impedance Z_1, where E_A is the generated voltage in phase A behind the positive-sequence impedance Z_1, and Z_1 may be the subtransient, transient, or equivalent steady-state positive-sequence impedance of the machine. In the negative-sequence network, there is no generated voltage. At the machine terminals,

$$\begin{aligned} V_{A1} &= E_A - I_{A1}Z_1 \\ V_{A2} &= -I_{A2}Z_2 \end{aligned} \qquad [43]$$

Replacing $Z_{11}(90°)$ and Z_{22} in [41] by Z_1 and Z_2, respectively, with $Z_{12} = Z_{21} = 0$, the self- and mutual impedances in the positive- and zero-sequence symmetrical systems are

$$\begin{aligned} Z_{11} &= Z_{00} = \tfrac{1}{2}(Z_1 + Z_2) \\ Z_{10} &= -Z_{01} = -\frac{j}{2}(Z_2 - Z_1) \end{aligned} \qquad [44]$$

The phase self- and mutual impedances from [42] are

$$\begin{aligned} Z_{AA} &= Z_{BB} = \tfrac{1}{2}(Z_1 + Z_2) \\ Z_{AB} &= -Z_{BA} = -\frac{j}{2}(Z_2 - Z_1) \end{aligned} \qquad [45]$$

From [44] and [45], if $Z_1 \neq Z_2$, the mutual impedances between the symmetrical positive- and zero-sequence networks and between the phases are non-reciprocal. If Z_1 and Z_2 can be assumed equal, all mutual impedances disappear and

$$Z_{11} = Z_{00} = Z_{AA} = Z_{BB} = Z_1 \qquad [46]$$

Even with $Z_1 = Z_2$, the synchronous machine is more simply represented in terms of positive- and negative-sequence right-angle components than in terms of either positive- and zero-sequence symmetrical components or phase quantities because of the simpler components of generated voltage. (See [37] and [39].)

Impedances of a Two-Phase Three-Wire Transmission Circuit. In the two-phase, three-wire circuit of Fig. 6(*b*), there is a neutral conductor N and loads are taken off between A and N and B and N. There may also be loads between phases. The sum of the currents in phases A and B returns through the neutral conductor when N is ungrounded; through N in parallel with the ground, when N is multigrounded. Phase voltages at any point in an ungrounded system are referred to the neutral conductor at that point; in the grounded system, to the ground at the point. The two-phase three-wire circuit is most simply represented by its positive- and zero-sequence symmetrical components of impedance as explained under the single-phase system. (See [24] and [25].) In a three-wire circuit with the neutral conductor equidistant from the phase conductors there is no mutual impedance between the positive- and zero-sequence networks. Let Z_{11} and Z_{00} represent the positive- and zero-sequence self-impedances, with $Z_{10} = Z_{01} = 0$. Substituting these values in [40], the positive- and negative-sequence right-angle components of impedance are

$$\begin{aligned} Z_{11}(90°) &= Z_{22} = \tfrac{1}{2}(Z_{11} + Z_{00}) \\ Z_{12} &= -Z_{21} = \frac{j}{2}(Z_{00} - Z_{11}) \end{aligned} \qquad [47]$$

From [47], the mutual impedances are non-reciprocal.

The phase self- and mutual impedances in terms of Z_{11} and Z_{00} are given in [28]. The mutual impedances between phases are reciprocal.

Impedances of a Two-Phase Four-Wire Transmission Circuit. In the two-phase, four-wire transmission circuit shown in Fig. 6(*a*), there is no interconnection between phases. The self-impedances in phases A and B are two-conductor single-phase loop impedances, which will be indicated by Z_{AA} and Z_{BB}. When the loads can be represented by impedances, these impedances can be included in Z_{AA} and Z_{BB}. Mutual impedance between phases in the transmission circuit will depend upon the conductor arrangement. If the conductors are placed at the corners of a rhombus or square, with conductors of the same phase at opposite corners, there will be no mutual impedances between phases. In any case, the mutual impedance between phases will be small relative to the self-impedances.

With no mutual impedances between phases, $Z_{AB} = Z_{BA} = 0$. Making this substitution in [40], the positive- and negative-sequence right-angle components of self- and mutual impedances are

$$\begin{aligned} Z_{11}(90°) &= Z_{22} = \tfrac{1}{2}(Z_{AA} + Z_{BB}) \\ Z_{12} &= Z_{21} = \tfrac{1}{2}(Z_{AA} - Z_{BB}) \end{aligned} \qquad [48]$$

Substituting $Z_{AB} = Z_{BA} = 0$ in [41], the positive- and zero-sequence symmetrical components are

$$\begin{aligned} Z_{11} &= Z_{00} = \tfrac{1}{2}(Z_{AA} + Z_{BB}) \\ Z_{10} &= Z_{01} = \tfrac{1}{2}(Z_{AA} - Z_{BB}) \end{aligned} \qquad [49]$$

Choice of Components or Phase Quantities in Two-Phase System Calculations. Unfortunately, neither of the systems of components nor the phase quantities are well suited to all types of two-phase circuits. If the positive- and negative-sequence right-angle impedances are unequal, currents and voltages at the machine terminals are most simply determined by positive- and negative-sequence right-angle components. These components, however, are not well suited to three-wire transmission circuits because of the non-reciprocal mutual coupling between the component networks.

Positive- and zero-sequence symmetrical components are well adapted to three-wire transmission circuits and are useful in determining voltage regulation in two-phase three-wire circuits. When $Z_1 \neq Z_2$ in the two-phase synchronous machines of the system, the positive- and zero-sequence symmetrical components have non-reciprocal mutual coupling between the component networks. If $Z_1 = Z_2$, and the two-phase system is operated without connection to a three-phase system, the positive- and zero-sequence symmetrical components will be most suitable when there are appreciable lengths of three-wire transmission circuits.

The phase quantities are best suited to the study of two-phase four-wire transmission circuits and are better adapted to three-wire transmission circuits than positive- and negative-sequence right-angle components. If $Z_1 \neq Z_2$ in the rotating machines, there is non-reciprocal mutual coupling between the phases.

For any given two-vector system, operated without connection to a three-phase system, either of the component systems or the phase quantities can be used in a system study. In cases where mutual impedances are non-reciprocal, solution by means of equations is always possible, as illustrated in Chapter VIII for unsymmetrical three-phase systems.

When a Scott-connected transformer or autotransformer bank connects the three-phase and two-phase parts of a power system, and current and voltage conditions in both parts of the system are to be determined, either positive- and negative-sequence right-angle components or phase quantities can be used on the two-phase side. If symmetrical components are used on the three-phase side, positive- and negative-sequence right-angle components will be used on the two-phase side. If the positive- and negative-sequence impedances of the rotating machines on both the two-phase and three-phase sides of the Scott-connected transformer bank can be assumed equal, phase quantities can be advantageously used on the two-phase side with α, β, and 0 components, described in the following chapter, on the three-phase side. A further discussion of two-phase and three-phase systems connected by Scott-connected transformers is given in Chapter X.

Admittances of Two-Vector Systems

In calculations involving shunt circuits between phases or between phases and ground in single-phase and two-phase circuits, the positive-and zero-sequence self- and mutual admittances of the circuit may be found more convenient than the impedances.

Sequence Admittances of Two-Vector Circuits

Following the notation and definitions of Chapter VIII, the positive-sequence admittances of phases a and b of a single-phase or two-phase circuit will be indicated by Y_{a1} and Y_{b1}, respectively, and defined as the ratios of the phase currents I_a and I_b entering the circuit to the corresponding applied phase voltages with positive-sequence voltages only applied to the circuit; similarly, the zero-sequence admittances of phases a and b will be indicated by Y_{a0} and Y_{b0}, respectively, and defined as the ratios of the phase currents to the corresponding phase voltages with zero-sequence voltages only applied to the circuit. The phase currents I_a and I_b in terms of the positive- and zero-sequence phase admittances of the circuit and the applied voltages V_a and V_b, replaced by their positive- and zero-sequence symmetrical components, are then

$$\begin{aligned} I_a &= V_{a1}Y_{a1} + V_{a0}Y_{a0} \\ I_b &= V_{b1}Y_{b1} + V_{b0}Y_{b0} = -V_{a1}Y_{b1} + V_{a0}Y_{b0} \end{aligned} \qquad [50]$$

Substituting I_a and I_b from [50] in [8],

$$\begin{aligned} I_{a1} &= V_{a1}\frac{Y_{a1} + Y_{b1}}{2} + V_{a0}\frac{Y_{a0} - Y_{b0}}{2} \\ I_{a0} &= V_{a1}\frac{Y_{a1} - Y_{b1}}{2} + V_{a0}\frac{Y_{a0} + Y_{b0}}{2} \end{aligned} \qquad [51]$$

Positive- and Zero-Sequence Self- and Mutual Admittances. With phase voltages V_a and V_b at the circuit terminals and phase currents I_a and I_b flowing into the circuit, the positive- and zero-sequence components I_{a1} and I_{a0} are expressed in terms of the positive- and zero-sequence components of voltage V_{a1} and V_{a0} by the following equations:

$$\begin{aligned} I_{a1} &= V_{a1}Y_{11} + V_{a0}Y_{10} \\ I_{a0} &= V_{a1}Y_{01} + V_{a0}Y_{00} \end{aligned} \qquad [52]$$

where Y's with two subscripts represent the sequence self- and mutual admittances, the first subscript referring to the sequence of the current given by the equation and the second to the sequence of the voltage associated with the admittance.

Equating the coefficients of V_{a1} and V_{a0} in the corresponding current equations of [51] and [52],

$$\begin{aligned}
Y_{11} &= \tfrac{1}{2}(Y_{a1} + Y_{b1}) = \text{self-admittance to positive-sequence voltages} \\
Y_{00} &= \tfrac{1}{2}(Y_{a0} + Y_{b0}) = \text{self-admittance to zero-sequence voltages} \\
Y_{10} &= \tfrac{1}{2}(Y_{a0} - Y_{b0}) = \text{ratio of positive-sequence current produced by } V_{a0} \text{ to } V_{a0} \\
Y_{01} &= \tfrac{1}{2}(Y_{a1} - Y_{b1}) = \text{ratio of zero-sequence current produced by } V_{a1} \text{ to } V_{a1}
\end{aligned} \qquad [53]$$

Sequence Admittances in Terms of Sequence Impedances. The positive- and zero-sequence self- and mutual admittances are given by [53] in terms of the positive- and zero-sequence phase admittances. They can also be expressed in terms of the positive- and zero-sequence self- and mutual *impedances* of the circuit by solving [21] for I_{a1} and I_{a0} and then equating coefficients of V_{a1} and V_{a0} in the resulting equations to the coefficients of V_{a1} and V_{a0}, respectively, in the corresponding current equations of [52]. This gives

$$\begin{aligned}
Y_{11} &= \frac{Z_{00}}{Z_{11}Z_{00} - Z_{10}Z_{01}} \\
Y_{00} &= \frac{Z_{11}}{Z_{11}Z_{00} - Z_{10}Z_{01}} \\
Y_{10} &= \frac{-Z_{10}}{Z_{11}Z_{00} - Z_{10}Z_{01}} \\
Y_{01} &= \frac{-Z_{01}}{Z_{11}Z_{00} - Z_{10}Z_{01}}
\end{aligned} \qquad [54]$$

In a *symmetrical circuit*, $Z_{01} = Z_{10} = 0$. Making this substitution in [54],

$$\begin{aligned}
Y_{11} &= \frac{1}{Z_{11}} = \frac{1}{Z_1} \\
Y_{00} &= \frac{1}{Z_{00}} = \frac{1}{Z_0} \\
Y_{10} &= Y_{01} = 0
\end{aligned} \qquad [55]$$

In a symmetrical circuit, the positive- and zero-sequence self-admittances are the reciprocals of the corresponding self-impedances and there are no mutual admittances between the sequence networks. It is only in a symmetrical circuit that this is true.

In many circuits, the self- and mutual sequence impedances are more readily obtained than the self- and mutual sequence admittances. When this is the case, the sequence impedances can first be determined and from them the sequence admittances by using [54].

Formulas for the positive- and zero-sequence self- and mutual capacitive susceptances of two- and three-wire transmission circuits in terms of circuit dimensions are given in Chapter XII.

Problem 1. Check the general equations [40], [41], and [42].

Problem 2. Rewrite these equations for static circuits in which $Z_{AB} = Z_{BA}$.

Problem 3. In Fig. 5(*a*), the transformer is rated 1000 kva, 4000 volts–230 volts. The turn ratio is 4000/230. Leakage reactance based on rating is 6%. The reactance of the two half windings in parallel is 0.3% based on 500 kva and 115 volts. $V_p = 4100$ volts. A phase-to-ground fault occurs at the transformer terminal. Determine the current in the primary and secondary windings in amperes.

Problem 4. A two-phase, 60-cycle, unloaded generator, rated 5000 kva and 13,800 volts per phase, with positive- and negative-sequence right-angle subtransient reactances of 9% based on its rating, is directly connected to a symmetrically spaced three-wire transmission circuit (with identical wires) five miles in length. The two phases of the generator are connected to the ungrounded neutral conductor, as in Fig. 6(*b*). The reactance of the transmission circuit to positive-sequence symmetrical components of current is 0.72 ohm per mile. (*a*) What is the zero-sequence reactance? (*b*) A fault occurs between one phase and the neutral conductor at the terminals of the five-mile transmission circuit. What is the initial symmetrical rms fault current in amperes? Neglect resistance and solve by any two of the three different methods given in this chapter.

Problem 5. Solve Problem 4 with the fault at the generator terminals.

Problem 6. Solve Problems 4 and 5 with a generator rated 200 kva and 2300 volts per phase with positive- and negative-sequence right-angle subtransient reactances of 24% and 32%, respectively.

CHAPTER X

ALPHA, BETA, AND ZERO COMPONENTS OF THREE-PHASE SYSTEMS

It is pointed out in Chapter II that a set of three voltage or current vectors pertaining to the phases of a three-phase system can be replaced by any one of a number of different systems of component vectors. The symmetrical component system is one of such systems. The positive-plus-negative, positive-minus-negative, and zero-sequence system of components discussed in Chapter V is another. The present chapter deals with a third system of components, here called α, β, and 0 components.

With phase a as reference phase in a three-phase system, the α, β, and 0 components of current and voltage are defined as follows:

α components in phases b and c are equal; they are opposite in sign and of half the magnitude of the α component of phase a.

β components in phases b and c are equal in magnitude and opposite in sign; in phase a they are zero.

0 components are equal in the three phases.

α components of current flow into a three-phase circuit in phase a and return one-half in phase b and one-half in phase c. β components of current are circulating currents in phases b and c. 0 components are zero-sequence components taken over from symmetrical components without change except in notation; they are here written 0 components for brevity and also to indicate that they are to be used with α and β components.

Relations between Phase Currents and Voltages and Their α, β, and 0 Components. Referring to Chapter II, equations [1]–[3], let $V_1 = V_\alpha$, $V_2 = V_\beta$, $V_3 = V_0$. The constant coefficients required to express a set of three vectors V_a, V_b, V_c of a three-phase system in terms of their α, β, 0 components are:

$$1, \quad -\tfrac{1}{2}, \quad -\tfrac{1}{2} \text{ for } \alpha \text{ components}$$

$$0, \quad \frac{\sqrt{3}}{2}, \quad -\frac{\sqrt{3}}{2} \text{ for } \beta \text{ components}$$

$$1, \quad 1, \quad 1 \text{ for 0 components}$$

A set of three voltage vectors V_a, V_b, and V_c are expressed in terms of their α, β, and 0 components by the equations

$$V_a = V_\alpha + V_0 \qquad [1]$$

$$V_b = -\tfrac{1}{2}V_\alpha + \frac{\sqrt{3}}{2}V_\beta + V_0 \qquad [2]$$

$$V_c = -\tfrac{1}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta + V_0 \qquad [3]$$

Equations [1]–[3] satisfy the required condition that the determinant[1] made up of the coefficients is not zero.

The three voltage vectors V_α, V_β, and V_0 are expressed in terms of the vectors V_a, V_b, and V_c by solving the simultaneous equations [1]–[3]:

Subtracting one-half the sum of [2] and [3] from [1] and solving for V_α,

$$V_\alpha = \tfrac{2}{3}\left(V_a - \frac{V_b + V_c}{2}\right) \qquad [4]$$

Subtracting [3] from [2] and solving for V_β,

$$V_\beta = \frac{1}{\sqrt{3}}(V_b - V_c) \qquad [5]$$

Adding the three equations and solving for V_0,

$$V_0 = \tfrac{1}{3}(V_a + V_b + V_c) \qquad [6]$$

The corresponding current equations are

$$I_a = I_\alpha + I_0 \qquad [7]$$

$$I_b = -\tfrac{1}{2}I_\alpha + \frac{\sqrt{3}}{2}I_\beta + I_0 \qquad [8]$$

$$I_c = -\tfrac{1}{2}I_\alpha - \frac{\sqrt{3}}{2}I_\beta + I_0 \qquad [9]$$

$$I_\alpha = \tfrac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) \qquad [10]$$

$$I_\beta = \frac{1}{\sqrt{3}}(I_b - I_c) \qquad [11]$$

$$I_0 = \tfrac{1}{3}(I_a + I_b + I_c) \qquad [12]$$

Equations [1]–[3] and [7]–[9] express any set of three voltage or current vectors, respectively, pertaining to the three phases of a three-phase system in terms of their α, β, and 0 components. Equations [4]–[6] and [10]–[12] express α, β, and 0 components in terms of phase voltages and currents, respectively. Equations [1]–[12] are analogous to the fundamental symmetrical component equations developed in Chapter II.

Line-to-Line Voltages. If V_a, V_b, and V_c in [1]–[3] represent the phase voltages to ground at a system point, the line-to-line voltages at the same point in terms of the α, β, 0 components of phase voltages to ground are

$$\begin{aligned} V_{ba} &= V_a - V_b = \tfrac{3}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta \\ V_{ac} &= V_c - V_a = -\tfrac{3}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta \\ V_{cb} &= V_b - V_c = \sqrt{3}V_\beta \end{aligned} \qquad [13]$$

If V_α and V_β are expressed in per unit of base line-to-neutral voltage, the line-to-line voltages will also be in per unit of base line-to-neutral voltage.

History of α, β, 0 Components. α, β, 0 components of current are not new. Components of current answering to the description of α, β, and 0, although not so named, were used in a method[2] developed by Dr. W. W. Lewis, and published in 1917, to determine system currents and voltages during line-to-ground faults. In Fig. 2 of the paper, which is similar to Fig. 1 of this chapter, phase currents are repre-

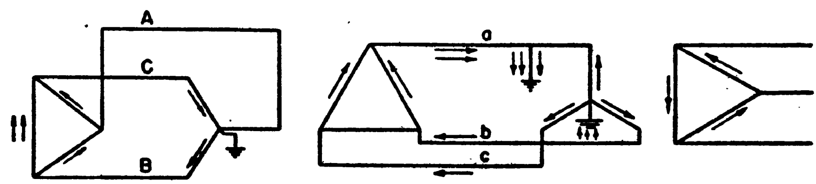


FIG. 1. Phase currents represented by arrows in direction and magnitude, number of arrows showing relative magnitudes of currents in any circuit.

sented by arrows both in direction and magnitude, the number of arrows indicating relative magnitudes of currents in each circuit. Applying the definitions given above for α, β, and 0 currents to Fig. 1, it may be seen that all three components of current are present. Currents in the Y–Δ transformer bank and in the line to the right of the fault are 0 currents; currents in the transmission line to the left of the fault are α currents; currents in the second Y–Δ transformer bank and in the line at the generator terminals are β currents; currents in the generator are α currents. In the method as used before symmetrical components were applied to unsymmetrical short circuits, each component of phase current met its respective impedance, but calculations were made with phase voltages and currents, not with *component networks*, and therefore were time consuming if many circuits operated at different voltages had to be considered.

In problems involving unsymmetrical three-phase circuits, and in particular circuits with two of the phases symmetrical with respect to the third phase, the use of components of current which flow in one phase and divide equally between the other two phases, and components of current which circulate in two phases, is a logical development. Such components, as yet unnamed, were used[3] in 1931; in 1938 they appeared under the names of α and β components in two papers,[4,5] both of which deal with transient conditions in rotating machines where the development is materially simplified by their use. Two papers have been devoted exclusively to these components. In one paper,[6] they are called α, β, and 0 components and the system *Modified Symmetrical Components*. In the other paper,[7] entitled "Two-Phase Co-ordinates of a Three-Phase System," by Dr. E. W. Kimbark, the components are called x, y, and z. Comparing these two sets: x and α components are identical; y and β components differ only in sign; z components of voltage are 0 (zero-sequence) components of voltage, z components of currents are twice 0 (zero-sequence) components of currents, and z impedances are one-half 0 (zero-sequence) impedances. At present, definitions and notation for the components (here called α, β, 0) are not definitely established[8] by usage. The choice of the sign for β or y components is arbitrary. The use of z components as defined in reference 7 has advantages which will be pointed out later. On the other hand, the familiar zero-sequence network, modified as required before interconnecting the component networks to satisfy unsymmetrical system conditions, is of advantage in analytic calculations. This is illustrated in Chapter V, Figs. 1 and 3, where two different modifications of the zero-sequence network are made. Dr. Kimbark's paper[7] and the discussions[8] by Messrs. Boyajian, Helwith, and Sligant in terms of matrix and tensor concepts should be read for a comprehensive view of these important components.

α, β, AND 0 ONE-LINE DIAGRAMS

When components of phase currents and voltages instead of phase quantities are used in calculations, each set of components is conveniently represented by a separate one-line diagram or component network from which the components of current and voltage in the three phases can be obtained. To draw component networks it is necessary to determine: (1) references for the components of voltage, (2) components of generated voltage, and (3) the impedances offered to the components of current, or the admittances associated with the components of voltage. Of interest also are the components of current present in a symmetrical system during normal operating conditions.

Reference for α, β, and 0 Voltages. The neutral of a Y-connected circuit is common to the three phases; therefore, in the limit as the neutral is approached, $V_a = V_b = V_c$. From [1]–[3], this condition is satisfied if $V_\alpha = 0$ and $V_\beta = 0$. All neutral points are therefore points of zero potential in the α and β networks, and α and β voltages are referred to neutral. As all neutrals are at zero potential in the α and β networks, the expressions "voltage to neutral" and "voltage to ground" can be used interchangeably for α and β voltages just as they are used interchangeably for positive- and negative-sequence voltages. 0 voltages at any point in a grounded system will be referred to ground at that point. In an ungrounded system with a neutral conductor, they will be referred to the neutral conductor.

Generated α, β, and 0 Voltages. In a synchronous machine with generated voltages E_a, E_b, and E_c, the generated α, β, and 0 voltages E_α, E_β, and E_0 obtained by substituting E_a, E_b, and E_c for V_a, V_b, and V_c, respectively, in [4]–[6] are

$$
\begin{aligned}
E_\alpha &= \tfrac{2}{3}\left(E_a - \frac{E_b + E_c}{2}\right) \\
E_\beta &= \frac{1}{\sqrt{3}}(E_b - E_c) \\
E_0 &= \tfrac{1}{3}(E_a + E_b + E_c)
\end{aligned}
\qquad [14]
$$

If the generated voltages are balanced, $E_b = a^2E_a$, $E_c = aE_a$, and [14] becomes

$$
\begin{aligned}
E_\alpha &= E_a \\
E_\beta &= -jE_a \\
E_0 &= 0
\end{aligned}
\qquad [15]
$$

With balanced generated voltages in a synchronous machine, the generated voltage in the α network is E_a, the generated voltage of phase a. In the β network it is $-jE_a$, the generated voltage of phase a turned backward 90°. There is no generated voltage in the 0 network.

α and β Currents in a Balanced System. In a symmetrical system operating under balanced conditions, the currents in phases b and c at any point of the system are $I_b = a^2 I_a$, $I_c = aI_a$. Substituting these values for I_b and I_c in [10]–[12],

$$\begin{aligned} I_\alpha &= I_a \\ I_\beta &= -jI_a \\ I_0 &= 0 \end{aligned} \qquad [16]$$

Equations [15] and [16] show that generated voltages and load currents are present in both the α and β networks of a symmetrical system during normal operation. Because two networks must be considered instead of one, α, β, and 0 components are not as convenient as symmetrical components for the study of symmetrical systems during normal operation or during three-phase faults.

α, β, and 0 Networks. Figure 2(a) shows a symmetrical three-phase system with balanced applied voltages and equal self-impedances Z in the three phases. I_α, flowing in phase a and returning one-half in each of phases b and c, flows in a loop circuit. The voltage applied to this loop, as shown in Fig. 2(a), is $E_a - (-E_a/2) = \frac{3}{2}E_a$. The α loop impedance for a symmetrical three-phase circuit of equal self-impedance Z in the three phases is $\frac{3}{2}Z$. The current I_α in phase a is

$$I_\alpha = \frac{\frac{3}{2}E_a}{\frac{3}{2}Z} = \frac{E_a}{Z}$$

The impedance met by I_α is Z. The equivalent circuit for phase a in the α system is shown in Fig. 2(b), with the applied voltage E_a and the self-impedance Z. In this equivalent circuit, voltages are referred to neutral, base voltage is line-to-neutral voltage, and base current is line current. Since the α currents and voltages in phases b and c at any point in the system are $-\frac{1}{2}$ those of phase a at the same point, it is unnecessary to have additional equivalent circuits for these phases. The equivalent circuit for phase a in the α system will be called the *α network.*

β currents, flowing in phase b and returning in phase c, flow in a loop circuit. The voltage applied to this loop, as shown in Fig. 2(a), is $-j\sqrt{3}E_a$. The β loop impedance for the symmetrical three-phase circuit of equal self-impedances Z in the three phases is $2Z$. The β current flowing in phase b in the direction indicated by arrow is $(\sqrt{3}/2)I_\beta$. Therefore

$$(\sqrt{3}/2)I_\beta = -j(\sqrt{3}E_a/2Z), \quad \text{and} \quad I_\beta = -j(E_a/Z)$$

The impedance met by I_β is Z. The equivalent circuit for the β system is shown in Fig. 2(c), with the applied voltage $-jE_a$ and the self-impedance Z. In this equivalent circuit, which will be called the β *network*, voltages are referred to neutral, base voltage is line-to-neutral voltage, and base current is line current. The β voltages and currents in phases b and c are the voltages and currents in the β network multiplied by $\sqrt{3}/2$ and $-\sqrt{3}/2$, respectively. *The β network does not give directly the β voltages and currents in either phase b or phase c.*

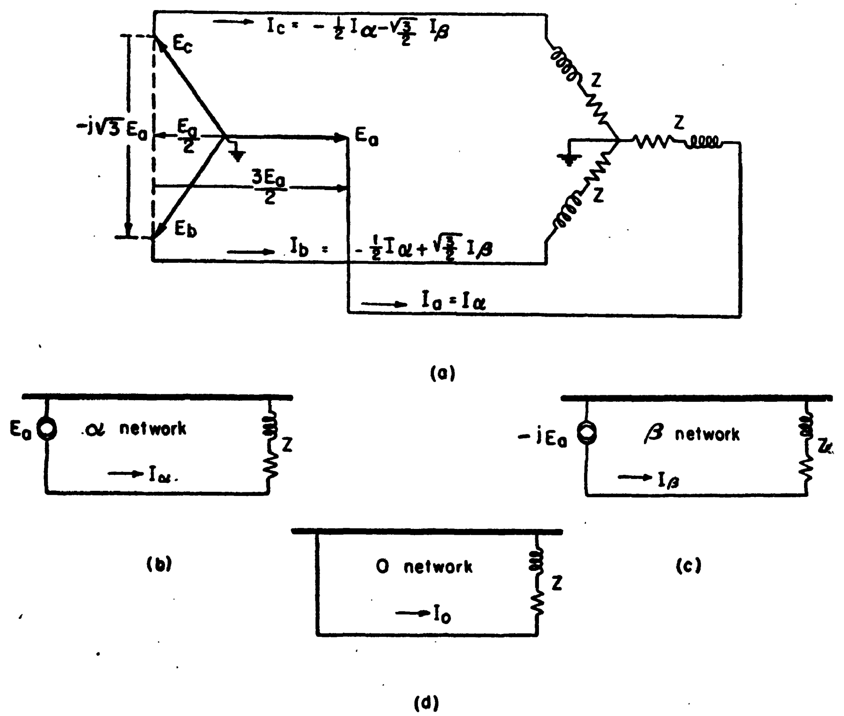


FIG. 2(a). Flow of α and β currents in balanced system with equal self-impedances in the three phases and balanced applied voltages. (b) α network for system shown in (a). (c) β network for system shown in (a). (d) 0 network for system shown in (a).

This slight disadvantage is more than offset by the convenience of having the same line-to-neutral voltage and line current as base quantities in the β as in the α and 0 networks.

With a path for 0 currents through the circuit of equal self-impedances Z in the three phases, the impedance met by I_0 is Z. The 0 network for the system of Fig. 2(a) is shown in Fig. 2(d).

α, β, and 0 equivalent circuits to replace the various equipment, machines, and transmission circuits of a three-phase power system in the α, β, and 0 networks can be determined when the α, β, and 0 self-and mutual impedances of the circuits are known. α, β, and 0 impedances, just as positive-, negative-, and zero-sequence impedances, can be obtained by calculation or test. Before developing equivalent circuits for use in the α, β, and 0 networks, relations between symmetrical components and α, β, and 0 components will be established.

α, β, and 0 Components of Voltage and Current in Terms of Symmetrical Components of Voltage and Current. From [1]–[3] and [7]–[9] of this chapter and [1]–[6] of Chapter V,

$$\begin{aligned} V_\alpha &= V_{a1} + V_{a2} \\ V_\beta &= -j(V_{a1} - V_{a2}) \\ V_0 &= V_{a0} \\ I_\alpha &= I_{a1} + I_{a2} \\ I_\beta &= -j(I_{a1} - I_{a2}) \\ I_0 &= I_{a0} \end{aligned} \tag{17}$$

From [17], α components are positive-plus-negative components, β components are positive-minus-negative components turned backward 90°.

Symmetrical Components of Voltage and Current in Terms of α, β, and 0 Components of Voltage and Current. Solving the simultaneous voltage and current equations of [17],

$$\begin{aligned} V_{a1} &= \tfrac{1}{2}(V_\alpha + jV_\beta) \\ V_{a2} &= \tfrac{1}{2}(V_\alpha - jV_\beta) \\ V_{a0} &= V_0 \\ I_{a1} &= \tfrac{1}{2}(I_\alpha + jI_\beta) \\ I_{a2} &= \tfrac{1}{2}(I_\alpha - jI_\beta) \\ I_{a0} &= I_0 \end{aligned} \tag{18}$$

α, β, and 0 Self- and Mutual Impedances

In [4]–[6] of Chapter VIII, the symmetrical components of voltage drop in an unsymmetrical three-phase series circuit without internal voltages are expressed in terms of the symmetrical components of current flowing in the circuit and the self- and mutual impedances of the sequence networks. Equations for the α, β, and 0 voltage drops in terms of the α, β, and 0 currents flowing in the circuit and the α, β, and 0 self- and mutual impedances of the circuit likewise will be written. In these equations, as in the corresponding symmetrical component equations, the effects of saturation are neglected and linear relations between currents and voltages assumed.

Finite 0 Self-Impedance. Let V_α, V_β, V_0 and V'_α, V'_β, V'_0 be the α, β, 0 components of voltage to ground at P and Q, respectively, and I_α, I_β, and I_0 the components of line current flowing from P to Q. Then

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + I_0 Z_{\alpha 0} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + I_0 Z_{\beta 0} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + I_0 Z_{00} \end{aligned} \qquad [19]$$

where $Z_{\alpha\alpha}$, $Z_{\beta\beta}$, and Z_{00} are the α, β, and 0 self-impedances, respectively, of the circuit. The Z's with two unlike subscripts represent mutual impedances, the first subscript referring to the voltage and the second to the current associated with the impedance. $Z_{\alpha\alpha}$ is the ratio of the voltage drop in the α network produced by I_α to I_α; $Z_{\alpha\beta}$ is the ratio of the voltage drop in the α network produced by I_β to I_β; $Z_{\beta\alpha}$ is the ratio of the voltage drop in the β network produced by I_α to I_α, etc. If $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha 0} = Z_{0\alpha}$, and $Z_{\beta 0} = Z_{0\beta}$, the mutual impedances between the component networks are reciprocal. If the mutual impedances between any two networks are zero, there is no mutual coupling between these networks.

Infinite 0 Self-Impedance. If there is no path for zero-sequence currents through the circuit, $I_0 = 0$, and no voltages are induced in the α and β networks by I_0. $I_0 Z_{\alpha 0}$ and $I_0 Z_{\beta 0}$ are zero, $Z_{00} = \infty$, and [19] becomes

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + 0 \cdot \infty \text{ (indeterminate)} \end{aligned} \qquad [20]$$

The voltage drop v_0 between P and Q is indeterminate from [20] but can be evaluated when the 0 impedance diagram and operating conditions are known. (See [36]–[39] for an evaluation of v_0.)

Equations [19] and [20], written for a series circuit between P and Q, can be applied to a circuit connected at one point of the system. If currents flow *out of the circuit*, the components of *series voltage drop* in the circuit between ground (or neutral) and terminals in the direction of current flow are given by [19] or [20]. If currents flow *into a circuit* without internal voltages, the components of voltage to ground or to neutral *at the circuit terminals* are given by the right-hand sides of equations [19] or [20].

The α, β, and 0 self- and mutual impedances in [19] and [20] will be expressed in terms of the self- and mutual impedances of the sequence networks, and also in terms of the self- and mutual impedances of the phases.

α, β, and 0 Self- and Mutual Impedances in Terms of the Sequence Impedances. Replacing I_{a1}, I_{a2}, and I_{a0} in [4]–[6] of Chapter VIII by their values in terms of I_α, I_β, and I_0 given by [18], v_{a1}, v_{a2}, and v_{a0} are expressed in terms of I_α, I_β, and I_0. Substituting these equations for v_{a1}, v_{a2}, and v_{a0} in [17], v_α, v_β, and v_0 are expressed in terms of I_α, I_β, I_0. Equating the coefficients of I_α, I_β, and I_0 in the resultant equations for v_α, v_β, and v_0 to the corresponding coefficients in [19], the α, β, and 0 self- and mutual impedances are

$$\begin{aligned}
Z_{\alpha\alpha} &= \tfrac{1}{2}(Z_{11} + Z_{22} + Z_{21} + Z_{12})\\
Z_{\beta\beta} &= \tfrac{1}{2}(Z_{11} + Z_{22} - Z_{12} - Z_{21})\\
Z_{00} &= Z_{00}\\
Z_{\beta\alpha} &= -j\tfrac{1}{2}(Z_{11} - Z_{22} + Z_{12} - Z_{21})\\
Z_{\alpha\beta} &= j\tfrac{1}{2}(Z_{11} - Z_{22} + Z_{21} - Z_{12})\\
Z_{0\alpha} &= \tfrac{1}{2}(Z_{01} + Z_{02})\\
Z_{\alpha 0} &= (Z_{10} + Z_{20})\\
Z_{0\beta} &= j\tfrac{1}{2}(Z_{01} - Z_{02})\\
Z_{\beta 0} &= -j(Z_{10} - Z_{20})
\end{aligned} \qquad [21]$$

Self- and Mutual Impedances of the Sequence Networks in Terms of α, β, 0 Self- and Mutual Impedances. Proceeding in a manner analogous to that used to determine [21] or by solving [21] for Z_{11}, Z_{22}, etc.:

$$\begin{aligned}
Z_{11} &= \tfrac{1}{2}[Z_{\alpha\alpha} + Z_{\beta\beta} - j(Z_{\alpha\beta} - Z_{\beta\alpha})]\\
Z_{22} &= \tfrac{1}{2}[Z_{\alpha\alpha} + Z_{\beta\beta} + j(Z_{\alpha\beta} - Z_{\beta\alpha})]\\
Z_{00} &= Z_{00}\\
Z_{12} &= \tfrac{1}{2}[Z_{\alpha\alpha} - Z_{\beta\beta} + j(Z_{\alpha\beta} + Z_{\beta\alpha})]\\
Z_{21} &= \tfrac{1}{2}[Z_{\alpha\alpha} - Z_{\beta\beta} - j(Z_{\alpha\beta} + Z_{\beta\alpha})]\\
Z_{10} &= \tfrac{1}{2}(Z_{\alpha 0} + jZ_{\beta 0})\\
Z_{01} &= (Z_{0\alpha} - jZ_{0\beta})\\
Z_{20} &= \tfrac{1}{2}(Z_{\alpha 0} - jZ_{\beta 0})\\
Z_{02} &= (Z_{0\alpha} + jZ_{0\beta})
\end{aligned} \qquad [22]$$

Equations [17] and [21] can be used to pass from symmetrical components to α, β, 0 components. Equations [18] and [22] give symmetrical components in terms of α, β, 0 components. If a problem is to be solved by α, β, 0 components and the sequence self- and mutual impedances are known, the α, β, 0 self- and mutual impedances are obtained by substituting the sequence impedances in [21]. If the problem is to be solved by symmetrical components, but the α, β, 0 self- and mutual impedances of an unsymmetrical circuit are more readily obtained than the sequence self- and mutual impedances, [22] will be found useful.

Symmetrical Circuit with Equal Positive- and Negative-Sequence Impedances. In a circuit with equal positive- and negative-sequence self-impedances and no mutual impedances between the sequence networks, the α, β, and 0 self- and mutual impedances obtained from [21] are

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = Z_{11} = Z_1 \\ Z_{00} &= Z_{00} = Z_0 \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \end{aligned} \qquad [23]$$

When there are no mutual impedances between the sequence networks, the positive-, negative-, and zero-sequence self-impedances are customarily indicated by Z_1, Z_2, and Z_0, respectively, instead of Z_{11}, Z_{22}, and Z_{00}.

Unsymmetrical Static Circuits. If $Z_{11} = Z_{22}$, $Z_{10} = Z_{02}$, and $Z_{20} = Z_{01}$, [21] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{11} + \tfrac{1}{2}(Z_{21} + Z_{12}) \\ Z_{\beta\beta} &= Z_{11} - \tfrac{1}{2}(Z_{21} + Z_{12}) \\ Z_{00} &= Z_{00} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = j\tfrac{1}{2}(Z_{21} - Z_{12}) \\ Z_{\alpha 0} &= 2Z_{0\alpha} = (Z_{10} + Z_{20}) \\ Z_{\beta 0} &= 2Z_{0\beta} = -j(Z_{10} - Z_{20}) \end{aligned} \qquad [24]$$

If $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$, and $Z_{10} = Z_{02} = Z_{20} = Z_{01}$, [21] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{11} + Z_{12} \\ Z_{\beta\beta} &= Z_{11} - Z_{12} \\ Z_{00} &= Z_{00} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = 2Z_{10} \end{aligned} \qquad [25]$$

If the sequence self- and mutual impedances of the unsymmetrical static circuits developed in Chapter VIII are substituted in [21], [24], or [25], their α, β, and 0 self- and mutual impedances will be obtained.

Modified 0 Network. In circuits in which $Z_{0\alpha} = \frac{1}{2}Z_{\alpha 0}$ and $Z_{0\beta} = \frac{1}{2}Z_{\beta 0}$, equations [19] are conveniently expressed in terms of a *modified* 0 network in which the voltage is 0 voltage, the current is $2I_0$, and the impedance is one-half the 0 impedance. Rewriting [19] in terms of $2I_0$,

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + (2I_0)\frac{Z_{\alpha 0}}{2} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + (2I_0)\frac{Z_{\beta 0}}{2} \\ v_0 &= V_0 - V'_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + (2I_0)\frac{Z_{00}}{2} \end{aligned} \quad [26]$$

Equations [26] can be used instead of [19] in cases where $\frac{Z_{\alpha 0}}{2} = Z_{0\alpha}$ and $\frac{Z_{\beta 0}}{2} = Z_{0\beta}$, thereby giving reciprocal mutual coupling between the α (or β) network and a modified 0 network in which the current is $2I_0$, the impedances are one-half 0 impedances, and the voltages are 0 voltages. This is the *z* network used in reference 7.

An alternate modification of the 0 network is obtained by rewriting [19] in terms of $2v_0$. Retaining the equations for v_α and v_β in [19] and multiplying the equation for v_0 by 2,

$$\begin{aligned} v_\alpha &= V_\alpha - V'_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + I_0 Z_{\alpha 0} \\ v_\beta &= V_\beta - V'_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + I_0 Z_{\beta 0} \\ 2v_0 &= 2(V_0 - V'_0) = I_\alpha(2Z_{0\alpha}) + I_\beta(2Z_{0\beta}) + I_0(2Z_{00}) \end{aligned} \quad [27]$$

Equations [27] apply to a modified 0 network in which currents are 0 currents, voltages are twice 0 voltages, and impedances are twice 0 impedances.

In unsymmetrical circuits in which $2Z_{0\alpha} = Z_{\alpha 0}$ and $2Z_{0\beta} = Z_{\beta 0}$, either of the modified 0 networks defined in [26] and [27] is mutually coupled with the α and β networks. Where either of the modified networks can be used equally well, the one corresponding to the *z* network[7] will be chosen in the work which follows.

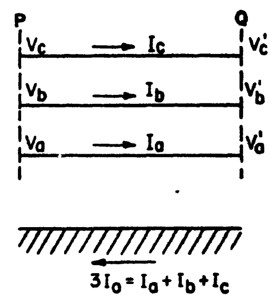


FIG. 3. Unsymmetrical series circuit between P and Q.

α, β, 0 Self- and Mutual Impedances in Terms of the Phase Impedances. Let Fig. 3 represent a general three-phase static circuit composed of bilateral circuit elements without internal voltages between points P and Q, with a return path for 0 currents. With phase voltages at P and Q referred to ground or to a neutral conductor at P and Q, respectively, the voltage drops

v_a, v_b, and v_c in phases a, b, and c in the direction of current flow are

$$\begin{aligned} v_a &= V_a - V_a' = I_aZ_{aa} + I_bZ_{ab} + I_cZ_{ac} \\ v_b &= V_b - V_b' = I_aZ_{ab} + I_bZ_{bb} + I_cZ_{bc} \\ v_c &= V_c - V_c' = I_aZ_{ac} + I_bZ_{bc} + I_cZ_{cc} \end{aligned} \qquad [28]$$

Equations [28] are general equations expressing phase voltage drops in terms of phase currents after all other currents in the circuit have been eliminated. For example, in a three-phase transmission circuit with a neutral conductor or ground wires, Z_{aa}, $Z_{ab} = Z_{ba}$, etc., may include the effects of neutral conductor or ground wires.

Replacing I_a, I_b, and I_c in [28] by their α, β, 0 components given by [7]–[9], v_a, v_b, and v_c are expressed in terms of I_α, I_β, and I_0. Substituting these equations for v_a, v_b, and v_c in [4]–[6], v_α, v_β, and v_0 are expressed in terms of I_α, I_β, I_0. Equating the coefficients of I_α, I_β, and I_0 in these resultant equations for v_α, v_β, and v_0 to the corresponding coefficients in [19], the α, β, 0 self- and mutual impedances in terms of phase impedances are

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left[Z_{aa} + \frac{Z_{bb} + Z_{cc}}{4} - \left(Z_{ab} + Z_{ac} - \frac{Z_{bc}}{2}\right)\right] \\ Z_{\beta\beta} &= \tfrac{1}{2}(Z_{bb} + Z_{cc} - 2Z_{bc}) \\ Z_{00} &= \tfrac{1}{3}[Z_{aa} + Z_{bb} + Z_{cc} + 2(Z_{ab} + Z_{ac} + Z_{bc})] \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = \frac{1}{2\sqrt{3}}[Z_{cc} - Z_{bb} + 2(Z_{ab} - Z_{ac})] \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{1}{3}[2Z_{aa} - Z_{bb} - Z_{cc} + (Z_{ab} + Z_{ac} - 2Z_{bc})] \\ Z_{\beta 0} &= 2Z_{0\beta} = \frac{1}{\sqrt{3}}(Z_{bb} - Z_{cc} + Z_{ab} - Z_{ac}) \end{aligned} \qquad [29]$$

Two Phases with Equal Self-Impedances and Equal Mutual Impedances with the Third Phase. Let $Z_{bb} = Z_{cc}$ and $Z_{ac} = Z_{ab}$. Equations [29] then become

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left[Z_{aa} + \frac{Z_{bb}}{2} - \left(2Z_{ab} - \frac{Z_{bc}}{2}\right)\right] \\ Z_{\beta\beta} &= \tfrac{1}{2}(Z_{bb} + Z_{cc} - 2Z_{bc}) \\ Z_{00} &= \tfrac{1}{3}[Z_{aa} + 2Z_{bb} + 2(2Z_{ab} + Z_{bc})] \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{2}{3}[Z_{aa} - Z_{bb} + (Z_{ab} - Z_{bc})] \end{aligned} \qquad [30]$$

Symmetrical Circuit. With all self-impedances equal to Z_{aa} and all mutual impedances equal to Z_{ab}, [29] or [30] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = Z_{aa} - Z_{ab} \\ Z_{00} &= Z_{aa} + 2Z_{ab} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = Z_{\alpha 0} = Z_{0\alpha} = 0 \end{aligned} \tag{31}$$

Unsymmetrical Three-Phase Self-Impedance Circuit with Finite 0 Self-Impedance. In a three-phase series circuit between P and Q, let the self-impedances of phases a, b, and c be Z_a, Z_b, and Z_c, respectively, with no mutual impedance between phases. The α, β, and 0 self- and mutual impedances in terms of the phase impedances can be obtained by replacing Z_{aa}, Z_{bb}, and Z_{cc} in [29] by Z_a, Z_b, and Z_c, respectively, and equating all mutual impedances between phases to zero. Then,

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left(Z_a + \frac{Z_b + Z_c}{4}\right) \\ Z_{\beta\beta} &= \frac{Z_b + Z_c}{2} \\ Z_{00} &= \frac{Z_a + Z_b + Z_c}{3} \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = \frac{Z_c - Z_b}{2\sqrt{3}} \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \frac{2Z_a - Z_b - Z_c}{3} = 2(Z_a - Z_{\alpha\alpha}) \\ Z_{\beta 0} &= 2Z_{0\beta} = \frac{Z_b - Z_c}{\sqrt{3}} \end{aligned} \tag{32}$$

Two Phases with Equal Self-Impedances. Let $Z_b = Z_c$, then [32] becomes

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left(Z_a + \frac{Z_b}{2}\right) \\ Z_{\beta\beta} &= Z_b \\ Z_{00} &= \tfrac{1}{3}(Z_a + 2Z_b) \\ Z_{\alpha\beta} &= Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \\ Z_{\alpha 0} &= 2Z_{0\alpha} = \tfrac{2}{3}(Z_a - Z_b) = 2(Z_a - Z_{\alpha\alpha}) \end{aligned} \tag{33}$$

Symmetrical Self-Impedance Circuit. Let $Z_a = Z_b = Z_c = Z$. From [32] or [33],

$$Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{00} = Z \tag{34}$$

and there are no mutual couplings between the α, β, and 0 networks. Figure 2(a), used to illustrate the flow of α and β currents and to determine α and β impedances from the α and β loop circuits, is a symmetrical self-impedance circuit.

Z_{00} *Infinite.* If there is no path for 0 currents in the unsymmetrical circuit, the α and β components of voltage drop in the circuit are given by [20]. The α and β self-impedances and the mutual impedances between the α and β networks are not affected by the presence or absence of 0 currents, nor are the mutual impedances $Z_{0\alpha}$ and $Z_{0\beta}$. The 0 voltage drops $I_\alpha Z_{0\alpha}$ and $I_\beta Z_{0\beta}$ caused by I_α and I_β flowing through an unsymmetrical circuit can be determined by calculation, if required, after I_α and I_β are known.

With $Z_{00} = \infty$, if two of the phases have equal self-impedances and equal mutual impedances (including no mutual impedance) with the other phase, substituting [30] or [33] in [20],

$$\begin{aligned} V_\alpha &= I_\alpha Z_{\alpha\alpha} \\ V_\beta &= I_\beta Z_{\beta\beta} \end{aligned} \tag{35}$$

This case is of special interest as there is no mutual coupling between the α, β, and 0 networks because of the unsymmetrical circuit.

Unsymmetrical Y-Connected Static Circuits. The α, β, and 0 self- and mutual impedances given in terms of the phase impedances by [29]–[34] apply to an unsymmetrical Y-connected circuit with grounded neutral, if I_α, I_β, and I_0 in [19] are the components of currents flowing into the circuit and v_α, v_β, and v_0 the components of voltages to neutral at the circuit terminals. Z_{00} in these equations is the 0 self-impedance between circuit terminals and neutral. If the neutral is grounded through Z_n, and Z_{00} is replaced by $Z_{00} + 3Z_n$, v_0 will be referred to ground.

If the neutral is ungrounded, equations [20] apply. The voltage at the neutral of the ungrounded Y may be evaluated as follows: α and β currents flowing in the unsymmetrical circuit produce a voltage drop v_0 between circuit terminals T and neutral N, where

$$v_0 = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} \tag{36}$$

If the 0 voltage at T is $V_{0(T)}$, the voltage V_N at N is

$$V_N = V_{0(T)} - v_0 = V_{0(T)} - I_\alpha Z_{0\alpha} - I_\beta Z_{0\beta} \tag{37}$$

$V_{0(T)}$ can be evaluated when the 0 impedance diagram and operating conditions are given. If the 0 voltage at T is zero, the voltage at the neutral is

$$V_N = -I_\alpha Z_{0\alpha} - I_\beta Z_{0\beta} \tag{38}$$

where $Z_{0\alpha}$ and $Z_{0\beta}$ are defined in [29]–[33].

If $Z_b = Z_c$; $Z_{0\beta} = 0$. Then, if $V_{0(T)} = 0$,

$$V_N = -I_\alpha Z_{0\alpha} \tag{39}$$

α and β Self-Impedances from α and β Loop Impedances. I_α flows in phase a and one-half I_α flows in phase b and one-half in phase c. The one-line α impedance diagram for determining α currents and voltages in phase a of the system is also the impedance diagram for phases b or c because $-\frac{1}{2}I_\alpha$ flows in phases b and c and the applied α voltage in these phases is $-\frac{1}{2}E_\alpha$. The α self-impedance $Z_{\alpha\alpha}$ is therefore two-thirds the α loop impedance, regardless of the type of circuit. I_β, flowing in phases b and c in series, meets an impedance which is twice the β impedance. The β self-impedance is therefore one-half the β loop impedance. In certain unsymmetrical circuits, $Z_{\alpha\alpha}$ and $Z_{\beta\beta}$ are more readily determined from the α and β loop impedances than from the phase impedances. In a circuit in which the self-impedances of phases b and c are equal and α currents induce no voltages in the β loop and β currents induce no voltages in the α loop, the α loop impedance may be determined by connecting the three phases at one terminal and applying a voltage at the other terminal between phase a and phases b and c connected to a common point; and the β loop impedance by applying a voltage between phases b and c with phase a open. In either case, the loop impedances are the ratios of the applied voltages to the resultant α and β currents, respectively, flowing in the α and β loops. Then

$$Z_{\alpha\alpha} = \tfrac{2}{3}(\alpha \text{ loop impedance}) \tag{40}$$

$$Z_{\beta\beta} = \tfrac{1}{2}(\beta \text{ loop impedance}) \tag{41}$$

In determining the α loop impedance for an unsymmetrical self-impedance static circuit in which the self-impedances are unequal in phases b and c, it should be noted (see [32]) that the α loop impedance is the sum of Z_a and $(Z_b + Z_c)/4$. Z_b and Z_c are *not* paralleled.

α and β Applied Voltages Determined from α and β Loop Voltages. Equations [14] give α, β, 0 voltages in terms of the applied phase voltages. In unsymmetrical circuits involving transformers, the voltages applied to the α and β loops may be more readily obtained than the phase voltages. In such cases, the applied α voltage is two-thirds the voltage applied to the α loop; the applied β voltage is $1/\sqrt{3}$ times the voltage applied to the β loop.

EQUIVALENT CIRCUITS TO REPLACE AN ACTUAL CIRCUIT IN THE α, β, AND 0 NETWORKS

Synchronous Machine with Equal Positive- and Negative-Sequence Impedances. From [23], the α and β self-impedances are equal to Z_1, and the 0 self-impedance to Z_0. There are no mutual impedances between the α, β, and 0 networks. With balanced generated voltages in the machine, the generated voltage in the α network from [15] is E_a; in the β network it is $-jE_a$. The α, β, and 0 equivalent circuits for a synchronous machine with balanced generated voltages and equal positive- and negative-sequence impedances are shown in Fig. 4.

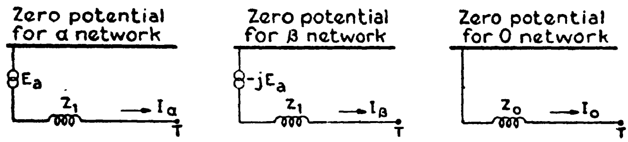


FIG. 4. α, β, and 0 equivalent circuits for a synchronous machine with balanced generated voltages and equal positive- and negative-sequence impedances.

Points T are the terminals of the machine to which the equivalent α, β, and 0 circuits for the rest of the system are to be connected.

In a three-phase power system consisting of symmetrical circuits with equal positive- and negative-sequence impedances, the one-line impedance diagrams for the α and β systems are the same as the positive-sequence impedance diagram. Generated α voltages are positive-sequence generated voltages; the α network is the same as the positive-sequence network. The β network differs from the positive-sequence network only in its generated voltages, which are positive-sequence voltages multiplied by $-j$.

Symmetrical Circuit with Unequal Positive- and Negative-Sequence Impedances. From [21] with $Z_1 \neq Z_2$ and all sequence mutual impedances zero,

$$\begin{aligned} Z_{\alpha\alpha} &= Z_{\beta\beta} = \tfrac{1}{2}(Z_1 + Z_2) \\ Z_{00} &= Z_0 \\ Z_{\alpha\beta} &= -Z_{\beta\alpha} = j\tfrac{1}{2}(Z_1 - Z_2) \\ Z_{\alpha 0} &= Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \end{aligned} \qquad [42]$$

When the positive- and negative-sequence self-impedances of a circuit are unequal and there are no sequence mutual impedances, the α and β self-impedances are the average of the positive- and negative-sequence impedances. There is no mutual coupling with the 0 network; but the α and β networks are coupled through *non-reciprocal* mutual impedances. Because of this non-reciprocal coupling between the α and β networks in rotating machines in which $Z_1 \neq Z_2$, the α, β, 0 components are not convenient for determining fundamental-frequency currents and voltages in systems in which the positive- and negative-sequence impedances cannot be assumed equal. However, if there is but one machine or group of machines in which $Z_1 \neq Z_2$, [19] can be rewritten to give a reciprocal mutual coupling between the α network and a *modified* β network, from which an equivalent circuit can be obtained.

Modified β Network. Substituting $Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$ from [42] in [19], the α, β, and 0 components of voltage drop in the circuit in the direction of current flow are

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} = I_\alpha \tfrac{1}{2}(Z_1 + Z_2) + jI_\beta \tfrac{1}{2}(Z_1 - Z_2) \\ v_\beta &= I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} = -jI_\alpha \tfrac{1}{2}(Z_1 - Z_2) + I_\beta \tfrac{1}{2}(Z_1 + Z_2) \\ v_0 &= I_0 Z_0 \end{aligned} \qquad [43]$$

Rewriting v_α and v_β in [43] in terms of $(-I_\beta)$, with $-Z_{\alpha\beta}$ replaced by $Z_{\beta\alpha}$,

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + (-I_\beta)(-Z_{\alpha\beta}) = I_\alpha Z_{\alpha\alpha} + (-I_\beta)Z_{\beta\alpha} \\ &= I_\alpha(Z_{\alpha\alpha} - Z_{\beta\alpha}) + (I_\alpha - I_\beta)Z_{\beta\alpha} \\ v_\beta &= I_\alpha Z_{\beta\alpha} + (-I_\beta)(-Z_{\beta\beta}) \\ &= (-I_\beta)(-Z_{\beta\beta} - Z_{\beta\alpha}) + (I_\alpha - I_\beta)Z_{\beta\alpha} \end{aligned} \qquad [44]$$

Retaining the equation for v_α in [43] but rewriting that for v_β,

$$\begin{aligned} v_\alpha &= I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} = I_\alpha(Z_{\alpha\alpha} - Z_{\alpha\beta}) + (I_\alpha + I_\beta)Z_{\alpha\beta} \\ -v_\beta &= I_\alpha(-Z_{\beta\alpha}) + I_\beta(-Z_{\beta\beta}) = I_\alpha Z_{\alpha\beta} + I_\beta(-Z_{\beta\beta}) \\ &= I_\beta(-Z_{\beta\beta} - Z_{\alpha\beta}) + (I_\alpha + I_\beta)Z_{\alpha\beta} \end{aligned} \qquad [45]$$

In [44] and [45] the mutual impedances between the α network and a modified β network are reciprocal. In [44], v_β has been retained, but I_β has been replaced by $(-I_\beta)$ flowing in the direction assumed as positive for I_β. In [45], I_β has been retained but v_β has been replaced by $-v_\beta$ measured in the direction of v_β.

Equivalent Circuits for a Synchronous Machine with $Z_1 \neq Z_2$. Figures 5(*a*) and (*b*) give equivalent circuits to replace a synchronous machine with balanced generated voltages in the α and β networks when $Z_1 \neq Z_2$. The α and β components of balanced generated voltage are given by [15]. Figures 5(a) and (b) satisfy the equations for the α and β components of voltage drop in the direction of current flow given by [44] and [45], respectively, where $Z_{\alpha\alpha}$, $Z_{\beta\beta}$, $Z_{\alpha\beta}$, and $Z_{\beta\alpha}$ are defined in [42]. In both equivalent circuits, currents and voltages in

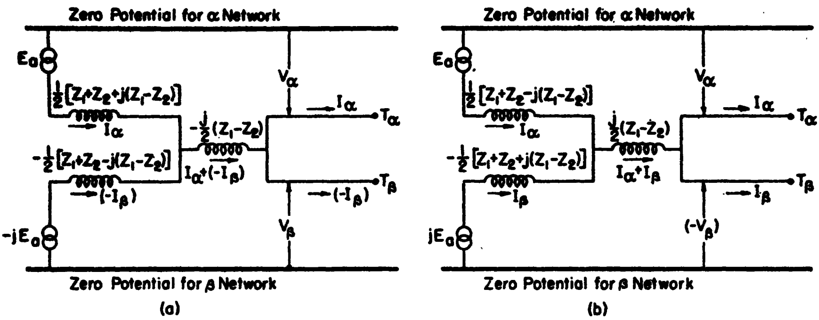


FIG. 5. Equivalent circuits to replace a synchronous machine with $Z_1 \neq Z_2$ in the α and modified β network. T_α and T_β are the machine terminals to which the α and β networks for the rest of the system are to be connected after all β impedances have been multiplied by -1. (a) Currents in the β network are negative β currents; voltages are β voltages. (b) Voltages in the β network are negative β voltages; currents are β currents.

the α network are correctly represented. In Fig. 5(a) and equations [44], current in the β network is $-I_\beta$, giving a modified β network in which voltages are β voltages. In Fig. 5(b) and equations [45], voltages in the β network are negative β voltages, giving a modified β network in which currents are β currents. The generated voltage in the modified β network of Fig. 5(b) becomes $-(-jE_a) = jE_a$, as indicated. The points T_α and T_β are the terminals of the synchronous machine to which the α and β networks, respectively, for the system exclusive of the synchronous machine are to be connected, after all impedances in the β network have been multiplied by -1. If the impedances in the β network include resistances, negative resistances will be present in the network; capacitive reactances will become inductive reactances, and vice versa. The modification of the β network presents no difficulties in an analytic solution.

The equivalent circuit for the synchronous machine with $Z_1 \neq Z_2$ in the 0 network is the same as that given in Fig. 4.

As α, β, and 0 components will be used instead of symmetrical components only if calculations are simplified by their use, the equivalent synchronous machine circuits of Figs. 5(a) and (b), which are much less simple than the symmetrical component equivalent circuits, have but limited application. They can sometimes be used to advantage in solutions of special problems involving unsymmetrical circuits and unsymmetrical faults, where the conditions of the problem do not require that the β network be coupled with the 0 network, or have a second coupling with the α network.

In approximate solutions, where the mutual impedance $\pm j(Z_1 - Z_2)$ is omitted, $\frac{1}{2}(Z_1 + Z_2)$ should be used as the self-impedance in both the α and β networks.

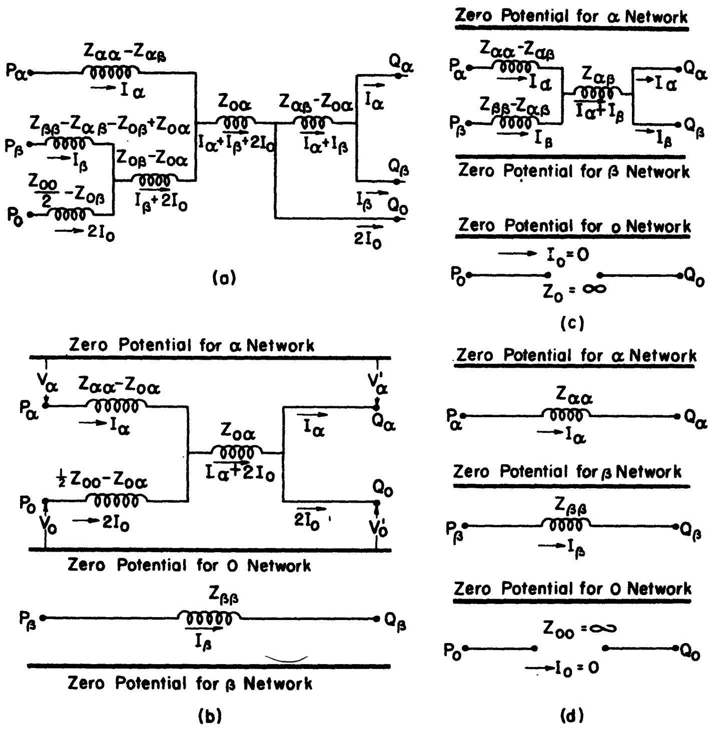


FIG. 6. Equivalent circuits for unsymmetrical series circuit in which (a) $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha0} = 2Z_{0\alpha}$, $Z_{\beta0} = 2Z_{0\beta}$. (b) $Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta0} = Z_{0\beta} = 0$, and $Z_{\alpha0} = 2Z_{0\alpha}$. (c) $Z_{00} = \infty$, $Z_{\alpha\beta} = Z_{\beta\alpha}$. (d) $Z_{00} = \infty$, $Z_{\alpha\beta} = Z_{\beta\alpha} = 0$.

Equivalent Circuits for Unsymmetrical Three-Phase Static Circuits. In [29] and [32], $Z_{\alpha\beta} = Z_{\beta\alpha}$, $Z_{\alpha0} = 2Z_{0\alpha}$, and $Z_{\beta0} = 2Z_{0\beta}$. An equivalent circuit which satisfies equations [26] for the general case of unequal self- and mutual impedances (including no mutual impedances) is shown in Fig. 6(*a*). *P* and *Q* with subscripts α, β, and 0 indicate the terminals of the equivalent circuit in the α, β, and 0 networks, respectively, to which the equivalent α, β, 0 networks for the rest of the system are to be connected after all 0 impedances have been divided by 2. Figure 6(a) is tested for correct self-impedances in each network by opening both of the other networks at P or Q. It is tested for correct mutual impedance between any two networks by opening the third network at P or Q. When an a-c network analyzer is available, mutual coupling between the networks can be obtained by either mutual coupling transformers or direct connections as in Fig. 6(a).

For the special case of two phases symmetrical with respect to the third phase, the α, β, 0 self- and mutual impedances are given by [30] and [33]. In these equations there is no mutual coupling between the β network and either the α or 0 networks. The equivalent circuits for this case are shown in Fig. 6(b).

With $Z_{00} = \infty$, the zero network is open between P and Q so that no 0 current flows into or out of the circuit at either P or Q. The equivalent circuits for this case can be determined from Figs. 6(a) and (b) by opening the 0 network at P or Q, giving the equivalent circuits shown in Figs. 6(c) and (d), respectively.

Comparing equations [29], [30], [32], and [33], with equations [13], [17], [18], and [19], respectively, of Chapter VIII, it may be seen that non-reciprocal mutual impedances between the symmetrical component networks because of unsymmetrical static circuits become reciprocal mutual impedances between the α, β, 0 networks; while reciprocal mutual impedances between symmetrical component networks become zero between the β network and both the α and 0 networks. This may also be seen from equations [24] and [25].

Δ-CONNECTED CIRCUITS

Line Currents and Line-to-Neutral Voltages on Opposite Sides of a Δ–Y Transformer Bank. The difference in phase of positive-sequence line-to-neutral voltages on opposite sides of the bank at no load can be determined by inspection when the connection diagram is given. (See Chapter III, Fig. 19.) If positive-sequence components of line current and voltage to neutral are turned forward 90° and negative-sequence components of current and voltage backward 90° in passing through the bank,

$$\begin{aligned} V_\alpha &= (V_{a1} + V_{a2}) \text{ becomes } j(V_{a1} - V_{a2}) = -V'_\beta \\ V_\beta &= -j(V_{a1} - V_{a2}) \text{ becomes } (V_{a1} + V_{a2}) = V'_\alpha \end{aligned} \qquad [46]$$

If the shift in phase of positive-sequence components is backward 90°

and that of negative-sequence components forward 90°,

$$\begin{aligned} V_\alpha &= (V_{a1} + V_{a2}) \text{ becomes } -j(V_{a1} - V_{a2}) = V'_\beta \\ V_\beta &= -j(V_{a1} - V_{a2}) \text{ becomes } -(V_{a1} + V_{a2}) = -V'_\alpha \end{aligned} \qquad [47]$$

Replacing V by I in [46] and [47] the corresponding current equations are obtained. When the connection diagram is not given, it is immaterial whether [46] or [47] is used when the *relative phases* of currents and voltages on the two sides of the bank are not required. (See Chapter III, Problem 6.) In system studies in which the positive- and negative-sequence impedances of rotating machines can be assumed equal, the α and β impedance diagrams of the system, exclusive of unsymmetrical circuits, are the same as the positive-sequence impedance diagram. The shift of components from β to α and α to β presents no difficulty where the *same impedances* are met by α and β currents. Dissymmetries on opposite sides of a Δ-Y transformer bank will be discussed later.

Voltages and Currents of Δ-Connected Circuits. As the fundamental-frequency currents or voltages in the three phases of a Δ constitute a set of three vectors, they can be replaced by their α, β, 0 components. There will be no 0 components of line-to-line voltage, but there may be 0 components of Δ current. The three phases of the Δ will be indicated by A, B, and C, with A, B, and C opposite terminals a, b, and c, respectively, as in Fig. 7. Base Δ voltage is $\sqrt{3}$ times base line-to-neutral voltage, base Δ current is $1/\sqrt{3}$ times base line current, and base Δ impedance is 3 times base line-to-neutral impedance. In per unit of base line-to-line voltage, the Δ phase voltages in terms of their components are

$$\begin{aligned} V_A &= V'_\alpha \\ V_B &= -\tfrac{1}{2}V'_\alpha + \frac{\sqrt{3}}{2}V'_\beta \\ V_C &= -\tfrac{1}{2}V'_\alpha - \frac{\sqrt{3}}{2}V'_\beta \end{aligned} \qquad [48]$$

where V'_α and V'_β indicate components of line-to-line voltage in per unit of base line-to-line voltage. Using a similar notation for components of Δ current, the current equations are of the form of [7]–[9].

Relations between per Unit α and β Components of Current and Voltage in the Line and in the Δ. Line-to-line voltages are given by [13] in terms of α and β components of line-to-neutral voltage. Dividing equations [13] by $\sqrt{3}$ and replacing V_{cb} by V_A, V_{ac} by V_B, and

V_{ba} by V_C, in per unit of base line-to-line voltage,

$$\begin{aligned} V_A &= V_{cb} = V_\beta \\ V_B &= V_{ac} = -\frac{\sqrt{3}}{2} V_\alpha - \tfrac{1}{2} V_\beta \\ V_C &= V_{ba} = \frac{\sqrt{3}}{2} V_\alpha - \tfrac{1}{2} V_\beta \end{aligned} \qquad [49]$$

Comparing α and β components of line-to-neutral and line-to-line voltages, by eliminating V_A, V_B, and V_C in [48] and [49],

$$\begin{aligned} V'_\alpha &= V_\beta \\ V'_\beta &= -V_\alpha \end{aligned} \qquad [50]$$

where V'_α and V'_β are in per unit of base line-to-line voltage, and V_α and V_β in per unit of base line-to-neutral voltage.

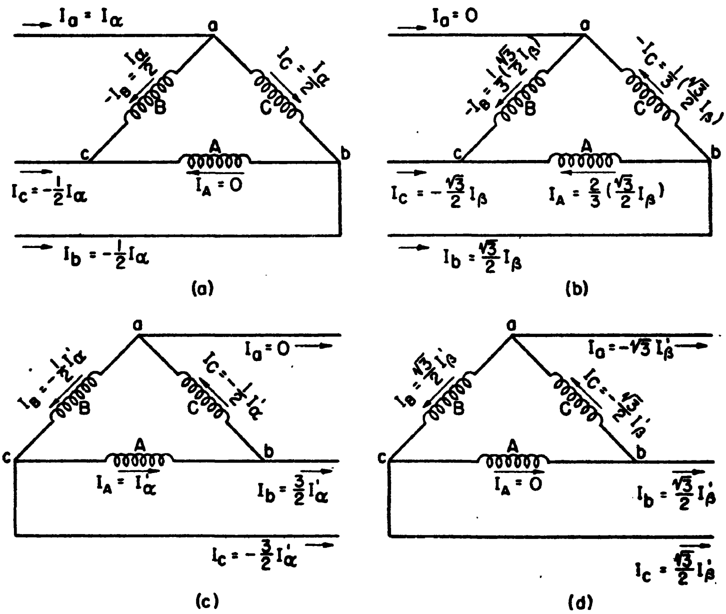


FIG. 7. (*a*) α components of line currents flowing into Δ circuit. (*b*) β components of line currents flowing into Δ circuit. (*c*) α components of Δ currents flowing into the line. (*d*) β components of Δ currents flowing into the line. In (*a*) and (*b*), base current is line current; in (*c*) and (*d*), base current is Δ current.

In Figs. 7(*a*) and (*b*), currents in per unit of *base line current* are shown in the line and in the Δ. In Fig. 7(*a*) there are only α components (I_α), in Fig. 7(*b*) only β components (I_β). In Figs. 7(*c*) and (*d*), currents in per unit of *base Δ current* are shown in the Δ and in the line.

In Fig. 7 (c) there are only α components (I'_α); in Fig. 7 (d) only β components (I'_β). Let line-to-line voltages in the Δ measured in the directions cb, ba, and ac represent voltage rises, the direction cba around the Δ corresponding to increasing potentials. Following the convention that positive direction for currents in a circuit is in the direction of increasing potential when currents flow *from the circuit*, and in the direction of decreasing potential when currents flow *towards a circuit*, a minus sign before a phase current in the Δ in Figs. 7 (a) or (b) means that the direction indicated by arrow is negative for the convention used.

Multiplying Δ currents in per unit of base line-to-neutral current by $\sqrt{3}$ to express them in per unit of base Δ current, I'_β in the Δ of Fig. 7 (a) is

$$I'_\beta = \frac{1}{\sqrt{3}}(I_B - I_C) = \frac{1}{\sqrt{3}}\left[-\frac{\sqrt{3}}{2}I_\alpha - \frac{\sqrt{3}}{2}I_\alpha\right] = -I_\alpha$$

Similarly, in Fig. 7 (b),

$$I'_\alpha = \tfrac{2}{3}\left(I_A - \frac{I_B + I_C}{2}\right) = \tfrac{2}{3}\sqrt{3}\left(\frac{\sqrt{3}}{2}I_\beta\right) = I_\beta$$

Multiplying line currents given in per unit of base line-to-line current in Figs. 7 (c) and (d) by $1/\sqrt{3}$ to express them in per unit of base line current,

$$I_\beta = \frac{1}{\sqrt{3}}(I_b - I_c) = \tfrac{1}{3}(3I'_\alpha) = I'_\alpha$$

$$I_\alpha = \tfrac{2}{3}\left(I_a - \frac{I_b + I_c}{2}\right) = \tfrac{2}{3} \times \frac{1}{\sqrt{3}}(-\tfrac{3}{2}\sqrt{3}I'_\beta) = -I'_\beta$$

Expressed in per unit each on their respective base voltages and currents, α and β components of Δ voltages and currents are equal to β and $-\alpha$ components of line-to-neutral voltages and line currents, respectively.

α and β Impedances of Δ-Connected Circuits. In a symmetrical static Δ, let the impedance of each phase be Z in per unit, based on system kva per phase and system line-to-line voltage; then, based on line-to-neutral voltage and system kva per phase, the Δ impedances are $3Z$. The α loop impedance in Fig. 7 (a) is $\frac{1}{2}(3Z) = \frac{3}{2}Z$. The self-impedance $Z_{\alpha\alpha}$ from [40] is $\frac{2}{3}(\frac{3}{2}Z) = Z$. The β loop impedance in Fig. 7 (b) is $3Z(6Z)/9Z = 2Z$. The self-impedance $Z_{\beta\beta}$ from [41] is $\frac{1}{2}(2Z) = Z$. This also follows directly from [23] in which $Z_{\alpha\alpha} = Z_{\beta\beta} = Z_1$, for the symmetrical circuit in which $Z_1 = Z_2$.

An *unsymmetrical* Δ*-connected self-impedance circuit* can be replaced by its equivalent Y with per unit impedances based on line-to-neutral voltage for determining voltages and currents at its terminals. After the α and β voltages at the terminals of the equivalent Y have been calculated, the line-to-line voltages in per unit of base line-to-neutral voltage can be obtained from [13]. These line-to-line voltages divided by the corresponding Δ impedances, expressed in per unit based on line-to-neutral voltage, give the Δ currents in per unit of base line current.

In considering unsymmetrical Δ-connected circuits in which two phases have equal self-impedances and equal mutual impedances (including no mutual impedance) with the other phase, the loop impedances offered to α and β line currents viewed from the circuit terminals are conveniently calculated in terms of base impedance in the line-to-neutral circuit. From [40] and [41], the α and β self-impedances can then be determined from the loop impedances. This is illustrated below.

Open-Δ Transformer Bank. Assume that exciting currents can be neglected and that the open phase is opposite terminal a, as in Fig. 12(a), Chapter VIII. Referring to Fig. 7(a), no α current flows in phase A opposite terminal a. The α impedance therefore is not changed because of the open phase. In Fig. 7(b) with phase A open, the β impedance is increased; the β self-impedance is one-half the impedance of phases C and B in series.

Bank of Two Identical Transformers. Let the per unit leakage impedance of the transformers be Z_t, based on system kva per phase and base line-to-line voltage. The α and β self-impedance in per unit, based on system kva per phase and base line-to-neutral voltage, are

$$\begin{aligned} Z_{\alpha\alpha} &= \tfrac{2}{3}\left(\frac{3Z_t}{2}\right) = Z_t \\ Z_{\beta\beta} &= \tfrac{1}{2}(3Z_t + 3Z_t) = 3Z_t \end{aligned} \qquad [51]$$

There is no mutual impedance between the α and β networks.

To illustrate the usefulness of [22] in passing from α and β components to symmetrical components, [51] will be substituted in [22], to obtain the positive- and negative-sequence self- and mutual impedances of the open-Δ transformer banks.

$$\begin{aligned} Z_{11} &= Z_{22} = 2Z_t \\ Z_{12} &= Z_{21} = -Z_t \end{aligned} \qquad [52]$$

Equations [52] check equations [61] of Chapter VIII.

Short Circuits on Systems Containing an Unsymmetrical Circuit

With phases a, b, and c specified in defining the unsymmetrical circuit, the connections of the component networks for faults of a given type will depend upon the phase or phases faulted. Table I gives relations between V_α, V_β, and V_0, the α, β, and 0 components of phase voltages to ground (or to the neutral conductor) at the fault, and between I_α, I_β, and I_0, the components of line currents flowing from the system into the fault for faults involving any phases. The following solution illustrates the procedure followed in determining the equations of Table I.

Double Line-to-Ground Fault on Phases a and b. Conditions at the Fault: $V_a = V_b = 0$; $I_c = 0$. Substituting $V_a = V_b = 0$ in [4]–[6] and $I_c = 0$ in [9],

$$V_\alpha = -V_0 = \frac{V_\beta}{\sqrt{3}} \qquad [53]$$
$$I_\alpha = -\sqrt{3}I_\beta + 2I_0$$

This is Case IV(b) of Table I.

TABLE I

SHORT CIRCUIT ON THREE-PHASE SYSTEM

Relations between α, β, and 0 Components of Voltages and Currents at the Fault

Case	Type of Fault	Phases Involved	Equations for Components of Voltage at Fault	Equations for Components of Currents Flowing into Fault
I(a)	Three-phase	a, b, c	$V_\alpha = 0$; $V_\beta = 0$	$I_0 = 0$
(b)		a, b, c and ground	$V_\alpha = 0$; $V_\beta = 0$, $V_0 = 0$	
II(a)	Line-to-ground	a and ground	$V_\alpha = -V_0$	$I_\beta = 0$; $I_\alpha = 2I_0$
(b)	Line-to-ground	b and ground	$V_\alpha = \sqrt{3}V_\beta + 2V_0$	$I_\alpha = -\dfrac{I_\beta}{\sqrt{3}}$; $I_\alpha = -I_0$
(c)	Line-to-ground	c and ground	$V_\alpha = -\sqrt{3}V_\beta + 2V_0$	$I_\alpha = \dfrac{I_\beta}{\sqrt{3}}$; $I_\alpha = -I_0$
III(a)	Line-to-line	b and c	$V_\beta = 0$	$I_\alpha = 0$; $I_0 = 0$
(b)	Line-to-line	a and b	$V_\alpha = \dfrac{V_\beta}{\sqrt{3}}$	$I_\alpha = -\sqrt{3}I_\beta$; $I_0 = 0$
(c)	Line-to-line	a and c	$V_\alpha = -\dfrac{V_\beta}{\sqrt{3}}$	$I_\alpha = \sqrt{3}I_\beta$; $I_0 = 0$
IV(a)	Two line-to-ground	b, c, and ground	$V_\beta = 0$; $V_\alpha = 2V_0$	$I_\alpha = -I_0$
(b)	Two line-to-ground	a, b, and ground	$V_\alpha = \dfrac{V_\beta}{\sqrt{3}}$; $V_\alpha = -V_0$	$I_\alpha = -\sqrt{3}I_\beta + 2I_0$
(c)	Two line-to-ground	a, c, and ground	$V_\alpha = -\dfrac{V_\beta}{\sqrt{3}}$; $V_\alpha = -V_0$	$I_\alpha = \sqrt{3}I_\beta + 2I_0$

Connections of the α, β, 0 Networks to Represent an Unsymmetrical Circuit and a Short Circuit — No Intervening Δ-Y Transformer Bank. Direct connections of the α, β, and 0 networks to satisfy the fault conditions of Table I are shown in Fig. 8. The fault is at F, the unsymmetrical circuit between P and Q. Mutual impedances between the component networks because of the unsymmetrical circuit are not indicated, but they may be present. For simplicity, two synchronous

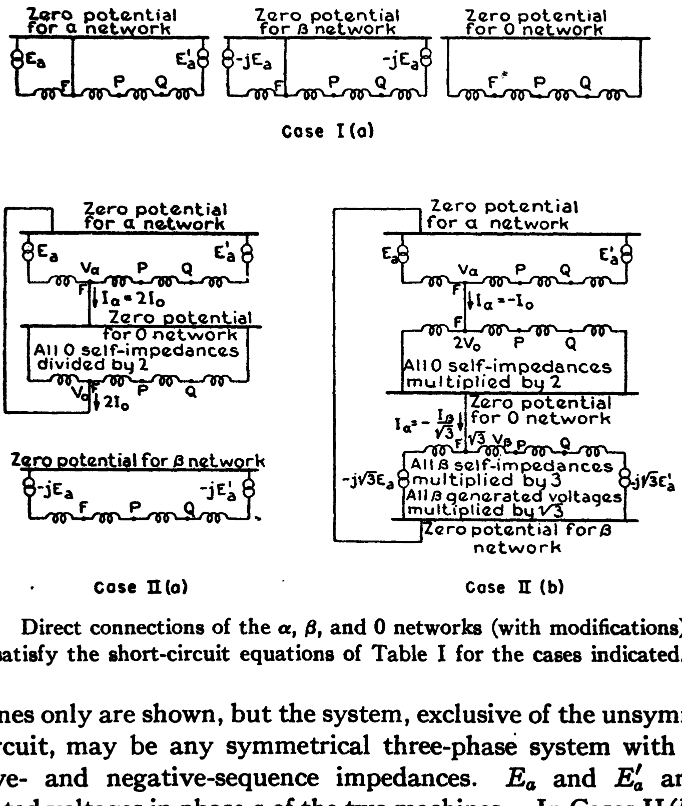


FIG. 8. Direct connections of the α, β, and 0 networks (with modifications) which satisfy the short-circuit equations of Table I for the cases indicated.

machines only are shown, but the system, exclusive of the unsymmetrical circuit, may be any symmetrical three-phase system with equal positive- and negative-sequence impedances. E_a and E_a' are the generated voltages in phase a of the two machines. In Cases II(b) and (c), and in Cases IV(b) and (c) of Table I, the β network as well as the 0 network is modified. Cases I(b), II(c), III(c) and IV(c) are not shown in Fig. 8. Case I(b) is similar to $I(a)$ except that F in the 0 network is shorted to the zero-potential bus for the 0 network. Connections for Cases II(c), III(c), and IV(c) are similar to those for Cases II(b), III(b), and IV(b), respectively, except that connections to F and the zero-potential bus of the β network are reversed. As alternate methods, the connections in Case II(b) may be retained for Case

II(c) but all generated β voltage multiplied by $-\sqrt{3}$; then currents in the β network are $-1/\sqrt{3}$ times β currents, and voltages are $-\sqrt{3}$ times β voltages. The connections for Cases III(b) and IV(b) may be

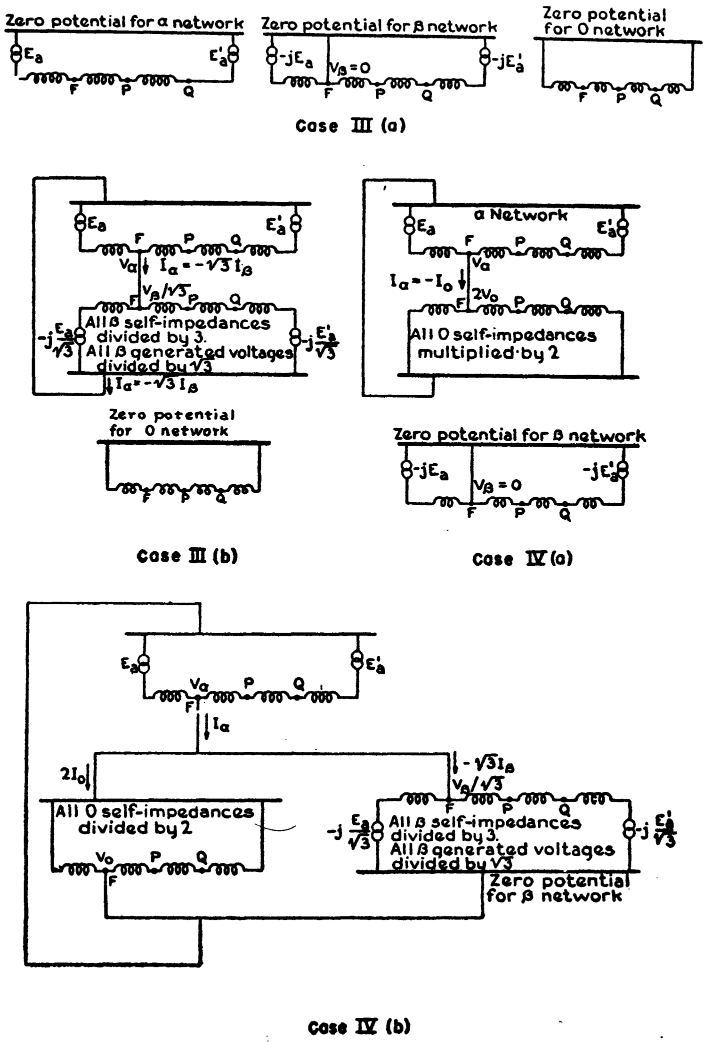


FIG 8 (*Continued*)

retained for Cases III(c) and IV(c), respectively, but all generated β voltages divided by $-\sqrt{3}$; then currents and voltages in the β network are $-\sqrt{3}$ and $-1/\sqrt{3}$ times β currents and voltages, respectively.

In any case where two networks are not directly connected to satisfy the unsymmetrical circuit conditions, they can be directly connected to satisfy fault conditions, and vice versa. For example, in Cases III (a), (b), and (c), the α and 0 networks are not connected to satisfy fault equations. These networks can therefore be directly connected to satisfy unsymmetrical circuit equations. A relatively large number of unsymmetrical circuits are symmetrical with respect to one phase.

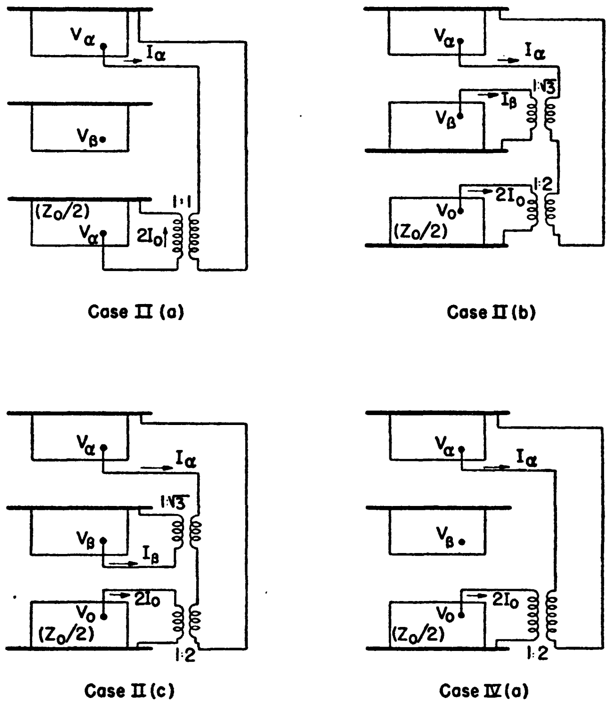


FIG. 9. Connections of the α, β, and 0 networks through isolating transformers on an a-c network analyzer which satisfy the short-circuit equations of Table I for the cases indicated.

When the zero-sequence impedances of such circuits is infinite, there is no mutual connection between the α, β, and 0 networks. The open-Δ transformer bank of two identical units is an example of a circuit requiring no connection between the component networks because of the unsymmetrical circuit.

In a system in which the positive- and negative-sequence impedances of the rotating machines can be assumed equal, the impedances of the β network differ from those of the α network only in the unsymmetrical circuit; therefore, whenever the β network is not connected with the α or 0 networks, positive-plus-negative and positive-minus-negative components described in Chapter V can be used instead of α and β components. The positive-minus-negative sequence network is the positive-plus-negative sequence network with $Z_{\beta\beta}$ replacing $Z_{\alpha\alpha}$ between P and Q.

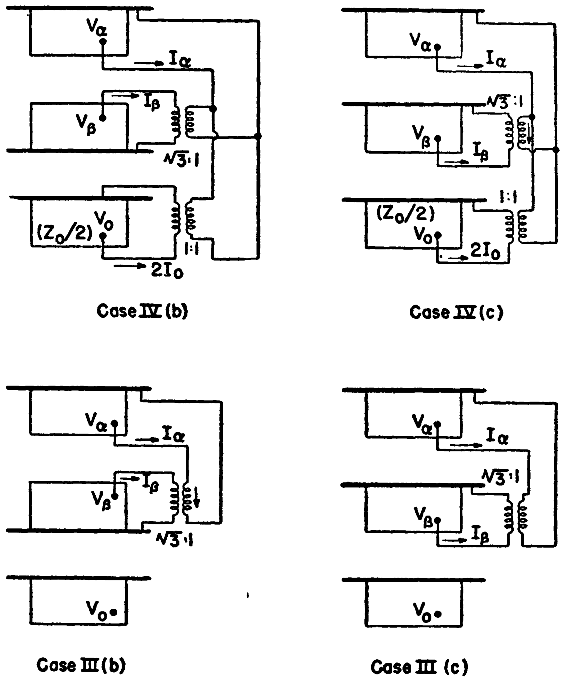


FIG. 9 (*Continued*)

Connections of the α, β, and 0 networks through isolating transformers[7] to satisfy the equations of Table I are shown in Fig. 9. These equivalent circuits are for use on an a-c network analyzer. No direct connections have been made between the component networks in these equivalent circuits. This makes it possible to apply direct connections to represent an unsymmetrical circuit or a second unsymmetrical fault. In the modified zero-sequence network of Fig. 9, all 0 impedances are divided by 2, voltages are 0 voltages, and currents are twice 0 currents. Connections to represent an unsymmetrical circuit or a second fault will require the same modification of the zero-sequence network. In addition to transformers of 1 : 1 turn ratio, transformers of $\sqrt{3}:1$ and $2:1$ turn ratios are required. Transformers of $1:1$ turn ratio in Fig. 9 can be replaced by direct connections between the α network and the modified 0 network when there are no other direct connections between these networks. With one dissymmetry represented by direct connections between two component networks, direct connections to represent a second dissymmetry can sometimes be made. This is illustrated in Chapter VII, Fig. 6, where no untrue relations are introduced by the second set of direct connections.

OPEN CONDUCTORS

Figures 10(a) and (b) show one and two open conductors, respectively. Let v and I' with appropriate subscripts represent voltage drop between F' and F, and line current flowing across the opening from F' to F, respectively. Here v is a series voltage drop and I' meets series impedance; v and I' are to be distinguished from V and I used to indicate voltages to ground at a fault and currents flowing from the phases into a fault. Table II gives equations expressing rela-

TABLE II

One or Two Open Conductors in a Three-Phase Circuit

Relations between α, β, and 0 Components of Series Voltages v across Openings and Line Current I' Flowing through Openings

Case	Open Phases	Equations Relating Components of Voltages across Openings	Equations Relating Components of Line Currents in Openings
I(a)	a	$v_\beta = 0;\quad v_\alpha = 2v_0$	$I'_\alpha = -I'_0$
I(b)	b	$v_\alpha = -\dfrac{v_\beta}{\sqrt{3}} = -v_0$	$I'_\alpha - \sqrt{3}I'_\beta - 2I'_0 = 0$
I(c)	c	$v_\alpha = \dfrac{v_\beta}{\sqrt{3}} = -v_0$	$I'_\alpha + \sqrt{3}I'_\beta - 2I'_0 = 0$
II(a)	b and c	$v_\alpha = -v_0$	$I'_\alpha = 2I'_0;\quad I'_\beta = 0$
II(b)	a and b	$v_\alpha + \sqrt{3}\,v_\beta - 2v_0 = 0$	$I'_\alpha = \dfrac{I'_\beta}{\sqrt{3}} = -I'_0$
II(c)	a and c	$v_\alpha - \sqrt{3}\,v_\beta - 2v_0 = 0$	$I'_\alpha = -\dfrac{I_\beta}{\sqrt{3}} = -I'_0$

tions between v_α, v_β, and v_0, the components of voltage drop between F' and F, and between I'_α, I'_β, and I'_0, the components of line current flowing from F' to F. The following solution illustrates the procedure followed in determining the equations of Table II.

Conductor b Open. Conditions at the fault: $v_a = v_c = 0$; $I_b = 0$. Substituting $v_a = v_c = 0$ in [1] and [3], and $I_b = 0$ in [8],

$$\begin{aligned} v_\alpha &= -\frac{v_\beta}{\sqrt{3}} = -v_0 \\ I'_\alpha &- \sqrt{3}I'_\beta - 2I'_0 = 0 \end{aligned} \qquad [54]$$

This is Case $I(b)$ of Table II.

Connection of the α, β, and 0 Networks to Represent Open Conductors. Figures 10(c) and (d) show *direct connections* of the α and 0 networks to satisfy Cases I(a) and II(a), respectively, of Table II.

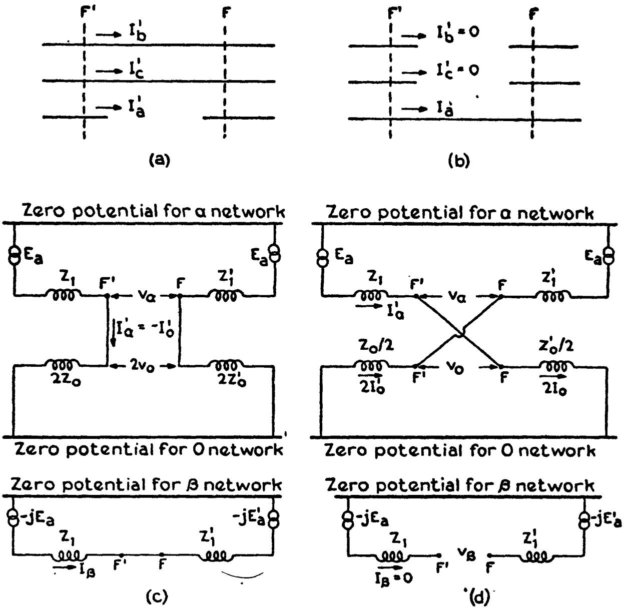


FIG. 10. (a) One open conductor. (b) Two open conductors. (c) Connection of α, β, and 0 equivalent circuits for open conductor in phase a. (d) Connection of α, β, and 0 equivalent circuits for open conductors in phases b and c.

For simplicity two generators only are indicated. The system could be any system with equal positive- and negative-sequence impedances. Direct connection of the α and 0 networks is made possible by multiplying the 0 impedances in Fig. 10(c) by 2 and dividing them by 2 in Fig. 10(d). In Fig. 10(c), the currents in the 0 network are 0 currents but the voltages are twice 0 voltages; in Fig. 10(d) the voltages are 0 voltages but the currents are twice 0 currents. The β network is unaffected by one open conductor (phase a). It is open for two open conductors (phases b and c).

Direct connections of the component networks to satisfy the equations of Table II, Cases I(b) and (c), and II(b) and (c) are not shown. They can be made if both the β and 0 networks are modified. (See Fig. 8.)

For the special case of *infinite series impedance in the zero-sequence network*, as when open conductors occur in a three-phase circuit connected at one set of terminals to a Δ or an ungrounded Y, $I_0' = 0$. From Table II, Cases I(a) and II(a), (b), and (c) if $I_0' = 0$, $I_\alpha' = 0$; the α and 0 networks are both open between F' and F. With two

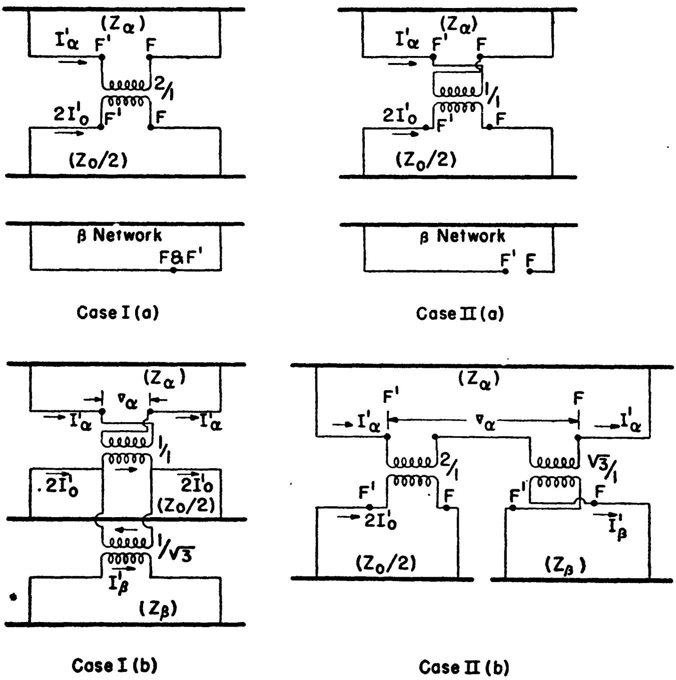


FIG. 11. Connections of α, β, and 0 networks through isolating transformers on an a-c network analyzer which satisfy the open-conductor equations of Table II for the cases indicated. I(c) and II(c) (not shown) are similar to I(b) and II(b), respectively, except that the connections to the β network are reversed.

conductors open, there is no current between F and F'. With conductor a open, currents flow between F and F' in the β circuit only. For these cases, there will be no interconnection of the α, β, and 0 networks because of open conductors.

Figure 11 gives equivalent circuits which satisfy[7] the equations of

Table II for use on an a-c network analyzer. The modified 0 network in these equivalent circuits is the same as that used in Fig. 9. No direct connections between component networks are indicated in Fig. 11. Direct connections between the component networks to represent an unsymmetrical circuit or a second fault can therefore be made.

SIMULTANEOUS SHORT CIRCUITS AND OPEN CONDUCTORS

If the positive- and negative-sequence impedances of the rotating machines of the system can be assumed equal and the effects of saturation neglected, all possible combinations of simultaneous dissymmetries — short circuits, open conductors, unsymmetrical circuits — can be represented by equivalent circuits on an a-c network analyzer. The use of isolating transformers to satisfy simultaneous equations, as illustrated in Figs. 9 and 11, makes it possible to interconnect the component networks to satisfy all the dissymmetries simultaneously without introducing untrue restrictions.

In analytic calculations, direct connections between the component networks to satisfy equations for one dissymmetry can be made. (See Figs. 8 and 10.) By using various modifications of the β and 0 networks, many simultaneous dissymmetries can be represented by direct connections of the component networks. When there are two dissymmetries and the modifications required to permit direct connections of the component networks for one dissymmetry are not those required for the second dissymmetry, an analytic solution can be made with equations. The use of equations is illustrated below in the case of an open conductor and a line-to-ground fault on the same phase, where an equivalent circuit to replace both these dissymmetries in the α network is developed.

Open Conductor and Fault to Ground through Impedance on the Same Phase. Let it be assumed that the conductor is open between F' and F and that the fault is at F on phase a through impedance Z_f to ground as in Fig. 12(a). The conditions at the fault are

$$I_b = I_c = 0; \quad V_a = V_\alpha + V_0 = I_a Z_f = (I_\alpha + I_0)Z_f$$

From these equations and those of Table II, Case I(a), the following relations exist between the components of I_a and V_a at the fault and between the components of v_a and I'_a between F' and F:

$$\begin{aligned} I_\beta &= 0 & v_\beta &= 0 \\ I_\alpha &= 2I_0 & v_\alpha &= 2v_0 \qquad [55a] \\ V_\alpha &= -V_0 + I_\alpha(\tfrac{3}{2}Z_f) & I'_\alpha &= -I'_0 \end{aligned}$$

The β network is unaffected by the open conductor and fault. In

Fig. 12(b), the 0 network has been replaced by an equivalent Y with impedances C_0, D_0, and S_0, the identity of points F and F' being retained. (See Chapter VII, Fig. 2(a).) If the 0 impedances viewed from F' and F with the three-phase circuit open between F' and F and the fault removed are not mutually related, $S_0 = 0$. In Fig. 12(b),

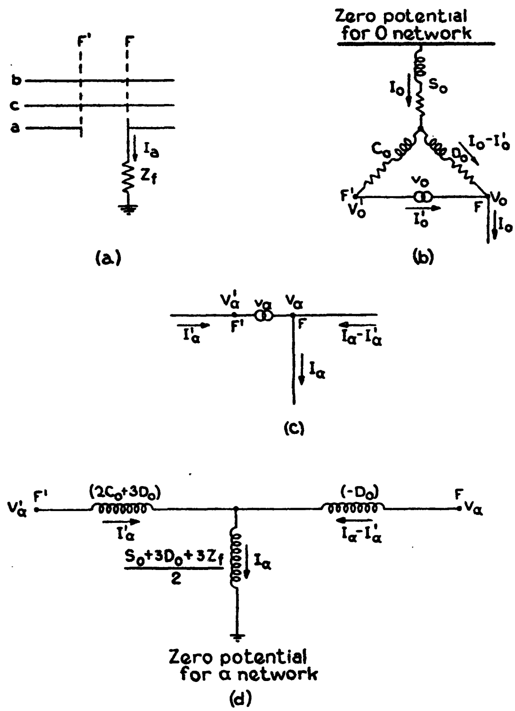


FIG. 12. (a) Open conductor and line-to-ground fault on phase a. (b) Equivalent Y to replace 0 network. (c) Section of α network including points F and F'. (d) Equivalent circuit to replace open conductor and fault in α network between F' and F.

v_0 is the 0 voltage drop between F' and F; V_0 is the 0 voltage to ground at F; I_0 and I_0' are the 0 currents flowing into the fault and from F' towards F, respectively. Figure 12(c) shows the portion of the α circuit between F and F'. V_α and V_α' are the α voltages at F and F', v_α is the α voltage drop between F' and F, I_α and I_α' are the α currents flowing into the fault and from F' towards F, respectively.

From Figs. 12(b) and (c),

$$
\begin{aligned}
V_0 &= -I_0(S_0 + D_0) + I_0'D_0 \\
v_0 &= V_0' - V_0 = I_0D_0 - I_0'(C_0 + D_0) \qquad [55b] \\
V_\alpha' &= V_\alpha + v_\alpha
\end{aligned}
$$

Eliminating V_0, I_0, I_0', v_0, and v_α from the simultaneous equations of [55a] and [55b], V_α and V_α' in terms of I_α and I_α' are obtained and may be written

$$\begin{aligned} V_\alpha' &= I_\alpha\left(\frac{S_0 + 3D_0 + 3Z_f}{2}\right) + I_\alpha'(2C_0 + 3D_0) \\ V_\alpha &= I_\alpha\left(\frac{S_0 + 3D_0 + 3Z_f}{2}\right) + (I_\alpha - I_\alpha')(-D_0) \end{aligned} \qquad [55c]$$

The equivalent Y shown in Fig. 12(*d*) satisfies equations [55*c*]. It can be used to replace the fault and open conductor in the α network. For a fault through zero impedance, $Z_f = 0$. If this equivalent circuit is substituted in the α network between F' and F, the α currents and voltages throughout the system can be determined. The β currents and voltages are unaffected by the fault and open conductor. In the 0 network: from [55*a*], $v_0 = \frac{1}{2}v_\alpha$; $I_0' = -I_\alpha'$; $I_0 = \frac{1}{2}I_\alpha$; and $V_0 = -V_\alpha + I_\alpha(\frac{3}{2}Z_f)$. Knowing the α components, with these relations and the 0 network, the 0 components of current and voltage throughout the system can be determined. The phase voltages and currents at any point are obtained by substituting the α, β, 0 components in [1]–[3] and [7]–[9], respectively.

An open conductor and fault to ground through impedance on the same phase can be represented on an a-c network analyzer as in Fig. 13(*b*), where a mutual coupling (or isolating) transformer is used to satisfy the equations for the open conductor and direct connections to satisfy the fault equations. In Fig. 13(*b*), impedance between the fault and open conductor is indicated. This impedance is zero in Fig. 12.

Single-Phase Load and Open Conductor in Three-Phase Four-Wire Circuit. Z_f in equations [55*a*] can be used to represent the impedance of a single-phase circuit between conductor *a* and the neutral conductor in a three-phase four-wire circuit. With an ungrounded neutral conductor, phase voltages and 0 voltages are referred to the neutral conductor. If conductor *a* is accidentally opened at the point where the load is taken off, the equivalent circuit of Fig. 12(*d*) for analytic work and the connections of Fig. 13(*b*) for the network analyzer apply to this case.

Line-to-Ground Fault and Two Openings in the Same Phase. Figure 13(*c*) gives a one-line diagram of a symmetrical three-phase power system. A fault to ground through impedance Z_f occurs on phase *a* of one of the two-parallel transmission lines at *F* very near its terminals. Single-phase breakers open phase *a* at both terminals, leaving the fault on the disconnected conductor. Assuming equal positive- and negative-sequence system impedances, Fig. 13(*d*) indicates the connections to be made on an a-c network analyzer between the α network and a modified 0 network in which voltages are 0 voltages, currents twice 0 currents, and impedance one-half 0 impedances. The α network for this system is the same as the positive-sequence network. As explained in Chapter VI, the mutual inductive and capacitive impedances between parallel transmission lines in the positive-

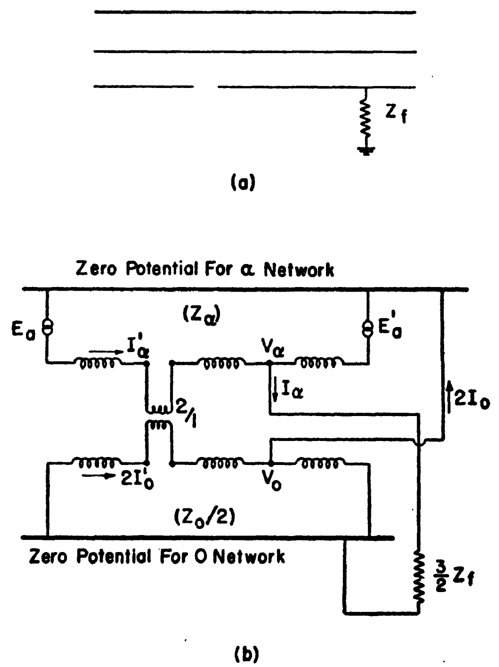


FIG. 13. (*a*) Open conductor and line-to-ground fault through impedance Z_f on same phase (phase *a*). (*b*) Connections of α and modified 0 networks on an a-c network analyzer to satisfy the conditions of (*a*).

sequence network are relatively unimportant; in general, they can be neglected and each of the parallel lines replaced by its nominal or equivalent Π or T. In the modified 0 network, the two parallel lines are replaced by the nominal equivalent circuit given in Fig. 8(*b*), Chapter VI, with all impedances divided by 2. If the parallel transmission lines are of such length that correcting factors should be applied to the nominal equivalent circuit in the 0 network, they may be divided into sections of length requiring no correcting factors, and each section replaced by its nominal equivalent Π circuit. Correcting factors for one line alone, given in Figs. 2(*a*) and (*b*) of Chapter VI, are applicable to parallel lines in the positive-sequence network. The β network, which is the same as the α network except that the generated voltages are turned through $-90°$, is not given in Fig. 13(d). It is unaffected by the line-to-ground fault and the two openings in the faulted conductor. As the β network is not connected with either the α or modified 0 network, positive-plus-negative and positive-minus-negative sequence components (discussed in Chapter V) can be used instead of α and β components in this problem. For faults at locations other than

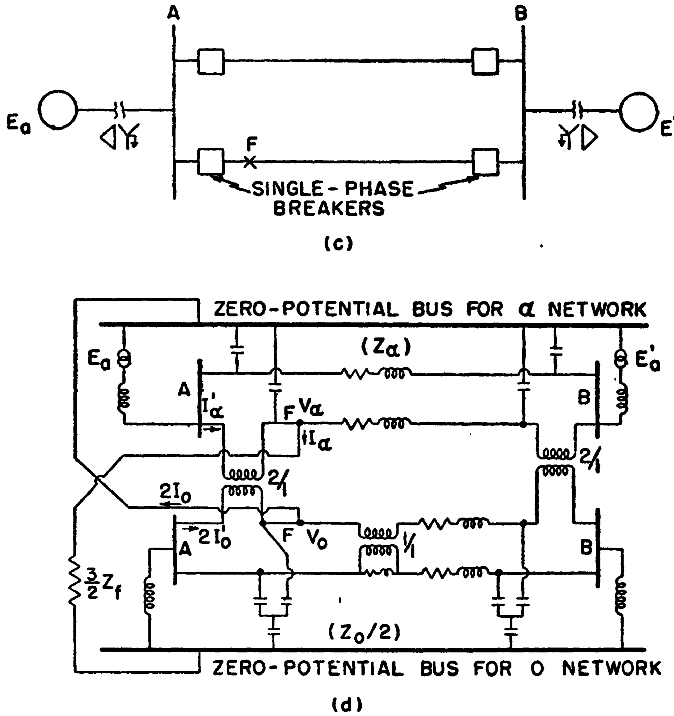


FIG. 13. (c) One-line diagram of a three-phase power system with two parallel transmission lines of appreciable capacitance. (d) Connections on a-c network analyzer of α and modified-0 networks for a line-to-ground fault on phase a of one transmission line at F through impedance Z_f with the faulted conductor open at A and B.

at a line terminal, the transmission lines are divided into two sections by the fault, each section being replaced by its equivalent circuits in the component networks and the fault applied between the two sections.

If the positive- and negative-sequence impedances of one of the synchronous machines (or equivalent synchronous machines) in Fig. 13(c) cannot be assumed equal, the equivalent circuit given in Fig. 5(a) or Fig. 5(b) may be used. Using either of these equivalent circuits, all impedances in the β network connected at their terminals are multiplied by -1. This turns inductive impedances in capacitive impedances, and vice versa. Resistances become negative resistances; but if they are small relative to the inductive and capacitive impedances, the error in neglecting them will likewise be small.

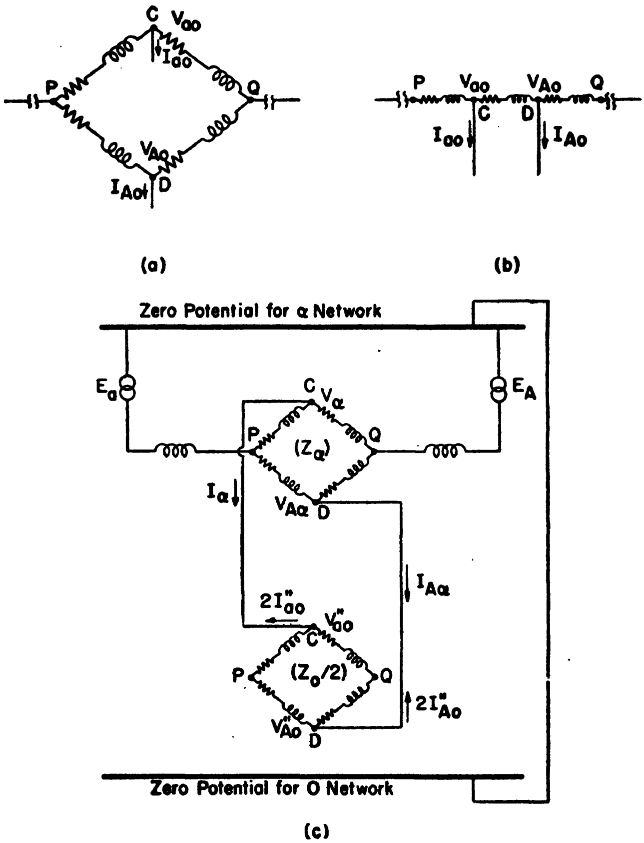


FIG. 14. (*a*) Ungrounded loop. (*b*) Ungrounded feeder in the 0 network with simultaneous ground faults at C and D isolated from the zero-sequence system by ungrounded transformers at P and Q. (*c*) Connection of the α network and a modified 0 network for line-to-ground faults at C and D both on phase a.

SIMULTANEOUS GROUND FAULTS ON UNGROUNDED SYSTEMS

The equivalent circuits developed for grounded systems can be applied to ungrounded systems of appreciable capacitance. In ungrounded systems of negligible capacitance, there will be little or no zero-sequence current with one ground fault. With two ground faults on an ungrounded section or loop isolated from the rest of the system by transformers, there is a path for 0 currents between the two fault points C and D. Figure 14(a) shows an ungrounded loop to which power is supplied at two points P and Q. Figure 14(b) shows points C and D on the same feeder between P and Q. Load may be taken off at various points of the loop or of the feeder.

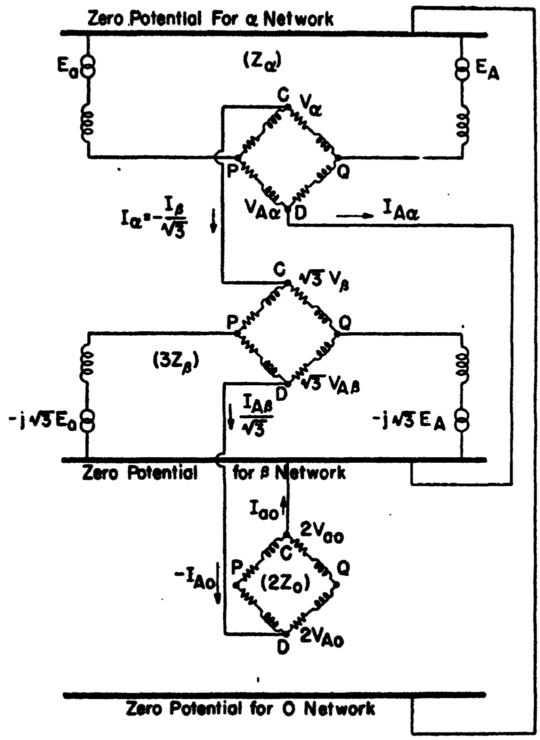


FIG. 14. (d) Connections for line-to-ground faults on phase b at C and phase c at D.

Let z_0 represent the 0 impedance between C and D, V_{a0} and V_{A0} the 0 voltages at C and D, and I_{a0} and I_{A0} the 0 currents flowing into the faults at C and D, respectively. Then,

$$\begin{aligned} I_{a0} &= -I_{A0} \\ V_{a0} &= V_{A0} - I_{a0}z_0 \end{aligned} \qquad [56]$$

Line-to-Ground Faults at C and D, Both on Phase a. The conditions at both faults are given by Case II(a) of Table I.

$$\begin{aligned} I_\alpha &= 2I_{a0}; \quad V_\alpha = -V_{a0} \text{ at } C \\ I_{A\alpha} &= 2I_{A0}; \quad V_{A\alpha} = -V_{A0} \text{ at } D \end{aligned} \qquad [57]$$

where the additional subscript A is used with α components at D. From [56] and [57],

$$V_\alpha - V_{A\alpha} = V_{A0} - V_{a0} = I_{a0}z_0 = I_\alpha \frac{z_0}{2} \qquad [58]$$

Figure 14(c) in which all 0 impedances are divided by two is an equivalent circuit for determining currents and voltages in the α network. It can also be used for determining 0 voltages and currents. Let $V_{a0} = -V''_{a0}$, $I_{a0} = -I''_{a0}$, $V_{A0} = -V''_{A0}$, $I_{A0} = -I''_{A0}$. Substituting these values in [57],

$$I_\alpha = -2I''_{a0}; \quad V_\alpha = V''_{a0}; \quad I_{A\alpha} = -2I''_{A0}; \quad V_{A\alpha} = V''_{A0} \qquad [59]$$

The equivalent circuit of Fig. 14(c) satisfies equations [59] for Fig. 14(a). The zero-potential bus of the zero-sequence network is connected to that of the α network to provide a reference for 0 voltages. Actual 0 voltages are those read from Fig. 14(c)(V''_{a0} and V''_{A0} at C and D, respectively) multiplied by -1. Actual 0 currents are those read from Fig. 14(c)($2I''_{a0}$ and $2I''_{A0}$ flowing into the fault at C and D, respectively) multiplied by $-\frac{1}{2}$. The β network is unaffected by the faults. If the α voltages at C and D were equal before the simultaneous faults, no fault currents will flow in the α and 0 networks. If the loop is supplied at one point only, the α network is open at P or Q. For the fault conditions indicated in Fig. 14(b), this circuit replaces Fig. 14(a) in the equivalent circuit of Fig. 14(c).

Line-to-Ground Faults at C and D on Different Phases. Let the phases be so named that the fault at C is on phase b and that at D on phase c. From Table I, Cases II(b) and (c),

$$\begin{aligned} I_\alpha &= -\frac{I_\beta}{\sqrt{3}} = -I_{a0}; \quad V_\alpha - \sqrt{3}V_\beta - 2V_{a0} = 0 \\ I_{A\alpha} &= \frac{I_{A\beta}}{\sqrt{3}} = -I_{A0}; \quad V_{A\alpha} + \sqrt{3}V_{A\beta} - 2V_{A0} = 0 \end{aligned} \qquad [60]$$

The equivalent circuit of Fig. 14(d) satisfies equations [56] and [60] without introducing untrue restrictions. If the ungrounded loop is supplied at one point only, the α and modified β networks will be open at P or Q. If C and D are on a feeder between P and Q, Fig. 14(b) replaces Fig. 14(a) in the equivalent circuit of Fig. 14(d).

In Fig. 14(d) direct connections between component networks have been made which require modifications of the β network as well as the 0 network. This problem can be solved on the a-c network analyzer by means of $\sqrt{3}:1$ and $2:1$ turn ratio transformers, as indicated in

Fig. 9, without modification of the β network and with a different modification of the 0 network.

DISSYMMETRIES ON OPPOSITE SIDES OF A Δ-Y TRANSFORMER BANK

Figures 15(*a*) and (*b*) give diagrams of two possible arrangements of the transformer windings, connected Δ–Y to form a three-phase transformer bank. (See discussion in Chapter VII under "Simultaneous Faults on Opposite Sides of a Δ–Y Transformer Bank.") The neutral of the Y may be grounded or ungrounded. The choice of the reference circuit is arbitrary. With the circuit at C as the reference circuit, let the components of currents and voltages at D referred to the circuit at C be indicated by double-primed symbols. For the transformer connection of Fig. 15(*a*), at no load with exciting current neglected, actual α voltages at D are in phase with β voltages at C. The actual α voltages and currents at D in terms of their values referred to circuit C are:

$$I_\alpha = I_\beta''; \quad V_\alpha = V_\beta'' \qquad [61a]$$

No-load β voltages at D are 180° out of phase with α voltages at C. They must be multiplied by -1 to be brought into phase with α voltages in circuit C. The actual β voltages and currents at D in terms of their values referred to circuit C are

$$I_\beta = -I_\alpha''; \quad V_\beta = -V_\alpha'' \qquad [61b]$$

For the transformer of Fig. 15(*b*), actual components of currents and voltages at D in terms of their values referred to circuit C are

$$I_\alpha = -I_\beta''; \quad V_\alpha = -V_\beta'' \qquad [62a]$$

$$I_\beta = I_\alpha''; \quad V_\beta = V_\alpha'' \qquad [62b]$$

(See also equations [46] and [47].) Zero-sequence currents and voltages at D are unchanged when referred to circuit C, assuming no connection between the zero-sequence impedances on opposite sides of the Δ–Y transformer bank.

By means of [61*a*] and [61*b*] or [62*a*] and [62*b*], equations from Tables I or II which give relations between actual components of currents and voltages at D can be expressed in terms of the components at D referred to circuit C. These new equations and the equations relating actual components of currents and voltages at C are the simultaneous equations to be satisfied. The new equations are no more difficult to represent by equivalent circuits than when the simultaneous dissymmetries are on the same side of a Δ–Y transformer bank.

The following problems illustrate some of the many simultaneous dissymmetries which can be represented by equivalent circuits in an analytic solution. It will be assumed that there is no connection in the system between the zero-sequence impedances viewed from C and D. Circuit C will be chosen as reference circuit and the system of Fig. 15(c) with the transformer connections of Fig. 15(a) will be assumed.

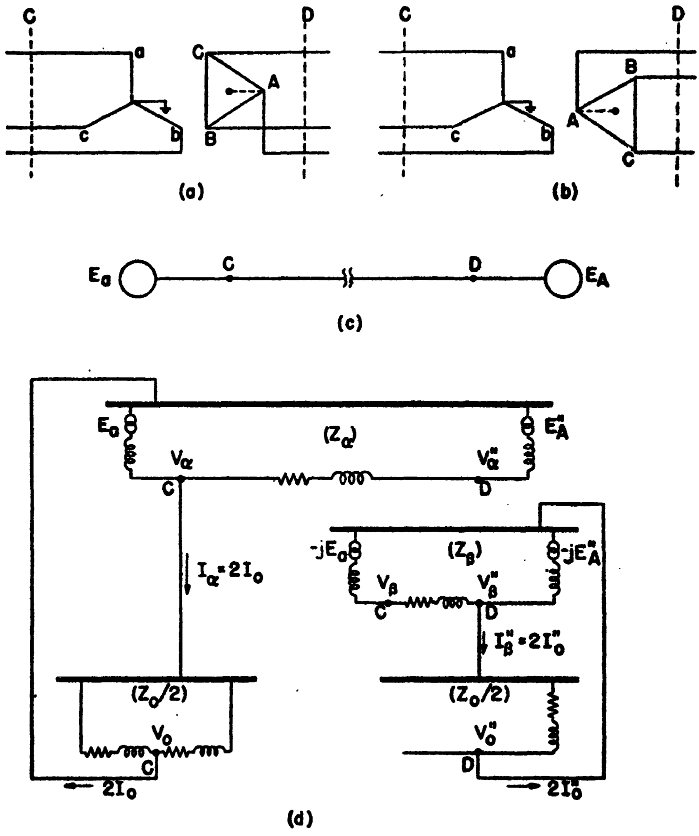


FIG. 15. (a) and (b) Transformer connection diagrams. (c) One-line diagram of a symmetrical three-phase power system. (d) Connections of component networks of (c) for line-to-ground faults on conductor A at D and on conductor a at C, with circuit C as reference circuit and transformer connection diagram (a).

Line-to-Ground Faults at C and D on Conductors a and A. At C and D, from Table I, Case II(a),

$$V_\alpha = -V_0; \quad I_\beta = 0; \quad I_\alpha = 2I_0 \tag{63}$$

Replacing V_α and I_α at D by V''_β and I''_β, and I_β at D by $-I''_\alpha$, [63] becomes

$$V''_\beta = -V''_0; \quad I''_\alpha = 0; \quad I''_\beta = 2I''_0 \tag{64}$$

The equivalent circuit of Fig. 15(d) satisfies equations [63] and [64]. Note that the zero-sequence network is open on the Δ side of the transformer bank at D. After V_α'', V_β'', I_β'', V_0'', and $2I_0''$ have been calculated, these components must be referred to circuit D to obtain actual components of currents and voltages at D. (See [61a] and [61b].) In the α and β networks of Fig. 15(d), the generated voltage E_A'' is the generated voltage E_A indicated in Fig. 15(c) referred to the circuit at C; at no load, E_A'' is in phase with E_a.

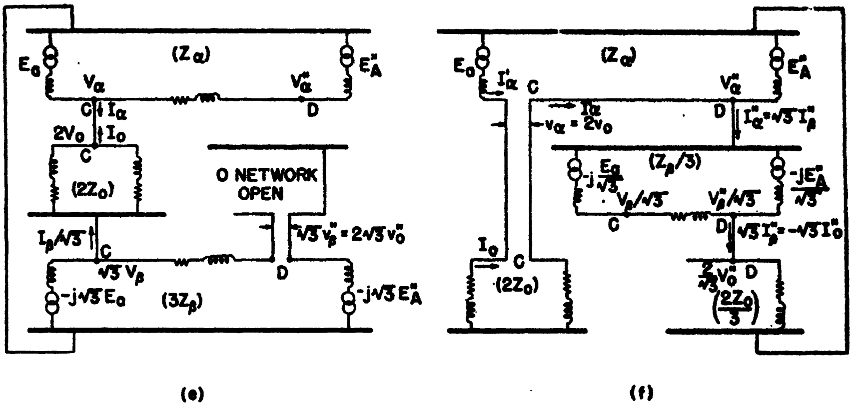


FIG. 15. (e) Conductor A open at D, line-to-ground fault on conductor b at C. (f) Line-to-ground fault on conductor B at D, conductor a open at C.

Conductor A Open at D, Line-to-Ground Fault on Conductor b at C. At C from Table I, Case II(b),

$$V_\alpha - \sqrt{3}V_\beta - 2V_0 = 0; \quad I_\alpha = -\frac{I_\beta}{\sqrt{3}} = -I_0 \qquad [65]$$

at D, from Table II, Case I(a),

$$v_\beta = 0; \quad v_\alpha = 2v_0; \quad I_\alpha' = -I_0' \qquad [66]$$

At the Δ, the current I_0' is zero. Therefore, $I_\alpha' = -I_0' = 0$. Replacing v_β by $-v_\alpha''$, v_α and I_α' by v_β'' and I_β'', respectively, [66] becomes

$$v_\alpha'' = 0; \quad v_\beta'' = 2v_0''; \quad I_\beta'' = -I_0'' = 0 \qquad [67]$$

Figure 15(e) is an equivalent circuit for analytic calculations which satisfies [65] and [67]. The connections for the line-to-ground fault on conductor b at C are the same as those of Fig. 8, Case II(b). All β impedances have been multiplied by 3. Because of infinite series impedance in the zero-sequence network at D, the 0 and β networks are both open at D. The α network is closed at D, since $v_\alpha'' = 0$.

Line-to-Ground Fault on Conductor B at D, Conductor a Open at C. Table I, Case II(b), and Table II, Case I(a), give the fault conditions at D and C, respectively. The equations of Table I, Case II(b), referred to the circuit at C become

$$\begin{gathered} V_\beta'' + \sqrt{3}V_\alpha'' - 2V_0'' = 0 \\ I_\beta'' = \frac{I_\alpha''}{\sqrt{3}} = -I_0'' \end{gathered} \qquad [68]$$

Equations [68] may be written

$$\begin{gathered} V_\alpha'' + \frac{V_\beta''}{\sqrt{3}} - \frac{2}{\sqrt{3}} V_0'' = 0 \\ I_\alpha'' = \sqrt{3}I_\beta'' = -\sqrt{3}I_0'' \end{gathered} \qquad [69]$$

Figure 15(f) satisfies the given conditions. The modification given the 0 network at D is necessary to permit direct connections which satisfy equations [69]. The connections at C for the open conductor in phase a are the same as in Fig. 10(c).

If E_A'' or E_a is zero, analytic calculations are appreciably simplified. When there is only one fault connection between a point in the α or β network and the zero-potential bus for the network, as in the α network of Fig. 15(e), the generated voltages in the network can be replaced by the voltage in phase a at the fault point before the dissymmetry occurred. This voltage is placed between the zero-potential bus for the network and the neutrals of the machines, as illustrated in Fig. 10(c) of Chapter IV. When such simplification cannot be made, the procedure is to apply each generated voltage separately and determine the currents in the system. Then, by superposition, the currents resulting from all voltages acting together are obtained.

Effects of Saturation. In the above discussion of simultaneous dissymmetries on the same or on opposite sides of transformer banks, transformer exciting impedances have not been included, nor has saturation been taken into account. Results obtained by neglecting exciting currents and the effects of saturation can be tested for accuracy by calculating voltages across transformer windings. When the voltage across any winding is below or only slightly above normal, its exciting impedance can usually be neglected with but slight error. If, on the other hand, the voltages are unbalanced and one or more well above normal, any method which neglects exciting currents and the effects of saturation may be appreciably in error. The effects of saturation in transformers upon fundamental-frequency currents and voltages are discussed in Volume II, where a method for including these effects is given.

SCOTT-CONNECTED TRANSFORMER BANK

A Scott-connected transformer bank used to transform from three-phase to two-phase, or vice versa, is shown in Fig. 16(*a*). The transformer *BC–RS* is called the main transformer and *AD–PQ* the teaser. The main and teaser windings are usually interchangeable, each being provided with a tap at the midpoint and at the 0.866 point. *D* is at the midpoint of the winding *BC*, and *A* is at the 0.866 point of the winding *DE*. If there is a neutral tap *N*, it will be at the one-third

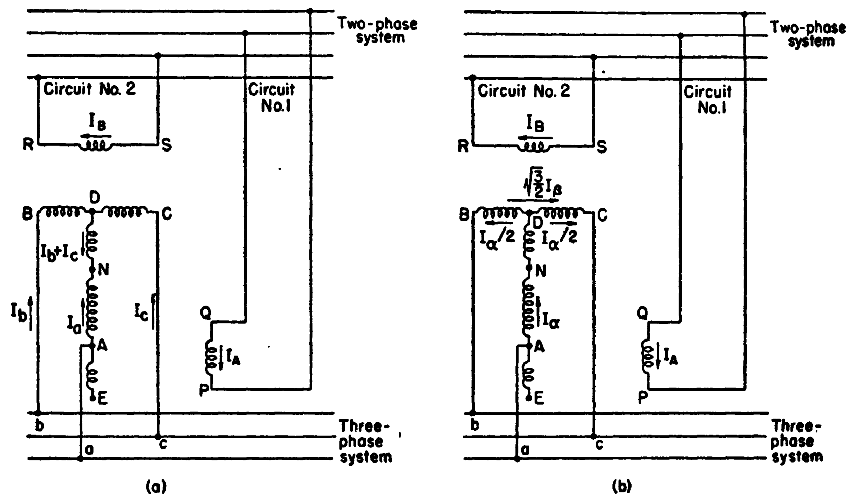


Fig. 16. Scott-connected transformer bank. (*a*) Phase currents on both sides of the bank. (*b*) α and β components on the three-phase side, phase currents on the two-phase side.

point of the winding *DA*. *N* may be grounded or ungrounded. Currents and voltages on the three-phase side of the bank are designated by the subscripts *a*, *b*, *c* and on the two-phase side by *A* and *B*. A two-phase four-wire system is indicated. For a three-wire system, points *Q* and *S* in Fig. 16 would be connected to a common wire.

In Fig. 16(*b*), the three-phase currents of Fig. 16(*a*) have been replaced by their α and β components. With the two halves of the main transformer winding on the three-phase side symmetrically placed with respect to the winding on the two-phase side, there will be no mutual connection between the α and β circuits. If 0 currents are present, I_0 will flow from B to D, C to D, and A to N; $2I_0$ will flow from D to N; 0 currents do not appear on the two-phase side.

Let Z_{mt} = per unit leakage impedance of main transformer BC–RS based on rating

Z_{tt} = per unit leakage impedance of teaser transformer AD–PQ based on rating [70]

Z_{it} = interlacing impedance — per unit impedance of windings BD and CD in parallel (with terminals B and C together) based on rated teaser voltage and current

If magnetizing currents are neglected, the impedance of a two-winding transformer is the same referred to either side when expressed in per unit on the same kva base and base voltages proportional to the number of turns in the two windings. The per unit transformer impedances defined in [70] are based on the rated kva of one transformer. With rated voltages proportional to the number of turns, base voltage on the two-phase side is rated two-phase voltage; on the three-phase side for the main transformer it is rated line-to-line voltage; for the teaser transformer and interlacing impedance it is 0.866 × line-to-line voltage (= 1.5 line-to-neutral voltage). Interlacing impedance is similar to an external impedance in series with the teaser. Since it is based on 0.866 × line-to-line voltage on the three-phase side, it will have the same per unit value viewed from the two-phase side.

If Z_{AA} and Z_{BB} are the per unit self-impedances of the transformer bank, referred to circuits A and B, respectively, on the two-phase side,

$$\begin{aligned} Z_{AA} &= Z_{tt} + Z_{it} \\ Z_{BB} &= Z_{mt} \end{aligned} \qquad [71]$$

and there is no mutual impedance between phases.

Rated two-phase kva and rated three-phase kva are each equal to the kva rating of the bank; but kva per phase on the two-phase side is the rating of one transformer and on the three-phase side two-thirds of that amount. If $Z_{\alpha\alpha}$ and $Z_{\beta\beta}$ are the per unit self-impedances of the transformer bank referred to the α and β circuits on the three-phase side, they must be based on line-to-neutral voltage and two-thirds of the kva per phase on which the impedances Z_{AA} and Z_{BB} are based. The self-impedance of the α circuit is two-thirds of the α loop impedances; that of the β circuit is one-half of the β loop impedance. Per unit impedances vary directly as the kva bases and inversely as the squares of the voltage bases, therefore in per unit

$$Z_{\alpha\alpha} = \tfrac{2}{3}\left[(Z_{tt} + Z_{it}) \times \left(\frac{1.5}{1}\right)^2 \times \tfrac{2}{3}\right] = Z_{tt} + Z_{it} = Z_{AA}$$

$$Z_{\beta\beta} = \tfrac{1}{2}\left[(Z_{mt}) \times \left(\frac{\sqrt{3}}{1}\right)^2 \times \tfrac{2}{3}\right] = Z_{mt} = Z_{BB} \qquad [72]$$

$$Z_{\beta\alpha} = Z_{\alpha\beta} = 0$$

From [71] and [72], the factors to convert per unit self-impedances in the A and B phases referred to the two-phase side to α and β impedances referred to the three-phase side, and vice versa, are unity.

Neutral of Bank Ungrounded. With the neutral of the transformer bank ungrounded, the α and β voltage drops through the bank in the direction of current flow are

$$\begin{aligned} v_\alpha &= (Z_{tt} + Z_{it})I_\alpha \\ v_\beta &= Z_{mt}I_\beta \end{aligned} \qquad [73]$$

The 0 voltage drop through the bank is indeterminate; but V_N, the voltage at N, is the 0 voltage at the terminals of the bank on the three-phase side (determined from the 0 diagram and given system conditions) minus $I_\alpha Z_{0\alpha}$ the 0 voltage drop produced by I_α. If the portion of the α loop impedance between A and N in Fig. 16 is considered the impedance of phase a to α currents, the equation $Z_{0\alpha} = Z_a - Z_{\alpha\alpha}$ given by [32] for the self-impedance circuit can be used to determine $Z_{0\alpha}$. The portion of the winding AD of the transformer $ADPQ$ in Fig. 16 between A and N includes two-thirds of the total number of turns in the winding. If the resistance and leakage reactance of the transformer with α currents flowing through it can be assumed to be distributed between A and N, and N and D in the same ratio as the number of turns in the windings AN and ND, the impedance of phase a to α currents will be two-thirds the transformer impedance. When expressed in per unit based on line-to-neutral voltage and the kva per phase on the three-phase side,

$$Z_a = \tfrac{2}{3}(Z_{tt})\left(\frac{1.5}{1}\right)^2 \times \tfrac{2}{3} = Z_{tt}$$

Replacing Z_a and $Z_{\alpha\alpha}$ in [32] by Z_{tt} and $(Z_{tt} + Z_{it})$, respectively,

$$Z_{0\alpha} = Z_{tt} - (Z_{tt} + Z_{it}) = -Z_{it} \qquad [74]$$

The voltage at the neutral N of the ungrounded Scott-connected transformer is

$$V_N = V_{0(T)} - I_\alpha Z_{0\alpha} = V_{0(T)} + I_\alpha Z_{it} \qquad [75]$$

where $V_{0(T)}$ is the 0 voltage at the terminals of the bank on the three-phase side, and the positive direction of I_a is toward N.

Ungrounded Scott-Connected Autotransformer Bank. The equations developed for the ungrounded Scott-connected transformer bank apply also to the ungrounded autotransformer bank if Z_{tt}, Z_{it}, and Z_{mt} are the teaser, interlacing, and main reactances, respectively, of the autotransformer bank.

Scott-connected transformers with grounded neutral are discussed in Volume II. The grounded neutral, by providing a path for 0 currents on the three-phase side, necessitates the determination of the self-impedance Z_{00} and the mutual impedance $Z_{\alpha 0}$. As these impedances depend upon leakage reactances between windings AN, ND, and PQ as a three-winding transformer, their determination will be delayed until transformers and autotransformers have been discussed in greater detail.

Positive- and Negative-Sequence Self- and Mutual Impedances of Ungrounded Scott-Connected Transformers. Substituting $Z_{\alpha\alpha}$ and $Z_{\beta\beta}$ from [72] with $Z_{\alpha\beta} = Z_{\beta\alpha} = 0$, in [22],

$$\begin{aligned} Z_{11} &= Z_{22} = \tfrac{1}{2}(Z_{\alpha\alpha} + Z_{\beta\beta}) = \tfrac{1}{2}(Z_{tt} + Z_{it} + Z_{mt}) \\ Z_{12} &= Z_{21} = \tfrac{1}{2}(Z_{\alpha\alpha} - Z_{\beta\beta}) = \tfrac{1}{2}(Z_{tt} + Z_{it} - Z_{mt}) \end{aligned} \qquad [76]$$

where Z_{tt}, Z_{it}, and Z_{mt} are defined in [70].

The equivalent circuits for the Scott-connected transformers in the positive- and negative-sequence networks are mutually coupled through the reciprocal mutual impedance $Z_{12} = Z_{21}$ given by [76]. In cases where $Z_{12} = Z_{21}$ is small enough to be neglected, the circuit becomes approximately a symmetrical one with equal positive- and negative-sequence impedances.

Three-Phase and Two-Phase Systems Interconnected through Ungrounded Scott-connected Transformer Bank

In Chapter IX, the phase quantities and two systems of components applicable to two-phase systems are discussed. The phase quantities of two-phase systems correspond to the α and β quantities of the three-phase system. The positive- and negative-sequence right-angle components of two-phase systems correspond to the positive- and negative-sequence symmetrical components of the three-phase system and will be called simply positive- and negative-sequence components when discussed in connection with Scott-connected transformers. The positive- and zero-sequence symmetrical components of the two-phase system have no corresponding components on the three-phase side.

The two-phase, three-wire transmission circuit is an unsymmetrical circuit when treated by either phase quantities or positive- and negative-sequence components. The coupling between phases is reciprocal, but it is non-reciprocal between the positive- and negative-sequence networks. Rotating machines of unequal positive- and negative-sequence impedances have non-reciprocal mutual coupling between phases, but none between the positive- and negative-sequence networks. If the positive- and negative-sequence impedances are equal, the mutual coupling between phases disappears. When two-phase and three-phase systems are connected through Scott-connected transformers, there is a choice of components for the system as a whole — symmetrical components or α, β, 0 components. With either system, base kva per phase on the three-phase side is two-thirds base kva per phase on the two-phase side.

$Z_1 \neq Z_2$. If the positive- and negative-sequence impedances of the rotating machines on either the two-phase or three-phase side, or both, cannot be assumed equal, positive- and negative-sequence components are preferable to α and β components. If the transformer mutual impedance $Z_{12} = Z_{21}$ defined in [76] can be neglected, and two-phase three-wire transmission circuits if present are of negligible length, the positive- and negative-sequence networks for the system will have no mutual coupling. The system as a whole can then be represented by positive-, negative-, and zero-sequence networks without mutual coupling (the zero-sequence network being for the three-phase system only) which are the same for all phases and can therefore be interconnected to represent conditions during faults which involve any phase or phases by the usual symmetrical component methods.

If, however, the mutual transformer impedance $Z_{12} = Z_{21}$ cannot be neglected, or the two-phase, three-wire transmission circuits are of importance, there are two different types of mutual coupling between the positive- and negative-sequence networks during normal operation. This is true whether the positive- and negative-sequence machine impedances are equal or unequal. Studies of system performance during normal operation or during faults can be made by means of equations. In such solutions, it must be remembered that the reference phase a has already been designated and therefore faults which may involve any phase are to be assigned to each phase in turn to cover all cases.

$Z_1 = Z_2$. With the positive- and negative-sequence impedances of the rotating machines of the system equal, there will be no mutual coupling between the α and β networks during normal operation, except as determined by the two-phase transmission circuits.

Three-Phase System and Two-Phase Four-Wire System. With $Z_1 = Z_2$ in all rotating machines, and negligible mutual coupling between phases in the two-phase four-wire transmission circuit, the α, β, and 0 equivalent circuits for a three-phase and a two-phase four-wire system connected by an ungrounded Scott-connected transformer or autotransformer bank between P and Q are shown in Fig. 17. There is no 0 network on the two-phase side. In Fig. 17 one synchronous

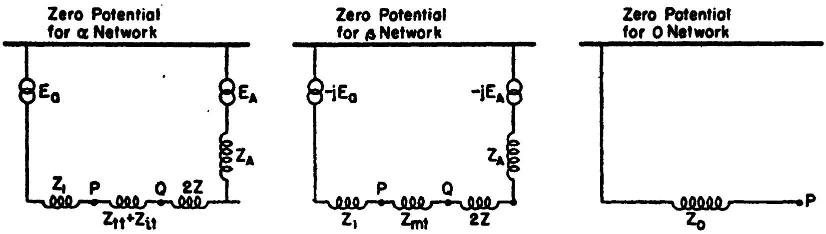


FIG. 17. Per unit α, β, and 0 equivalent circuits for three-phase and four-wire two-phase systems, connected by ungrounded Scott-connected transformer or autotransformer bank between P and Q and having equal positive- and negative-sequence machine reactances. The three-phase system is to the left of P, the two-phase system to the right of Q. Base voltage on the three-phase side is rated transformer line-to-neutral voltage, on the two-phase side it is rated two-phase transformer voltage (assuming rated voltages in proportion to the turns). Total kva is rated kva of the bank, kva per phase is one-half bank rating on the two-phase side and one-third on the three-phase side.

machine only is shown on the three-phase side and one on the two-phase side; there may be any number. Positive-, negative-, and zero-sequence impedances viewed from P are Z_1, Z_1, and Z_0; Z_A represents positive- or negative-sequence impedances of the two-phase machine. E_a is the equivalent generated voltage on the three-phase side in per unit of rated line-to-neutral transformer voltage, E_A that of the two-phase machine in per unit of rated transformer voltage on the two-phase side. $2Z$ represents the per unit loop impedance in each phase of the two-phase four-wire transmission circuit. $2Z = 0$ for transmission circuits of negligible length.

Faults on Three-Phase Side. The connections of the α, β, 0 networks of a three-phase system for various faults apply also to Fig. 17 for faults on the three-phase side.

Faults on Two-Phase Side. For faults on the two-phase side between the conductors of phase A, the α network at the fault point in Fig. 17 is short-circuited to "zero potential for α network." If the fault is between conductors of phase B, the β network is short-circuited to "zero potential for β network." If all four conductors are faulted, the α and β networks are both faulted to their zero potentials.

Three-Phase System and Two-Phase Three-Wire System. Figure 18 shows the α, β, and 0 equivalent circuits for the three-phase and two-phase three-wire systems connected by an ungrounded Scott-connected transformer or autotransformer bank. Two two-phase synchronous machines are indicated, one at Q and one at R, with a two-phase three-wire transmission line between Q and R. The α and β circuits are mutually coupled through the reciprocal mutual impedance

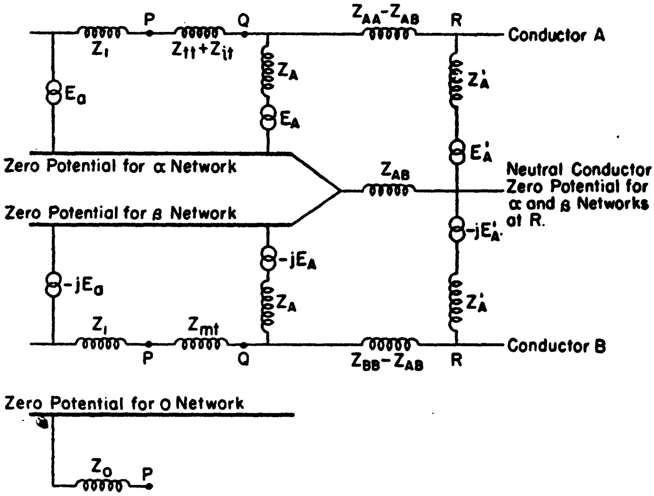


Fig. 18. Per unit α, β, and 0 equivalent circuits for three-phase and three-wire two-phase systems, connected by ungrounded Scott-connected transformer or autotransformer bank between P and Q, and having equal positive- and negative-sequence machine reactances.

Z_{AB} of the two-phase three-wire transmission circuit with grounded or ungrounded neutral conductor. Assuming $Z_1 = Z_2$ in the synchronous machines, there is no other coupling between the α and β networks. The self- and mutual phase impedances Z_{AA}, Z_{BB}, and $Z_{AB} = Z_{BA}$ of a three-wire two-phase transmission circuit with grounded or ungrounded neutral conductor are evaluated in Chapter XI, equations [107]–[108]. (See also Chapter IX, [22]–[28].)

Faults on the Two-Phase Side. As the equivalent circuit of Fig. 18 corresponds to the physical arrangement of the conductors on the two-phase side, faults involving conductors A, B, N or ground on the two-phase side can be represented by directly connecting the conductors involved. With a multigrounded neutral conductor, Z_{AB} represents the equivalent impedance of the grounded and neutral conductor in parallel.

Faults on Three-Phase Side. Figure 18 can also be used for determining currents and voltages during faults on the three-phase side. The connections given in Fig. 9 for studies on the a-c network analyzer are directly applicable. Instead of using Fig. 18 with the α and β circuits directly connected through the mutual impedance Z_{AB}, a mutual coupling transformer can be used to obtain this mutual connection. In an analytic study, all the connections given in Fig. 8 are applicable if the alternative connections of the networks for Cases II(c), III(c), and IV(c) and not the first-mentioned connections are used. These connections require direct connection of the zero-potential busses of the α and β networks which are already directly connected because of the mutual impedance Z_{AB}. Taking this precaution, no untrue current or voltage relations will be introduced by the fault connections of Fig. 8. The multiplier for generated β voltages and self-impedances in the β and 0 networks will be the same as those given in Fig. 8; the multiplier for the mutual impedance Z_{AB}, which occurs in three impedance branches in Fig. 18, will be the same as that for generated β voltages.

AVERAGE POWER

The *average power* at any point in a three-phase system in terms of per unit α, β, 0 components can be determined from [39] of Chapter II, which gives the average power in per unit of three-phase base power in terms of per unit symmetrical components of current and voltage. Replacing $|V||I| \cos \theta$ by the dot product $V \cdot I$ and the symmetrical components in this equation by their values in terms of α, β, 0 components from [18],

$$\begin{aligned} P &= V_{a1} \cdot I_{a1} + V_{a2} \cdot I_{a2} + V_{a0} \cdot I_{a0} \\ &= \tfrac{1}{4}(V_\alpha + jV_\beta) \cdot (I_\alpha + jI_\beta) + \tfrac{1}{4}(V_\alpha - jV_\beta) \cdot (I_\alpha - jI_\beta) + V_0 \cdot I_0 \end{aligned}$$

Expand the above equation, noting that if two vectors are rotated through the same angle their dot product is unchanged — for example, $(jV_\beta) \cdot (jI_\beta) = V_\beta \cdot I_\beta$. The average power is

$$\begin{aligned} P &= \tfrac{1}{2}(V_\alpha \cdot I_\alpha + V_\beta \cdot I_\beta) + V_0 \cdot I_0 \\ &= \tfrac{1}{2}|V_\alpha||I_\alpha| \cos \theta_\alpha + \tfrac{1}{2}|V_\beta||I_\beta| \cos \theta_\beta + |V_0||I_0| \cos \theta_0 \qquad [77] \end{aligned}$$

where P is the average power in per unit of three-phase base power, base power being numerically equal to base kva; θ_α, θ_β, and θ_0 are the phase angles by which I_α, I_β, and I_0 lead V_α, V_β, and V_0, respectively.

MERITS OF THE α, β, 0 SYSTEM OF COMPONENTS

In power systems in which the positive- and negative-sequence impedances of rotating machines can be assumed equal, use of α, β, and 0 components entails far less work in analytic solutions than the use of symmetrical components. They provide simpler equivalent circuits because the mutual impedances between component networks resulting from unsymmetrical static circuits, if not zero, are reciprocal. This is not the case with symmetrical components. When symmetrical components are to be used, the self- and mutual impedances in the sequence network can often be derived from α. β, and 0 self- and mutual impedances more simply than by direct determination. α, β, 0 components provide a point of view which is helpful in visualizing a problem even if it will eventually be solved by symmetrical components. In an unsymmetrical circuit, where the impedances of two phases are equal, or two phases are symmetrical with respect to the third phase, α and β components give an almost immediate solution to many simple problems which require appreciable time by other methods. This is illustrated in the following problem:

Problem 1. Balanced voltages are applied from a grounded source through zero impedance to the terminals of a circuit consisting of three capacitors connected in ungrounded Y. Two of the capacitors are equal. The three capacitive impedances are $-jx$, $-jy$, and $-jy$. Determine the currents in the three phases and the voltage above ground of the neutral of the ungrounded Y.

Solution. Let $Z_a = -jx$; $Z_b = Z_c = -jy$; and E_a = voltage applied to phase a.

The applied α voltage is E_a. The α impedance is $\frac{2}{3}\left(Z_a + \frac{Z_b}{2}\right)$.

$$I_a = I_\alpha = \frac{E_a}{\frac{2}{3}\left(Z_a + \frac{Z_b}{2}\right)} = \frac{3E_a}{2Z_a + Z_b} = j\frac{3E_a}{2x + y}$$

The applied β voltage is $-jE_a$. The β impedance is Z_b.

$$I_\beta = \frac{-jE_a}{Z_b} = \frac{E_a}{y}$$

$$I_b = -\tfrac{1}{2}I_\alpha + \frac{\sqrt{3}}{2}I_\beta = E_a\left[\frac{\sqrt{3}}{2y} - j\frac{3}{4x + 2y}\right]$$

$$I_c = -\tfrac{1}{2}I_\alpha - \frac{\sqrt{3}}{2}I_\beta = E_a\left[-\frac{\sqrt{3}}{2y} - j\frac{3}{4x + 2y}\right]$$

The voltage V_n at the neutral is

$$V_n = E_a - I_aZ_a = E_a\frac{Z_b - Z_a}{2Z_a + Z_b} = E_a\frac{y - x}{2x + y}$$

Problem 2. Figure 6(*a*) gives an equivalent circuit for the case of reciprocal mutual impedances between the α, β, 0 networks. (*a*) Draw a different equivalent circuit of this type for use in analytic work. (*b*) Make a diagram for this case with mutual impedances obtained with mutual coupling transformers for use on an a-c network analyzer.

Problem 3. Table I gives relations between the α, β, 0 voltages and currents at the fault for various types of fault through zero impedance. Prepare a similar table with impedance Z_f in the fault. Determine the phase currents flowing into the fault and the voltages to ground at the fault in terms of Z_f, V_f (the prefault voltage of phase *a* at the fault) and Z_α, Z_β, Z_0 the α, β, and 0 impedances, respectively, viewed from the fault. Compare with Table I, Chapter IV, with Z_2 replacing Z_1 and $Z_\alpha = Z_\beta = Z_1$.

Problem 4. Draw equivalent circuits showing direct connections between α, β, and 0 component networks (or modified networks) which satisfy the equations of Table II, Cases I(*b*) and (*c*) and II(*b*) and (*c*).

Problem 5. Solve Problem I, Chapter VIII, by means of α, β, and 0 components.

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CHAPTER XI

IMPEDANCES OF OVERHEAD TRANSMISSION LINES

If the conductors of a three-phase transmission line could be so arranged that they were equally distant from each other and at the same time equally distant from the ground and all other circuit elements carrying current, the circuit would be a symmetrical one and the positive-, negative-, or zero-sequence inductances and capacitances would be the same for each of the three phases. Although such an arrangement is not possible, the average values of the sequence constants for the three phases can be made approximately equal by means of transpositions. Even with complete transposition, where each conductor occupies each tower position for one-third the total distance, since currents and voltages vary along the line, *exact symmetry* is not obtained. However, the use of average constants for a completely transposed line will give satisfactory results for the usual transmission line problems. Moreover, in many system studies involving transmission line performance where the lines are untransposed, sufficient precision is obtained by assuming the sequence inductances and capacitances of each of the three phases equal to their average values.

Occasionally problems arise in which it is necessary to determine currents and voltages in an untransposed or incompletely transposed line to a greater degree of precision than is possible by the assumption of identical sequence constants for the three phases. For example, the negative- or zero-sequence currents in an unsymmetrical, untransposed three-phase transmission line under normal operation cannot be determined by the assumption of identical positive-sequence constants, as there would be no negative- and no zero-sequence currents under normal operation if the positive-sequence constants were identical for the three phases. As explained in Chapter VIII, positive-sequence currents flowing in an unsymmetrical three-phase circuit produce negative- and zero-sequence voltages; therefore, even during normal operation, the positive-sequence network is mutually coupled with the negative- and zero-sequence networks, the negative- and zero-sequence networks also being mutually coupled.

Unsymmetrical three-phase circuits are discussed in Chapter VIII, and unsymmetrical single-phase and two-phase circuits in Chapter IX.

The equations developed in those chapters for sequence self- and mutual impedances in terms of phase or conductor self- and mutual impedances will be evaluated for three-phase, two-phase, and single-phase overhead transmission circuits. These formulas, which give the impedances in ohms per mile, are applicable not only to short lines in which the effects of capacitance are negligible, but also to lines in which capacitance must be considered. Formulas for the sequence self- and mutual capacitances of overhead transmission lines are developed in the following chapter. In Chapter VI, lines with distributed constants are discussed and equivalent circuits developed to replace them in the sequence networks.

Self- and Mutual Impedances of Linear Conductors. In developing equations for the sequence impedances of overhead transmission lines and insulated cables, it is convenient to consider that each conductor (or circuit element carrying current) has a self-impedance which is independent of the positions of all other conductors, and a mutual impedance with each of the other conductors in the circuit or circuits under consideration. The self-impedance of a conductor, or the mutual impedance between two conductors, has but little significance when used alone. Current cannot flow in a conductor unless there is a return path. The self-impedance of a conductor, therefore, is always associated with the mutual impedance between the conductor and some other conductor or conductors.

Notation. Self- and mutual impedances will be indicated by Z with two subscripts. For self-impedance, two like subscripts, both referring to the conductor, will be used; for mutual impedance between two conductors, the subscripts will indicate the two conductors. The spacing s between two conductors will likewise be indicated by two subscripts referring to the conductors. Resistance and reactance will be indicated by r and x, respectively. The mutual impedance between two conductors a and b with no part of their paths in common has no resistance component; at a constant frequency f,

$Z_{aa} = r_a + jx_{aa} = r_a + j2\pi f L_{aa} =$ the self-impedance of conductor a

and

$Z_{ab} = jx_{ab} = j2\pi f M_{ab} =$ mutual impedance between conductors a and b

where L and M indicate self- and mutual inductances, respectively.

In the *Bulletin of the Bureau of Standards*, Vol. 4, No. 2, pages 302–306, expressions are derived for the self- and mutual inductances of straight, cylindrical, parallel, non-magnetic, solid wires in a uniform non-magnetic medium. These equations are based on a frequency low enough for the current distribution to be uniform over the cross section of the conductor, and are derived with the flux beyond the ends of the wires neglected, but with the variation in flux density along the length of the wire taken into account. When the length ℓ of the wires is great in comparison with the diameter d and the spacing between wires s, these inductances are given approximately by equations [1] and [2] below.

The *total self-inductance* in cgs units is

$$L = 2\ell\left(\log_e \frac{4\ell}{d} - \frac{3}{4}\right) \text{ abhenries} \tag{1}$$

The total self-inductance L of a conductor may be divided into two parts — the external inductance L_e resulting from the flux outside the conductor and the internal inductance L_i resulting from the flux within the conductor. For any cylindrical conductor of diameter d, the external self-inductance is

$$L_e = 2\ell\left(\log_e \frac{4\ell}{d} - 1\right) \text{ abhenries} \tag{1a}$$

For a solid non-magnetic conductor, the internal self-inductance is

$$L_i = \tfrac{1}{2} \text{ abhenry per cm} \tag{1b}$$

Multiplying abhenries per centimeter by $(2.540\times12\times5280\times10^{-9} =)$ 0.16093×10^{-3}, L_i in henries per mile is

$$L_i = 0.0805 \times 10^{-3} \tag{1c}$$

The internal self-reactance at a constant frequency f is

$$x_i = 2\pi f L_i = \left(\frac{f}{60}\right) 0.0303 \text{ ohm per mile} \tag{1d}$$

The internal self-reactance[1] of a hollow, non-magnetic, cylindrical conductor of internal and external diameters d_i and d_0, respectively, at a constant frequency f is

$$x_i = \frac{f}{60}\left[0.2794 \frac{d_i^4}{(d_0^2 - d_i^2)^2} \log_{10} \frac{d_0}{d_i} + 0.0303 \frac{d_0^2 - 3d_i^2}{d_0^2 - d_i^2}\right] \text{ ohms per mile} \tag{1e}$$

x_i given by [1e] is less than x_i given by [1d]. Equation [1e] reduces to [1d] for a solid conductor in which $d_i = 0$. For a tube of infinitesimal thickness, $x_i = 0$. Figure 4 of Appendix B gives x_i calculated from [1e] in terms of d_i/d_0.

The above formulas apply to conductors of non-magnetic materials, such as copper, aluminum, and lead, in which the permeability is unity. For magnetic or partly magnetic conductors, resistance and internal reactance at a given frequency and temperature vary with current. The wire tables of Appendix B give the internal reactance x_i, the resistance r at 25°C, and outside diameter d of various commonly used conductors at frequencies of 25, 50, and 60 cycles. Skin effects and the effects of spiraling are included, and, where resistances and internal reactances vary with current, r and x_i are given for several values of current.

The *mutual inductance* M between two parallel cylindrical wires of length ℓ in a non-magnetic medium with spacing s between their centers is

$$M = 2\ell\left(\log_e \frac{2\ell}{s} - 1\right)\text{abhenries} \tag{2}$$

The mutual inductance[1] between a hollow conductor of inner and outer diameters d_i and d_0, respectively, and a concentric interior conductor (solid or hollow) is

$$M = 2\ell\left[\log_e \frac{4\ell}{d_0} - \frac{d_i^2}{d_0^2 - d_i^2}\log_e \frac{d_0}{d_i} - \frac{1}{2}\right]\text{abhenries} \tag{2a}$$

For an outer conductor of *infinitesimal thickness*, $d_i = d_0$, the second term in [2a] becomes $-\frac{1}{2}$, and [2a] becomes

$$M = 2\ell\left(\log_e \frac{4\ell}{d_0} - 1\right)\text{abhenries} \tag{2b}$$

Comparing equation [2b] with [1a], the mutual inductance between the outer and inner conductors, neglecting thickness of outer conductor, is the same as the self-inductance of the outer conductor.

With an inner conductor a and an outer conductor w, current in w (with no current in a) produces flux external to w, which links both a and w. No flux is produced *within* a hollow conductor by current flowing in that conductor. The resultant flux in the cross-sectional area of the hollow conductor w produces the internal reactance given by [1e]. All this internal flux links a, but only part of it is effective in linking w. The mutual inductance between a and w is therefore larger than the self-inductance of w, but the difference is very slight. As a close approximation, the total inductance L_{ww} of w and the mutual inductance M_{aw} between a and w can be assumed equal and calculated from [1a], with d replaced by the average of the inner and outer diameters d_i and d_0 of w, giving

$$L_{ww} = M_{aw} = 2l\left(\log_e \frac{4l}{\frac{1}{2}(d_0 + d_i)} - 1\right) \text{abhenries} \qquad [2c]$$

The equations for self- and mutual inductances of conductors, given above and listed in electrical engineering handbooks, can be used to determine the self- and mutual impedances associated with (1) positive-sequence currents in a two-wire single-phase circuit, (2) positive- and zero-sequence currents in a single-phase or two-phase three-wire circuit with ungrounded neutral conductor, (3) positive- and negative-sequence currents in a three-wire, three-phase circuit without ground wires, (4) positive-, negative-, and zero-sequence currents in a four-wire three-phase circuit with ungrounded neutral conductor. In applying these formulas the effect of the presence of the earth on the impedances is neglected. It will be shown later that the earth has but little influence on impedances to currents of any sequence when the sum of the currents in all circuit elements, measured in the same direction, is zero.

Two-Wire Single-Phase Circuit. Neglecting the presence of the earth, with positive-sequence currents I_{a1} and I_{b1} ($= -I_{a1}$) flowing in the same direction in conductors a and b of a single-phase circuit, the impedance voltage drops in phases a and b in the direction of current flow are

$$V_a = I_{a1}Z_{aa} + I_{b1}Z_{ab} = I_{a1}(Z_{aa} - Z_{ab})$$

$$V_b = I_{b1}Z_{bb} + I_{a1}Z_{ab} = -I_{a1}(Z_{bb} - Z_{ab})$$

Resolving V_a and V_b into their positive- and zero-sequence symmetrical components by [6] of Chapter IX,

$$V_{a1} = \tfrac{1}{2}(V_a - V_b) = \tfrac{1}{2}(Z_{aa} + Z_{bb} - 2Z_{ab})I_{a1} = I_{a1}Z_{11}$$

$$V_{a0} = \tfrac{1}{2}(V_a + V_b) = \tfrac{1}{2}[(Z_{aa} - Z_{ab}) - (Z_{bb} - Z_{ab})]I_{a1} = I_{a1}Z_{01}$$

With identical conductors, $Z_{aa} = Z_{bb}$ and $Z_{01} = 0$; I_{a1} induces no voltage in the zero-sequence network. The positive-sequence self-impedance is

$$Z_{11} = Z_{aa} - Z_{ab} = r + j2\pi f(L_{aa} - M_{ab}) \qquad [3]$$

Replacing L_{aa} by its internal and external components L_i and L_e, substituting for L_e and M their values given by [1a] and [2], respectively, multiplying by 0.16093×10^{-3} and $2\pi f$, and replacing $\log_e$ by 2.3026 $\log_{10}$, Z_{11} in ohms per mile is

$$Z_{11} = r + j\left(0.2794 \frac{f}{60} \log_{10} \frac{2s}{d} + x_i\right) \qquad [4]$$

The spacing between conductors s and the diameter d are expressed in the same units. The resistance r, the internal reactance x_i, and the diameter d are given in Appendix B for commonly used conductors.

The *impedance of a single-phase loop* is twice the impedance Z_{11} given in [4].

Three-Wire Single-Phase or Two-Phase Circuit with Ungrounded Neutral Conductor. Comparing [3] above with Z_{11} in [23] of Chapter IX with $Z_{aa} = Z_{bb}$, the positive-sequence self-impedances of the two-wire and three-wire circuits are the same. The zero-sequence self-impedance and the mutual impedances between the positive- and zero-sequence networks, evaluated by means of [1] and [2], in ohms per mile are

$$Z_{00} = Z_{11} + 2\left[r_n + j\left(0.2794\,\frac{f}{60}\log_{10}\frac{2s_{an}s_{bn}}{d_n s_{ab}} + x_{in}\right)\right] \qquad [5]$$

$$Z_{10} = Z_{01} = j0.2794\,\frac{f}{60}\log_{10}\frac{s_{an}}{s_{bn}}$$

where Z_{11} is given by [4] and d_n, r_n, and x_{in} are the diameter, resistance, and internal reactance, respectively, of the neutral conductor n. If the neutral conductor n is equidistant from conductors a and b, $s_{an} = s_{bn}$, and there is no mutual coupling between the positive- and zero-sequence networks.

Three-Wire Three-Phase Circuit. Neglecting the presence of the earth, with currents I_a, I_b, and I_c flowing in the same direction in conductors a, b, and c, respectively, the impedance voltage drops in the direction of current flow are

$$V_a = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac}$$
$$V_b = I_a Z_{ab} + I_b Z_{bb} + I_c Z_{bc}$$
$$V_c = I_a Z_{ac} + I_b Z_{bc} + I_c Z_{cc}$$

With only positive-sequence current flowing,

$$I_a = I_{a1}, \quad I_b = a^2 I_{a1}, \quad \text{and} \quad I_c = aI_{a1}$$

Substituting these values in the above equations and solving for the positive-sequence impedances of the three phases (defined in Chapter VIII),

$$Z_{a1} = \frac{V_a}{I_{a1}} = Z_{aa} + a^2 Z_{ab} + aZ_{ac}$$

$$Z_{b1} = \frac{V_b}{a^2 I_{a1}} = Z_{bb} + a^2 Z_{bc} + aZ_{ab}$$

$$Z_{c1} = \frac{V_c}{aI_{a1}} = Z_{cc} + a^2 Z_{ac} + aZ_{bc}$$

Substituting Z_{a1}, Z_{b1}, and Z_{c1} from the above equations in [7] of Chapter VIII,

$$\begin{aligned} Z_{11} &= \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) - \tfrac{1}{3}(Z_{ab} + Z_{ac} + Z_{bc}) \\ Z_{21} &= \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2Z_{cc}) + \tfrac{2}{3}(a^2Z_{ab} + aZ_{ac} + Z_{bc}) \\ Z_{01} &= \tfrac{1}{3}(Z_{aa} + a^2Z_{bb} + aZ_{cc}) - \tfrac{1}{3}(aZ_{ab} + a^2Z_{ac} + Z_{bc}) \end{aligned} \qquad [6]$$

Proceeding as with positive-sequence currents, with only negative-sequence currents in the circuit,

$$\begin{aligned} Z_{22} &= \tfrac{1}{3}(Z_{aa} + Z_{bb} + Z_{cc}) - \tfrac{1}{3}(Z_{ab} + Z_{ac} + Z_{bc}) \\ Z_{12} &= \tfrac{1}{3}(Z_{aa} + a^2Z_{bb} + aZ_{cc}) + \tfrac{2}{3}(aZ_{ab} + a^2Z_{ac} + Z_{bc}) \\ Z_{02} &= \tfrac{1}{3}(Z_{aa} + aZ_{bb} + a^2Z_{cc}) - \tfrac{1}{3}(a^2Z_{ab} + aZ_{ac} + Z_{bc}) \end{aligned} \qquad [6a]$$

Replacing the impedances in [6] and [6*a*] by their resistance and reactance components and substituting for self- and mutual inductances their values from [1] and [2], the positive- and negative-sequence self-impedances and the mutual impedances associated with positive- and negative-sequence currents in an unsymmetrical three-phase transmission circuit are obtained. With identical conductors, $Z_{aa} = Z_{bb} = Z_{cc}$.

Positive- and Negative-Sequence Self-Impedances. Z_{11} and Z_{22} of the unsymmetrical untransposed circuit with identical conductors, in ohms per mile, are

$$Z_{11} = Z_{22} = r + j\left(0.2794 \frac{f}{60} \log_{10} \frac{2s}{d} + x_i\right) \qquad [7]$$

where

$s = \sqrt[3]{s_{ab}s_{ac}s_{bc}}$ = equivalent Δ spacing = geometrical mean distance between conductors.

The *mutual impedance* associated with the positive- and negative-sequence currents from [6] and [6*a*], in ohms per mile, are

$$Z_{12} = \tfrac{2}{3}\left[Z_{bc} - \tfrac{1}{2}(Z_{ab} + Z_{ac}) + j\frac{\sqrt{3}}{2}(Z_{ab} - Z_{ac})\right]$$

$$= j0.1863 \frac{f}{60}\left(\log_{10} \frac{\sqrt{s_{ab}s_{ac}}}{s_{bc}} + j\frac{\sqrt{3}}{2} \log_{10} \frac{s_{ac}}{s_{ab}}\right) \qquad [8]$$

$$Z_{21} = j0.1863 \frac{f}{60}\left(\log_{10} \frac{\sqrt{s_{ab}s_{ac}}}{s_{bc}} - j\frac{\sqrt{3}}{2} \log_{10} \frac{s_{ac}}{s_{ab}}\right) \qquad [9]$$

$$Z_{02} = -\tfrac{1}{2}Z_{21} = -j0.0931\,\frac{f}{60}\left(\log_{10}\frac{\sqrt{s_{ab}s_{ac}}}{s_{bc}} - j\frac{\sqrt{3}}{2}\log_{10}\frac{s_{ac}}{s_{bc}}\right) \quad [10]$$

$$Z_{01} = -\tfrac{1}{2}Z_{12} = -j0.0931\,\frac{f}{60}\left(\log_{10}\frac{\sqrt{s_{ab}s_{ac}}}{s_{bc}} + j\frac{\sqrt{3}}{2}\log_{10}\frac{s_{ac}}{s_{bc}}\right) \quad [11]$$

It will be shown later that $Z_{10} = Z_{02}$ and $Z_{20} = Z_{01}$.

Three-Phase Four-Wire Circuit with Ungrounded Neutral Conductor. Equations [13] of Chapter VIII give the sequence self- and mutual impedances of a three-phase circuit with neutral conductor in terms of the self-impedances of the conductors and the mutual impedances between them with the presence of the earth neglected. With identical conductors in the three phases but the neutral conductor not necessarily the same as the phase conductor, if the conductor self- and mutual inductances are replaced by their values given by [1] and [2] and the equations expressed in ohms per mile, the positive- and negative-sequence self-impedances $Z_{11} = Z_{22}$ and the mutual impedances Z_{12} and Z_{21} between the positive- and negative-sequence networks (which are independent of the neutral conductor) are given by [7], [8], and [9], respectively. The self-impedance Z_{00} and the mutual impedances $Z_{10} = Z_{02}$ and $Z_{20} = Z_{01}$ in ohms per mile may be conveniently expressed in terms of Z_{11}, Z_{12}, and Z_{21} as follows:

$$Z_{00} = Z_{11} + 3(r_n + jx_{in}) + j0.8382\left(\frac{f}{60}\right)\log_{10}\frac{2(s_{\overline{an}})^2}{s_{\overline{ab}}d_n} \quad [12]$$

$$Z_{10} = Z_{02} = -\frac{Z_{21}}{2} - j0.2794\left(\frac{f}{60}\right)\left[\log_{10}\frac{\sqrt{s_{bn}s_{cn}}}{s_{an}} + j\frac{\sqrt{3}}{2}\log_{10}\frac{s_{cn}}{s_{bn}}\right] \quad [13]$$

$$Z_{20} = Z_{01} = -\frac{Z_{12}}{2} - j0.2794\left(\frac{f}{60}\right)\left[\log_{10}\frac{\sqrt{s_{bn}s_{cn}}}{s_{an}} - j\frac{\sqrt{3}}{2}\log_{10}\frac{s_{cn}}{s_{bn}}\right] \quad [14]$$

where r_n, x_{in}, and d_n are the resistance, internal reactance, and diameter of the neutral conductor; $s_{\overline{an}} = \sqrt[3]{s_{an}s_{bn}s_{cn}}$; $s_{\overline{ab}} = \sqrt{s_{ab}s_{ac}s_{bc}}$; and Z_{11}, Z_{12}, and Z_{21} are given by [7]–[9]. Problem 7 deals with an unsymmetrical three-phase circuit with ungrounded or unigrounded neutral conductor.

Symmetrical Three-Phase Circuits. For identical phase conductors placed at the vertices of an equilateral triangle, $s_{ab} = s_{ac} = s_{bc} = s$.

In [8]–[11] and [13] and [14], with $\log_{10} s/s = \log 1 = 0$, there will be no mutual impedances between the sequence networks resulting from positive- and negative-sequence currents. The positive- and negative-sequence self-impedances in ohms per mile are given by [7], where s is the actual spacing between the phase conductors.

Completely Transposed Three-Phase Circuit. The positive- and negative-sequence impedances of a completely transposed circuit are the average impedances of the three phases to positive- or negative-sequence currents. They are, therefore, the same as the positive- and negative-sequence self-impedances of the unsymmetrical untransposed circuit given by [7]. Although there are mutual impedances between the sequence networks in sections between transpositions, the resultant mutual impedances between terminals are zero for the completely transposed circuit.

Comparing [7] and [4], the positive- or negative-sequence self-impedance per mile of a three-wire or four-wire three-phase transmission circuit with equivalent Δ spacing s between phase conductors is the same as the positive-sequence self-impedance of a two-wire single-phase circuit or a three-wire single-phase or two-phase circuit, with spacing s between the two phase conductors.

Reactance charts in terms of spacing s between conductors are given in Appendix B. These charts give the positive- or negative-sequence self-reactance of a three-phase circuit, the positive-sequence (or equivalent reactance of one conductor) of a two-wire single-phase circuit, and the positive-sequence self-reactance of a three-wire single-phase or two-phase circuit.

In many problems involving unsymmetrical untransposed transmission lines, the mutual impedances between the sequence networks can be neglected without appreciable error. This is illustrated in Problem 6. Since this problem involves zero-sequence self-impedances and the mutual impedances associated with zero-sequence currents which return in the earth, it will be considered after formulas for these impedances have been developed.

Self-Impedance of a Conductor with Earth Return and Mutual Impedance between Two Conductors with Common Earth Return. Any method of calculating currents and voltages during ground faults on transmission lines requires a knowledge of the impedances of the conductors with return paths through the earth. By means of tests, such knowledge was obtained and applied to the solution of unsymmetrical short-circuit problems before the advent of symmetrical components. Dr. W. W. Lewis[2] gives results of tests made in 1915, 1921, and 1922 to determine the impedances of one, two, and three conductors with earth return. These tests were made on three different power systems, with different sizes of conductors and spacings between conductors. When symmetrical components were first applied to the solution of unsymmetrical faults, empirical formulas[3] based on tests were developed giving the zero-sequence impedance of a transmission line as a function of the distance below the earth's surface of the return path for zero-sequence currents, this distance to be estimated from local soil conditions. The return path was taken farther below the earth's surface for dry or rocky soil than for moist soil.

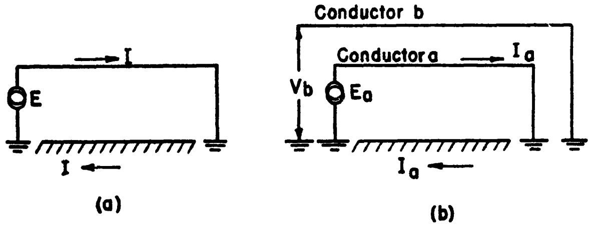


FIG. 1. Impedances with earth return from tests. (*a*) Self-impedance of a conductor with earth return. (*b*) Mutual impedance between two conductors with common earth return.

Carson's Formulas. In 1926, Dr. John R. Carson[4] gave equations for the self-impedance of a conductor with earth return and the mutual impedance between two conductors with common earth return, from which the various factors influencing these impedances can be evaluated. The equations are based on an earth of uniform conductivity, semi-infinite in extent, terminated by a plane parallel to the conductors.

Figure 1(*a*) shows how the impedance of a conductor with earth return is obtained by test. A voltage to ground E is applied at one end of the conductor a with the other end grounded, and the current is measured. Using the additional subscript g to indicate ground return, $Z_{aa-g} = E/I$. From Fig. 1(*b*), the mutual impedance between two conductors a and b with common earth return is obtained by applying a voltage to ground E_a at one end of conductor a with the other end grounded, and measuring the current in conductor a and the voltage V_b between conductor b and ground with one end of conductor b grounded. Then $Z_{ab-g} = V_b/I_a$.

Z_{aa-g} and Z_{ab-g} are expressed in terms of their resistance and reactance components by the following equations:

$$Z_{aa-g} = (r_c + R_{aa-g}) + j(x_i + X_{aa-g}) \qquad [15]$$

$$Z_{ab-g} = R_{ab-g} + jX_{ab-g} \qquad [16]$$

In [15], r_c and x_i are the resistance and internal reactance, respectively, of the conductor; R_{aa-g} and X_{aa-g} are the resistance and reactance, respectively, of the component of self-impedance with earth return external to the conductor (here conductor a); in [16], R_{ab-g} and X_{ab-g} are the resistance and reactance, respectively, of the mutual impedance with common earth return between two conductors (here conductors a and b).

If r_c, x_i, R_{aa-g}, X_{aa-g}, R_{ab-g}, and X_{ab-g} are expressed in ohms per mile, Z_{aa-g} and Z_{ab-g} will be in ohms per mile. R_{aa-g}, R_{ab-g}, X_{aa-g}, and X_{ab-g}, which are functions of frequency, earth resistivity, diameter of conductor, spacing between conductors, and heights of conductors above ground, are given in ohms per mile by equations [30]–[37].

Development of Equations [30]–[37] Which Express R_{aa-g}, R_{ab-g}, X_{aa-g}, and X_{ab-g} in Ohms per Mile. Carson's formulas[4] for self-impedance with earth return and mutual impedances with common earth return in absolute units are

$$Z_{aa-g} = z + j2\omega \log_e \frac{4h_a}{d} + 4\omega(P + jQ) = (r_c + R_{aa-g}) + j(X_{aa-g} + x_i) \qquad [15a]$$

$$Z_{ab-g} = j2\omega \log_e \frac{S}{s} + 4\omega(P + jQ) = R_{ab-g} + jX_{ab-g} \qquad [16a]$$

where

$z = r_c + jx_i$ = conductor internal impedance in abohms per centimeter

h_a, h_b = height above ground of conductors a and b, respectively, in centimeters

d = diameter of conductor in centimeters

s = distance between conductors in centimeters

S = distance from one conductor to the image of the other, assuming a perfectly conducting earth, in centimeters

$\omega = 2\pi f$

where f = frequency in cycles per second. Then

$$\begin{aligned} R_{aa-g} &= 4\omega P \text{ in abohms per centimeter} \\ R_{ab-g} &= 4\omega P \text{ in abohms per centimeter} \\ X_{aa-g} &= 2\omega \log_e \frac{4h_a}{d} + 4\omega Q \text{ in abohms per centimeter} \\ X_{ab-g} &= 2\omega \log_e \frac{S}{s} + 4\omega Q \text{ in abohms per centimeter} \end{aligned} \qquad [17]$$

P and Q are given in equations [32] and [33] of the reference paper[4] as functions of infinite series in terms of r (here replaced by k) and θ. P and Q have the same form for self- and mutual impedances, but k and θ have different values for the two cases.

For self-impedance: $k = 4\pi h_a\sqrt{2\lambda f}; \quad \theta = 0$ [18]

For mutual impedance: $k = 2\pi S_{ab}\sqrt{2\lambda f}; \quad \theta = \cos^{-1}\dfrac{h_a + h_b}{S_{ab}}$

$$= \sin^{-1}\frac{H_{ba}}{S_{ab}} \qquad [19]$$

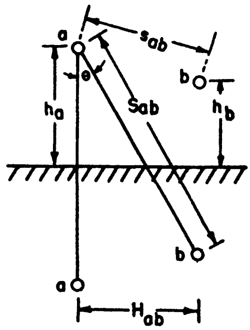


FIG. 2. Diagram showing conductors a and b with their images, the angle θ, and distances s, S, h, and H.

where λ is earth conductivity in abmhos per centimeter cube; h and S are in centimeters; θ for mutual impedances is the angle formed between lines drawn from a conductor to its own image and the image of the second conductor, assuming a perfectly conducting earth. (See Fig. 2 for θ, s, S, h, and H.)

The series in the equations for P and Q are rapidly converging for the values of k associated with overhead transmission lines at frequencies of 60 cycles or less. The following equations give P and Q to less than 1% error for values of k up to unity.

$$P = \frac{\pi}{8} - \frac{1}{3\sqrt{2}}k\cos\theta + \frac{k^2}{16}\cos 2\theta\left(0.6728 + \log_e\frac{2}{k}\right) + \frac{r^2}{16}\theta\sin 2\theta + \frac{k^3\cos 3\theta}{45\sqrt{2}} - \frac{\pi k^4\cos 4\theta}{1536} \qquad [20]$$

$$Q = -0.0386 + \tfrac{1}{2}\log_e\frac{2}{k} + \frac{1}{3\sqrt{2}}k\cos\theta - \frac{\pi k^2\cos 2\theta}{64} + \frac{k^3\cos 3\theta}{45\sqrt{2}} - \frac{k^4\,\theta\sin 4\theta}{384} - \frac{k^4\cos 4\theta}{384}\left(\log_e\frac{2}{k} + 1.0895\right) \qquad [21]$$

If h and S are expressed in feet, and λ, the earth conductivity in abmhos per centimeter cube, is replaced by $10^{-11}/\rho$, where ρ = resistivity of earth in ohms per meter cube, values of k for self- and mutual impedances are given by the following equations:

$$\text{For self-impedance: } k = 4\pi h_a(30.48)\sqrt{\frac{2f10^{-11}}{\rho}}$$

$$= 1.713 \times 10^{-3}\, h_a\sqrt{\frac{f}{\rho}} \qquad [22]$$

$$\text{For mutual impedance: } k = 2\pi S(30.48)\sqrt{\frac{2f10^{-11}}{\rho}}$$

$$= 1.713 \times 10^{-3}\,\frac{S_{ab}}{2}\sqrt{\frac{f}{\rho}} \qquad [23]$$

From [22] and [23], k varies directly with h or S, directly as $\sqrt{f}$, and inversely as $\sqrt{\rho}$. The following examples indicate the values of k which may be encountered in overhead transmission lines at 60 cycles.

$$\text{For } f = 60, \quad h \text{ or } \frac{S}{2} = 80 \text{ feet}, \quad \rho = 100: \quad k = 0.106$$

$$\text{For } f = 60, \quad h \text{ or } \frac{S}{2} = 40 \text{ feet}, \quad \rho = 5: \quad k = 0.238$$

$$\text{For } f = 60, \quad \frac{S}{2} = 150 \text{ feet}, \quad \rho = 10: \quad k = 0.63$$

For a lower frequency or higher earth resistivity, k will be lower than the value given above for the same h or S. For an earth resistivity of 100 ohms per meter cube or more and a frequency of 60 cycles or less, k for the conventional overhead transmission circuit will not be greater than 0.2 for self-impedances or for mutual impedance when the conductors are in the same circuit.

The number of terms that need be retained in P and Q for determining self- and mutual impedances depends upon the magnitude of k and the degree of precision required in the calculations involving them. In calculating mutual impedance, the value of θ must also be considered when k is relatively large. The large number of terms in [20] and [21] for P and Q are included for the rare cases where values of k of 1.0 or higher are encountered; as, for example, when self- and mutual impedances with ground return are required in the calculation of high harmonic currents and voltages. For frequencies of 60 cycles or less, it is convenient to use the first term in [20] and the first two terms in [21] to represent P and Q, respectively, the other terms in the equations to be applied as corrections. P and Q may then be written

$$P = \frac{\pi}{8} + \Delta P \qquad [24]$$

$$Q = -0.0386 + \tfrac{1}{2} \log_e \frac{2}{k} + \Delta Q \qquad [25]$$

Substituting [24] and [25] in [17] and replacing k by its values from [18] and [19], in absolute units

$$R_{aa-g} = 4\omega\left(\frac{\pi}{8}\right) + \Delta R_{aa-g} = \pi^2 f + \Delta R_{aa-g} \qquad [26]$$

$$R_{ab-g} = 4\omega\left(\frac{\pi}{8}\right) + \Delta R_{ab-g} = \pi^2 f + \Delta R_{aa-g} \qquad [27]$$

$$X_{aa-g} = 2\omega\left(\log_e \frac{2}{d\sqrt{f\lambda}\sqrt{2\pi}} - 0.0772\right) + \Delta X_{aa-g}$$

$$= 2\omega \log_e \frac{0.208}{\left(\frac{d}{2}\right)\sqrt{f\lambda}} + \Delta X_{aa-g} \qquad [28]$$

$$X_{ab-g} = 2\omega \log_e \frac{0.208}{s_{ab}\sqrt{f\lambda}} + \Delta X_{ab-g} \qquad [29]$$

To express [26]–[29] in ohms per mile, they are multiplied by 0.16093×10^{-3}; d in centimeters is replaced by its length in inches multiplied by 2.540, s in centimeters by its length in feet multiplied by 30.48; λ is replaced by $10^{-11}/\rho$, where ρ is in ohms per meter cube.

The values of R_{aa-g}, R_{ab-g}, X_{aa-g}, and X_{ab-g} in ohms per mile to be substituted in [15] and [16] to give Z_{aa-g} and Z_{ab-g} in ohms per mile are

$$R_{aa-g} = 10^{-3}\omega(0.2528) + \Delta R_{aa-g} \qquad [30]$$

$$R_{ab-g} = 10^{-3}\omega(0.2528) + \Delta R_{ab-g} \qquad [31]$$

$$X_{aa-g} = 10^{-3}\omega\left(0.74113 \log_{10} \frac{1}{d}\sqrt{\frac{\rho}{f}} + 3.4944\right) + \Delta X_{aa-g} \qquad [32]$$

$$X_{ab-g} = 10^{-3}\omega\left(0.74113 \log_{10} \frac{1}{s}\sqrt{\frac{\rho}{f}} + 2.4715\right) + \Delta X_{ab-g} \qquad [33]$$

where d = diameter of conductor in inches; s = spacing between conductors in feet; ρ = earth resistivity in ohms per meter cube;

$\omega = 2\pi f$; f = frequency in cycles per second; and the terms ΔR_{aa-g}, ΔR_{ab-g}, ΔX_{aa-g}, and ΔX_{ab-g} are given in ohms per mile by the following equations for values of k up to unity, where k is defined in [22] and [23]:

$$\Delta R_{aa-g} = 10^{-4}\omega\left[-\frac{2.599}{10^3}h_a\sqrt{\frac{f}{\rho}} + \frac{2.717}{10^6}h_a^2\frac{f}{\rho}\left(3.360 + \log_{10}\frac{1}{h_a}\sqrt{\frac{\rho}{f}}\right) + \frac{5.084}{10^{10}}h_a^3\frac{f}{\rho}\sqrt{\frac{f}{\rho}} - \frac{1.133}{10^{13}}h_a^4\frac{f^2}{\rho^2}\right] \quad [34]$$

$$\begin{aligned}\Delta R_{ab-g} = 10^{-4}\omega\Bigg[&-\frac{1.299}{10^3}\sqrt{\frac{f}{\rho}}\,S\cos\theta \\ &+ \frac{6.785}{10^7}\frac{f}{\rho}S^2\cos 2\theta\left(3.661 + \log_{10}\frac{1}{S}\sqrt{\frac{\rho}{f}}\right) \\ &+ \frac{2.951}{10^7}\frac{f}{\rho}S^2\theta\sin 2\theta + \frac{6.355}{10^{11}}\frac{f}{\rho}\sqrt{\frac{f}{\rho}}\,S^3\cos 3\theta \\ &- \frac{7.084}{10^{15}}\frac{f^2}{\rho^2}S^4\cos 4\theta\Bigg]\end{aligned} \quad [35]$$

$$\begin{aligned}\Delta X_{aa-g} = 10^{-4}\omega\Bigg[&\frac{2.599}{10^3}h_a\sqrt{\frac{f}{\rho}} - \frac{9.271}{10^7}h_a^2\frac{f}{\rho} + \frac{5.084}{10^{10}}h_a^3\frac{f}{\rho}\sqrt{\frac{f}{\rho}} \\ &- \frac{3.322}{10^{13}}h_a^4\frac{f^2}{\rho^2}\left(3.541 + \log_{10}\frac{1}{h_a}\sqrt{\frac{\rho}{f}}\right)\Bigg]\end{aligned} \quad [36]$$

$$\begin{aligned}\Delta X_{ab-g} = 10^{-4}\omega\Bigg[&\frac{1.299}{10^3}\sqrt{\frac{f}{\rho}}\,S\cos\theta - \frac{2.318}{10^7}\frac{f}{\rho}S^2\cos 2\theta \\ &+ \frac{6.355}{10^{11}}\frac{f}{\rho}\sqrt{\frac{f}{\rho}}\,S^3\cos 3\theta - \frac{9.020}{10^{15}}\frac{f^2}{\rho^2}S^4\theta\sin 4\theta \\ &- \frac{20.77}{10^{15}}\frac{f^2}{\rho^2}S^4\cos 4\theta\left(3.842 + \log_{10}\frac{1}{S}\sqrt{\frac{\rho}{f}}\right)\Bigg]\end{aligned} \quad [37]$$

where h and S are in feet and $\theta = \cos^{-1}\dfrac{h_a + h_b}{S_{ab}} = \sin^{-1}\dfrac{H_{ab}}{S_{ab}}$.

It will be noted that with the corrections given by [34]–[37] omitted, [30]–[33] are independent of conductor height above ground.

Neglecting the terms in ΔR_{aa-g} and ΔR_{ab-g}, after the first, results in an error of about 2% in R_{aa-g} and R_{ab-g} if $k = 0.2$, and an error of the

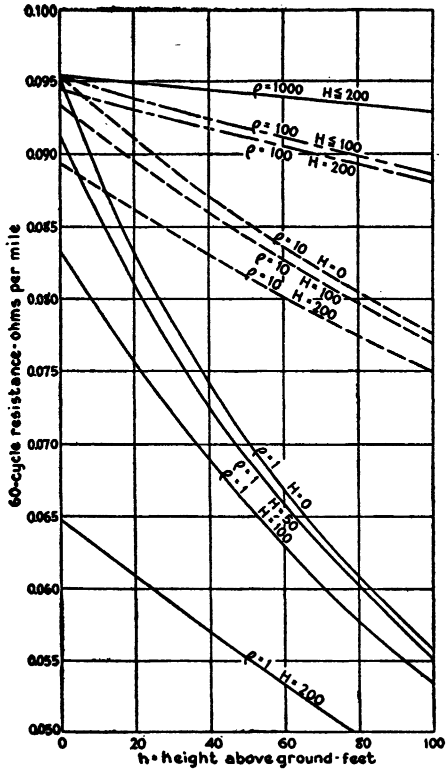


Fig. 3(*a*). 60-cycle self- and mutual resistances of the earth.

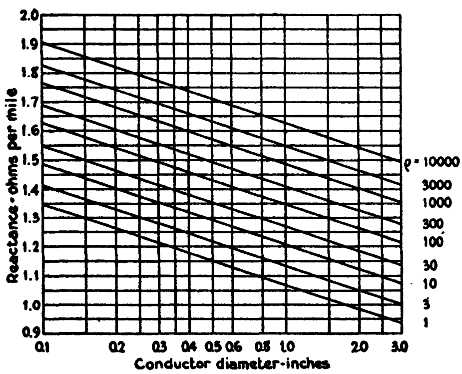


Fig. 3(*b*). 60-cycle self-reactance external to the conductor. Height of conductor above ground neglected.

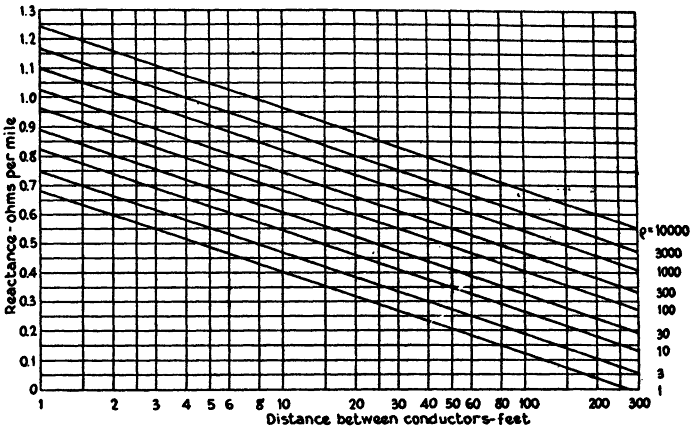


FIG. 3(*c*). 60-cycle mutual reactance between two conductors with earth return. Height of conductors above ground neglected.

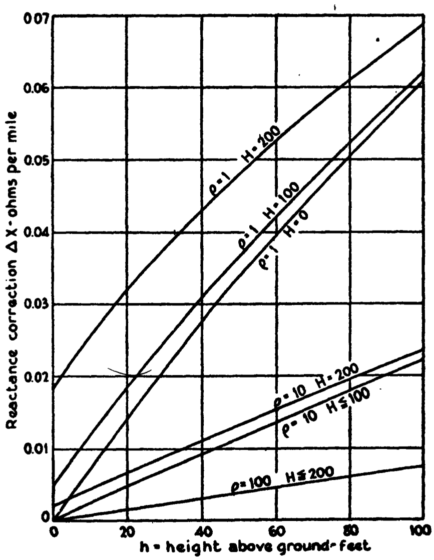


FIG. 3(*d*). Corrections to 60-cycle self- and mutual reactances to account for height of conductors above ground.

ρ = earth resistivity in ohms per meter cube
H = horizontal spacing between two conductors in feet
(H = 0 for self-impedance)

order of 15% if $k = 0.6$. R_{aa-g} and R_{ab-g}, for frequencies of 60 cycles or less, are less than 0.1 ohm per mile. In calculating Z_{aa-g}, R_{aa-g} is added to the line resistance r_c, which, except for lines with resistance less than 0.1 ohm per mile, reduces the error in the resistance component of Z_{aa-g} by more than half the errors given above. Moreover, the resistance components R_{aa-g} and R_{ab-g} must be combined with the reactance components X_{aa-g} and X_{ab-g} to give the self- and mutual

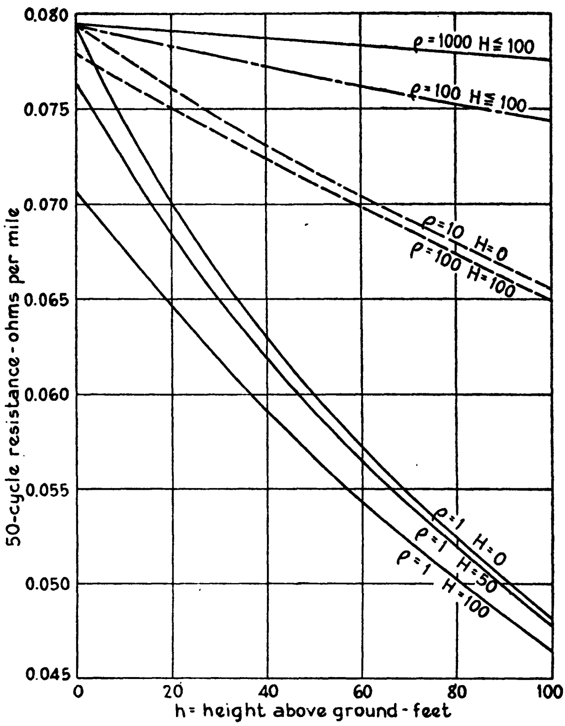


FIG. 4(*a*). 50-cycle self- and mutual resistances of the earth.

ρ = earth resistivity in ohms per meter cube
H = horizontal spacing between two conductors in feet
($H = 0$ for self-impedance)

impedances Z_{aa-g} and Z_{ab-g}, respectively. In systems with a frequency of 60 cycles or less, where the zero-sequence impedance of the transmission line is only part of the total zero-sequence impedance, the error in neglecting terms after the first in ΔR_{aa-g} and ΔR_{ab-g} is but slight. In fact, ΔR_{aa-g} and ΔR_{ab-g} may be neglected in most system studies with satisfactory results.

Neglecting the terms in ΔX_{aa-g} and ΔX_{ab-g}, after the first, results in an error of less than 0.1% in X_{aa-g} or X_{ab-g} if $k = 0.2$ and less than 3% if $k = 0.6$. If ΔX_{aa-g} and ΔX_{ab-g} are neglected entirely, the error in X_{aa-g} and X_{ab-g} with $k = 0.2$ will be approximately 2%; with $k = 0.6$, approximately 20%.

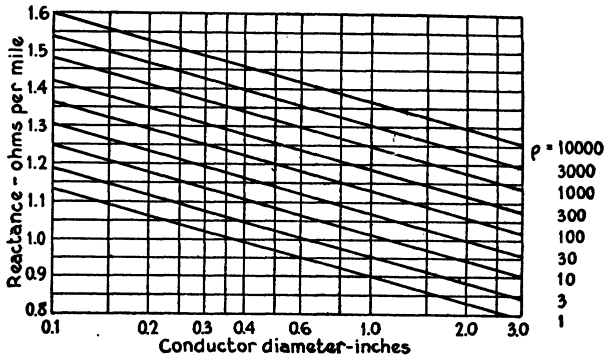


FIG. 4(*b*). 50-cycle self-reactance external to the conductor. Height of conductor above ground neglected.

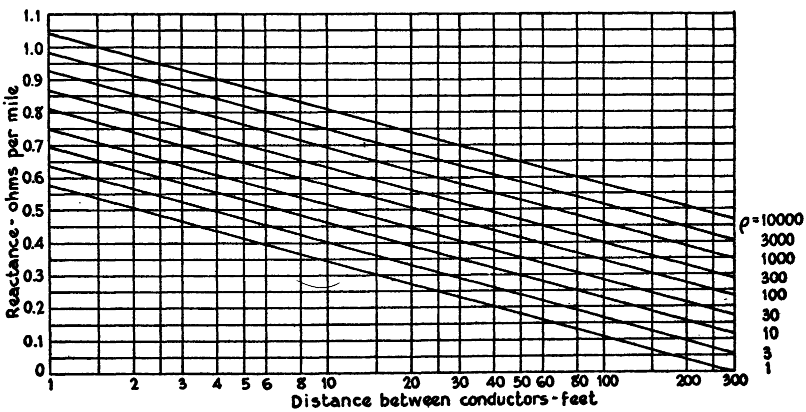


FIG. 4(*c*). 50-cycle mutual reactance between two conductors with earth return. Height of conductors above ground neglected.

Retaining the first term in ΔR_{aa-g}, ΔR_{ab-g}, ΔX_{aa-g}, and ΔX_{ab-g}, the following equations giving R_{aa-g}, R_{ab-g}, X_{aa-g}, and X_{ab-g} in ohms per mile, where d is in inches, h_a and h_b in feet, and ρ in ohms per meter cube, will be found satisfactory for the usual transmission line at frequency of 60 cycles or less.

$$R_{aa-g} = 0.00159f - 1.63 \times 10^{-6} h_a f \sqrt{\frac{f}{\rho}} \qquad [38]$$

$$R_{ab-g} = 0.00159f - 1.63 \times 10^{-6} \frac{h_a + h_b}{2} f \sqrt{\frac{f}{\rho}} \qquad [39]$$

$$X_{aa-g} = 2\pi f \times 10^{-3} \left[0.7411 \left(\log_{10} \frac{\sqrt{\rho}}{d} - \log_{10} \sqrt{f} \right) + 3.4944 + 0.00026 h_a \sqrt{\frac{f}{\rho}} \right] \qquad [40]$$

$$X_{ab-g} = 2\pi f \times 10^{-3} \left[0.7411 \left(\log_{10} \frac{\sqrt{\rho}}{s} - \log_{10} \sqrt{f} \right) + 2.4715 + 0.00026 \frac{h_a + h_b}{2} \sqrt{\frac{f}{\rho}} \right] \qquad [41]$$

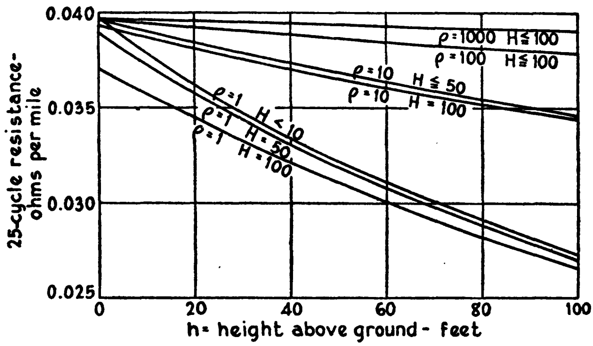


FIG. 5(*a*). 25-cycle self- and mutual resistances of the earth.

ρ = earth resistivity in ohms per meter cube
H = horizontal spacing between two conductors in feet
(H = 0 for self-impedance)

Resistance of the Earth from Curves. R_{aa-g} in [15] is the resistance of the return path through the earth when a single conductor with earth return is considered. R_{ab-g} in [16] is the resistance of the common earth return path when the mutual impedance between two conductors is considered. From [30] and [34], R_{aa-g} is a function of frequency f, earth resistivity ρ, and the height of the conductor above ground h. R_{ab-g}, given in [31] and [35], is a function of f and ρ, average height above ground of the conductors $\frac{1}{2}(h_a + h_b)$, and the horizontal spacing between conductors H, if θ is replaced by $\cos^{-1} (h_a + h_b)/S = \sin^{-1} H/S$. Figures 3($a$), 4($a$), and 5($a$) give R_{aa-g} and R_{ab-g} in ohms per mile at 60, 50, and 25 cycles, respectively, as influenced by earth resistivity and conductor arrangement, with all terms in ΔR_{aa-g} and ΔR_{ab-g} given by [34] and [35] included.

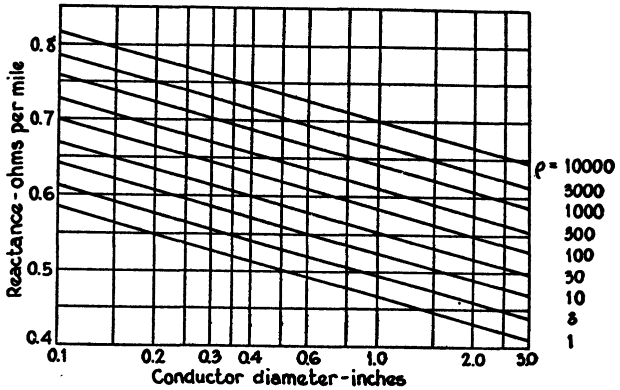


FIG. 5(b). 25-cycle self-reactance external to the conductor. Height of conductor above ground neglected.

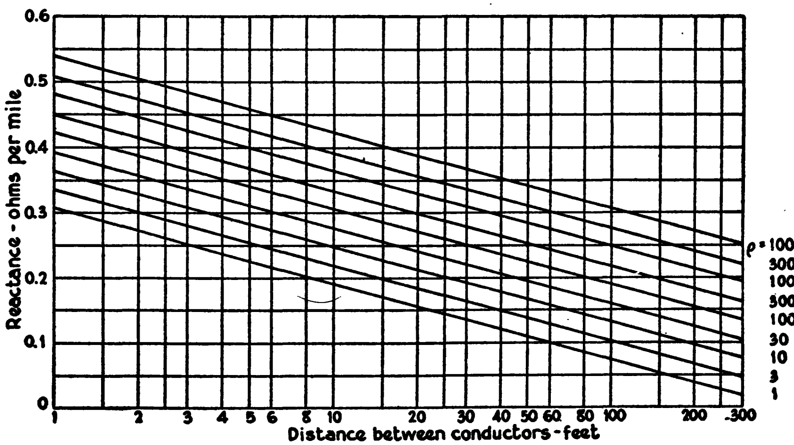


FIG. 5(c). 25-cycle mutual reactance between two conductors with earth return. Height of conductors above ground neglected.

Effect of Height of Conductors above Ground upon Self- and Mutual Reactances of Ground-Return Circuits. In [36] and [37] for ΔX_{aa-g} and ΔX_{ab-g}, respectively, are grouped terms in the self- and mutual reactances X_{aa-g} and X_{ab-g}, which involve height of conductors above ground. These equations, plotted in Fig. 3(d), for a frequency of 60 cycles, indicate the magnitudes of these terms as functions of earth resistivity and conductor arrangement. For frequencies lower than 60 cycles, ΔX_{aa-g} and ΔX_{ab-g} are approximately $(f/60)^{3/2}$ times their respective 60-cycle values.

Self-Reactance External to the Conductor of a Conductor with Earth Return. Figure 3(*b*) gives X_{aa-g}, the self-reactance external to the conductor of the loop circuit consisting of a conductor with earth return as a function of conductor diameter d and earth resistivity ρ at 60 cycles. Figure 3(*b*) was plotted from [32] with ΔX_{aa-g} neglected. The internal reactance x_i of the conductor and ΔX_{aa-g} from Fig. 3(*d*) are to be added to obtain total reactance of the loop circuit. Figures 4(*b*) and 5(*b*) are the corresponding figures for 50 and 25 cycles, respectively.

Mutual Reactance between Two Conductors with Common Earth Return. Figure 3(*c*) gives X_{ab-g}, the mutual reactance between two conductors with common earth return, as a function of spacing s between conductors in feet and earth resistivity ρ at 60 cycles, plotted from [33] with ΔX_{ab-g} neglected. Figures 4(*c*) and 5(*c*) are corresponding figures for 50 and 25 cycles, respectively.

Figures 3, 4, and 5, parts (*b*) and (*c*), are similar to charts for self- and mutual reactances drawn by Mr. J. E. Clem in 1931 but not published with his paper[5]; similar curves have been published[6] in which the terms involving conductor height above ground are included for an average height of 40 feet.

ZERO-SEQUENCE SELF- AND MUTUAL IMPEDANCES

Before determining the zero-sequence self-impedance and the mutual impedances between the zero-sequence network and the positive- and negative-sequence networks as defined in [7], Chapter VIII, reference for zero-sequence voltages and the zero-sequence equivalent circuit for a three-phase transmission circuit of negligible capacitance will be discussed.

Reference for Zero-Sequence Voltages. If the earth had infinite conductivity, its potential at all points would be the same. As its conductivity is finite, there will be a difference in potential between two given ground points when current flows between them. Zero-sequence equivalent circuits, based on Carson's equations for the self-impedances of conductors with earth return and the mutual impedances between two conductors with common earth return, do not give differences in potential of ground points. To refer zero-sequence voltages at various points of the system to a common ground point would require that the self- and mutual impedances of ground-return circuits be separated into equivalent self- and mutual impedances external to the earth and equivalent impedance of earth return. In Carson's equations as given, such a separation has not been made,[7] nor is it required for determining zero-sequence currents and zero-sequence voltages at points throughout the system *referred to ground at these points*. The voltage at any point referred to the ground at that particular point is usually of more interest than the voltage referred to some

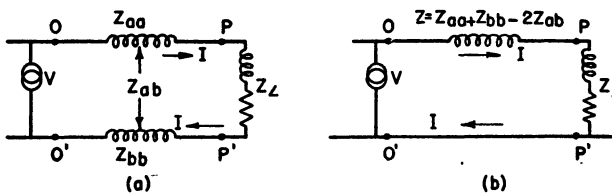


FIG. 6(*a*). Single-phase loop circuit. (*b*) Equivalent circuit for (*a*) to be used to determine I or the voltage across Z_L.

arbitrarily chosen ground a distance away. In the usual system problem, therefore, it is unnecessary to separate self- and mutual impedances with earth return into their component parts. Zero-sequence equivalent circuits with the *identity of the ground-return path retained* are discussed at the end of this chapter. In the following discussion, zero-sequence impedances and zero-sequence equivalent circuits are based on self- and mutual impedances of conductors with earth return.

In terms of self- and mutual impedances, the zero-sequence network is analogous to the loop circuit shown in Fig. 6(*a*), where a single-phase voltage V is applied between points O and O' and the same current I flows in both conductors. The current I and the voltage between P and P' are the same in Fig. 6(*b*) as in Fig. 6(*a*). Therefore, if the impedance $Z = (Z_{aa} + Z_{bb} - 2Z_{ab})$ is given, it is unnecessary to know the values of the separate impedances Z_{aa}, Z_{bb}, and Z_{ab} or the equivalent impedances $(Z_{aa} - Z_{ab})$ and $(Z_{bb} - Z_{ab})$ to determine the current or to determine the voltage at P referred to P'. It should be noted, however, that the voltage between O and P or between O' and P' cannot be determined from Fig. 6(*b*).

Zero-Sequence Equivalent Circuit in Which Voltage to Ground at Any System Point Is Referred to Ground at That Particular Point. A zero-sequence equivalent circuit similar to Fig. 6(*b*) is shown in Fig. 7

for a symmetrical three-phase transmission line between O and P. Z_0, the zero-sequence impedance per phase, is placed between O and P, with no impedance in the return path. The ground points at O and P are represented as zero-potential points. The voltages at O and P in Fig. 7, referred to the zero-potential bus, are the zero-sequence voltages at O and P referred to the ground at O and P, respectively.

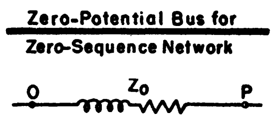


FIG. 7. Zero-sequence equivalent circuit for transmission circuit between O and P.

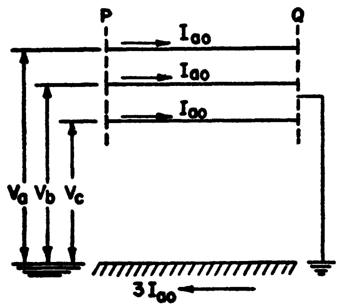


FIG. 8. Zero-sequence currents flowing from P to Q and returning in the earth.

Determination of Zero-Sequence Self-Impedances and Mutual Impedances between the Sequence Networks When Voltages to Ground at Any System Point Are Referred to Ground at That Particular Point

One Three-Phase Circuit. The zero-sequence impedances of the three phases a, b, and c of a three-phase circuit will be indicated by Z_{a0}, Z_{b0}, and Z_{c0}, respectively, and defined (see Chapter VIII) as the ratios of the voltage drops in the three phases to the currents in the corresponding phases with only zero-sequence currents flowing in the circuit. Figure 8 shows zero-sequence currents only flowing in the three phases of a circuit without ground wires. The three phases are connected at one end and grounded; V_a, V_b, and V_c are the voltage drops in phases a, b, and c, respectively, in the direction of current flow indicated by arrows.

No Ground Wires. The voltage drops in the three loop circuits, each consisting of conductor and ground, in Fig. 8 are

$$\begin{aligned} V_a &= I_{a0}(Z_{aa-g} + Z_{ab-g} + Z_{ac-g}) \\ V_b &= I_{a0}(Z_{bb-g} + Z_{ab-g} + Z_{bc-g}) \qquad [42] \\ V_c &= I_{a0}(Z_{cc-g} + Z_{ac-g} + Z_{bc-g}) \end{aligned}$$

From [42], the zero-sequence impedances of phases a, b, and c are

$$Z_{a0} = \frac{V_a}{I_{a0}} = Z_{aa-g} + Z_{ab-g} + Z_{ac-g}$$

$$Z_{b0} = \frac{V_b}{I_{a0}} = Z_{bb-g} + Z_{ab-g} + Z_{bc-g} \qquad [43]$$

$$Z_{c0} = \frac{V_c}{I_{a0}} = Z_{cc-g} + Z_{ac-g} + Z_{bc-g}$$

Substituting Z_{a0}, Z_{b0}, and Z_{c0} from [43] in [7] of Chapter VIII, the zero-sequence self-impedance Z_{00} and the mutual impedances Z_{10} and Z_{20} are

$$Z_{00} = \tfrac{1}{3}(Z_{aa-g} + Z_{bb-g} + Z_{cc-g}) + \tfrac{2}{3}(Z_{ab-g} + Z_{ac-g} + Z_{bc-g}) \qquad [44]$$

$$Z_{10} = \tfrac{1}{3}(Z_{aa-g} + aZ_{bb-g} + a^2Z_{cc-g}) - \tfrac{1}{3}(a^2Z_{ab-g} + aZ_{ac-g} + Z_{bc-g}) \qquad [45]$$

$$Z_{20} = \tfrac{1}{3}(Z_{aa-g} + a^2Z_{bb-g} + aZ_{cc-g}) - \tfrac{1}{3}(aZ_{ab-g} + a^2Z_{ac-g} + Z_{bc-g}) \qquad [46]$$

Zero-Sequence Self-Impedance of a Three-Phase Circuit without Ground Wires. Equation [44] gives Z_{00}, the zero-sequence self-impedance of an unsymmetrical untransposed circuit or the average zero-sequence impedance of a completely transposed circuit, in terms of self- and mutual impedances of ground return circuits. With identical conductors in the three-phase circuit, Z_{aa-g}, Z_{bb-g}, and Z_{cc-g}, which are functions of frequency, conductor diameter, earth resistivity, and height of conductor above ground, differ only in the terms involving height above ground h. For equal conductor heights, they are equal. Figures 3(a) and (d) show the effect on earth resistance and self-reactance, respectively, of varying h (with $H = 0$) for a given earth resistivity at a frequency of 60 cycles. Since the curves are approximately straight lines, the average value of the three self-impedances with ground return when conductors are not at the same height above ground will be substantially that corresponding to the average height above ground of the three conductors. Therefore,

$$\tfrac{1}{3}(Z_{aa-g} + Z_{bb-g} + Z_{cc-g}) = Z_{\overline{aa}-g} = (r_c + jR_{\overline{aa}-g}) + j(X_{\overline{aa}-g} + x_i)$$

where $Z_{\overline{aa}-g}$ indicates the average value of the three self-impedances with earth return and $R_{\overline{aa}-g}$ and $X_{\overline{aa}-g}$ are determined for a conductor with $h = \tfrac{1}{3}(h_a + h_b + h_c)$, $H = 0$.

The second term in [44] is twice the average value of the three mutual impedances Z_{ab-g}, Z_{ac-g}, and Z_{bc-g}. If ΔX_{ab-g} (which is the sum of the terms involving height above ground) is neglected in

[33], at a given frequency and earth resistivity, the mutual reactance depends upon s, the geometric mean spacing between conductors.

The average value of the terms in Z_{ab-g}, Z_{ac-g}, and Z_{bc-g} involving height above ground will be approximately that corresponding to the average height of the three conductors and the average horizontal spacing. Therefore, $\frac{2}{3}(Z_{ab-g} + Z_{ac-g} + Z_{bc-g}) = 2Z_{\overline{ab}-g} = 2(R_{\overline{ab}-g} + jX_{\overline{ab}-g})$ ohms per mile, where $Z_{\overline{ab}-g}$ is the average value of the three mutual impedances with earth return and is determined with $s = \sqrt[3]{s_{ab}s_{ac}s_{bc}}$, $h = \frac{1}{3}(h_a + h_b + h_c)$, and $H = \frac{1}{3}(H_{ab} + H_{ac} + H_{bc})$.

The zero-sequence self-impedance in ohms per mile may be written

$$\begin{aligned} Z_{00} &= r_{00} + jx_{00} = Z_{\overline{aa}-g} + 2Z_{\overline{ab}-g} \\ &= (r_c + R_{\overline{aa}-g} + 2R_{\overline{ab}-g}) + j(X_{\overline{aa}-g} + 2X_{\overline{ab}-g} + x_i) \quad [47] \end{aligned}$$

where bars over the subscripts indicate average values.

At 60 cycles, $R_{\overline{aa}-g}$ and $R_{\overline{ab}-g}$ can be read from Fig. 3(a) corresponding to $h = \frac{1}{3}(h_a + h_b + h_c)$, with $H = 0$ for $R_{\overline{aa}-g}$ and $H = \frac{1}{3}(H_{ab} + H_{ac} + H_{bc})$ for $R_{\overline{ab}-g}$. $X_{\overline{aa}-g}$ with conductor height neglected can be read from Fig. 3(b) corresponding to conductor diameter d and earth resistivity ρ. $X_{\overline{ab}-g}$ with conductor heights neglected can be read from Fig. 3(c) corresponding to earth resistivity ρ and the geometric mean spacing between conductors s. Figure 3(d) indicates the correction to $X_{\overline{aa}-g}$ and $X_{\overline{ab}-g}$ for various values of h and H at 60 cycles. These terms are relatively small, and the difference in retaining or neglecting them in most problems is insignificant.

The following problem illustrates the procedure in calculating Z_{00} from [47].

Problem 1. Calculate the zero-sequence self-impedance in ohms per mile of a three-phase, 60-cycle overhead transmission circuit of 4/0 copper, 19 strands, 10 feet horizontal spacing between adjacent conductors, height above ground of all conductors 40 feet, earth resistivity 100 ohms per meter cube.

Solution. From Table II, Appendix B:

$$\begin{aligned} d &= 0.528 \text{ inch} \\ r_c &= 0.276 \text{ ohm per mile} \\ x_i &= 0.034 \text{ ohm per mile} \end{aligned}$$

From the given configuration:

$$\begin{aligned} s &= \sqrt[3]{10 \cdot 10 \cdot 20} = 12.6 \text{ feet} \\ h &= 40 \text{ feet} \\ H &= \tfrac{1}{3}(10 + 10 + 20) = 13.3 \text{ feet} \end{aligned}$$

From Fig. 3(a), with $\rho = 100$, $h = 40$ feet, and $H = 0$ and 13.3 feet for self- and mutual reactance, respectively,

$$R_{aa-g} = 0.0924 \text{ ohm per mile}$$
$$R_{ab-g} = 0.0924 \text{ ohm per mile}$$

From Fig. 3(*b*), with $\rho = 100$ and $d = 0.528$ inch,

$$X_{\overline{aa}-g} \text{ (conductor height neglected)} = 1.425 \text{ ohms per mile}$$

From Fig. 3(*c*), with $\rho = 100$ and $s = 12.6$ feet,

$$X_{\overline{ab}-g} \text{ (conductor height neglected)} = 0.655 \text{ ohm per mile}$$

The corrections to allow for conductor height above ground read from Fig. 3(*d*), with $\rho = 100$, $h = 40$, and $H = 0$ and 13.3 feet for self- and mutual reactances, respectively, are

$$\Delta X_{\overline{aa}-g} = 0.003 \text{ ohm per mile}$$
$$\Delta X_{\overline{ab}-g} = 0.003 \text{ ohm per mile}$$

Applying these corrections,

$$X_{\overline{aa}-g} = 1.428 \text{ ohms per mile}$$
$$X_{\overline{ab}-g} = 0.658 \text{ ohm per mile}$$

Substituting in [47],

$$\begin{aligned} Z_{00} &= 0.276 + 3(0.0924) + j[1.428 + 2(0.658) + 0.034] \\ &= 0.55 + j2.78 \text{ ohms per mile} \end{aligned}$$

Zero-Sequence Reactance Charts for Three-Phase Circuits without Ground Wires. Charts have been published[8] giving the zero-sequence self-reactance without ground wires of three-phase circuits at 25, 50, and 60 cycles as functions of earth resistivity and circuit geometric mean radius of separation in feet, defined as the cube root of the product of the geometric mean radius of the conductors and the square of the geometric mean distance between conductors.

From [47], the zero-sequence self-reactance x_{00} in ohms per mile is

$$x_{00} = X_{\overline{aa}-g} + 2X_{\overline{ab}-g} + x_i \qquad [48]$$

X_{aa-g} and X_{ab-g} are given by [32] and [33], respectively, with d in inches and s in feet. If Δ_{aa-g} and Δ_{ab-g}, the terms involving conductor heights above ground, are neglected and the diameter of the conductors d in inches is replaced by twice its radius r in feet multiplied by 12, the average self- and mutual impedances in ohms per mile are, respectively,

$$X_{\overline{aa}-g} = 2\pi f 10^{-3}\left(0.7411 \log_{10} \frac{1}{r}\sqrt{\frac{\rho}{f}} + 2.4715\right) \qquad [49]$$

$$X_{\overline{ab}-g} = 2\pi f 10^{-3}\left(0.7411 \log_{10} \frac{1}{s}\sqrt{\frac{\rho}{f}} + 2.4715\right) \qquad [50]$$

where r = conductor radius in feet; s = geometric mean spacing between conductors in feet.

The internal reactance x_i of a conductor in ohms per mile is expressed[9] in terms of the ratio of its radius r to its *equivalent self geometric mean radius gmr* by the following equation in which r and gmr are in the same unit.

$$x_i = 2\pi f 10^{-3}\left(0.7411 \log_{10}\frac{r}{gmr}\right) \qquad [51]$$

Wire tables in Appendix B give the internal reactance x_i in ohms per mile and the conductor self geometric mean radius gmr in feet, either of which may be calculated from the other by means of [51].

Substituting [49], [50], and [51] in [48], x_{00} in ohms per mile, with terms involving heights of conductors above ground neglected, is given by the following equation, where gmr and s are in feet.

$$x_{00} = 2\pi f 10^{-3}\left(0.7411 \log_{10}\frac{1}{s^2(gmr)}\left(\frac{\rho}{f}\right)^{3/2} + 7.414\right)$$

$$= f\left(0.01397 \log_{10}\frac{\sqrt{\frac{\rho}{f}}}{\sqrt[3]{s^2(gmr)}} + 0.0466\right) \text{ ohms per mile} \qquad [52]$$

Figures 9(*a*), (*b*), and (*c*) (taken from reference 10) give x_{00}, the zero-sequence self-reactance of a single three-phase circuit without ground wires at frequencies of 60, 50, and 25 cycles, respectively, as influenced by conductor sizes and materials, geometric mean spacing between conductors, and earth resistivity. In calculating Fig. 9, the average value of the conductor heights above ground h was taken as 50 feet and the average horizontal spacing H as zero. Figure 3(*d*) indicates that the correction for other values of h or H at frequencies of 60 cycles or less is insignificant for conventional three-phase circuits and the usual values of earth resistivity. The total zero-sequence reactance is given in the charts of Fig. 9 for a large number of the commonly used conductors. These include stranded and hollow copper conductors, aluminum cable steel reinforced (A.C.S.R.) of both multiple- and single-layer types of conductors, and Copperweld conductors. For each of the conductors listed, there is a short horizontal mark or index which corresponds to the conductor equivalent self geometric mean radius in feet. The index for a conductor not listed can be determined from its geometric mean radius in feet by interpolation between conductors of known geometric mean radii (see Appendix B)

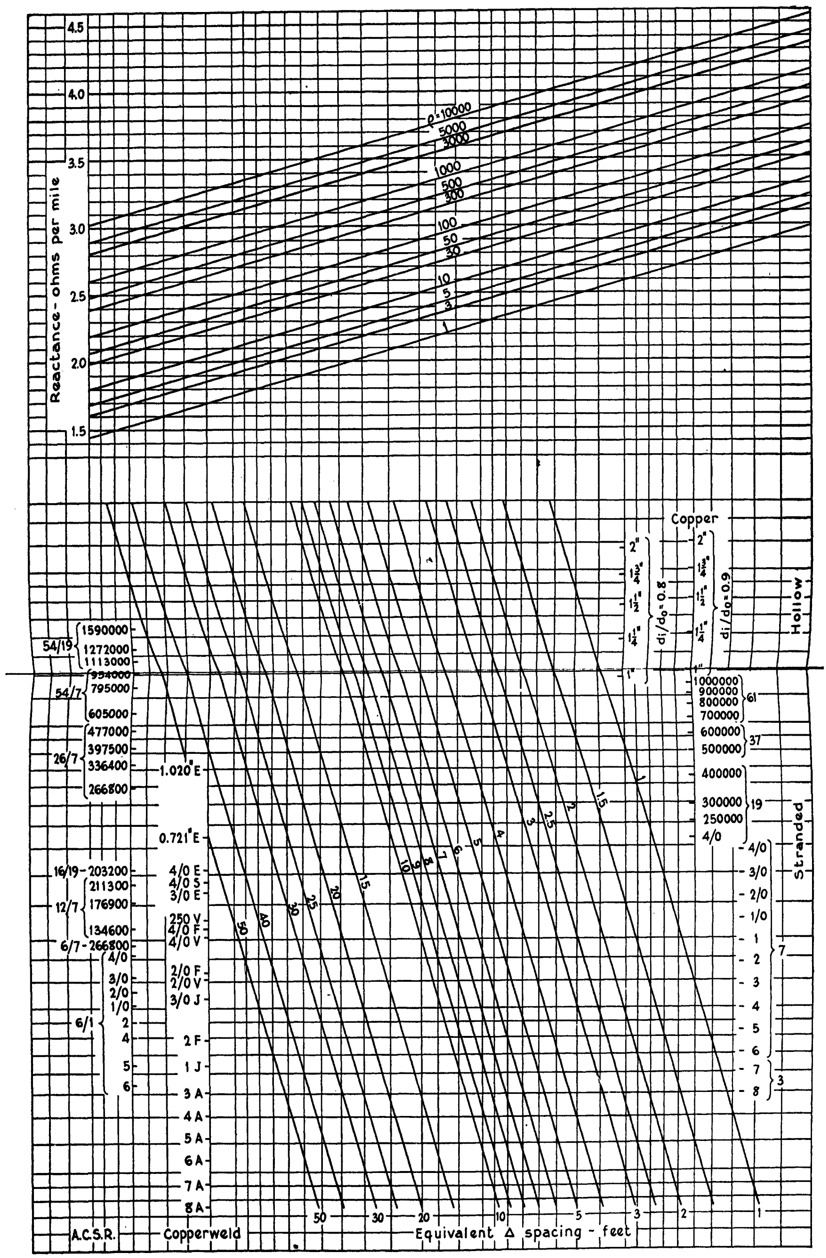


FIG. 9(a). 60-cycle, zero-sequence reactance of overhead transmission lines without ground wires.

ρ = earth resistivity in ohms per meter cube

Average height of conductors above ground, 50 ft.

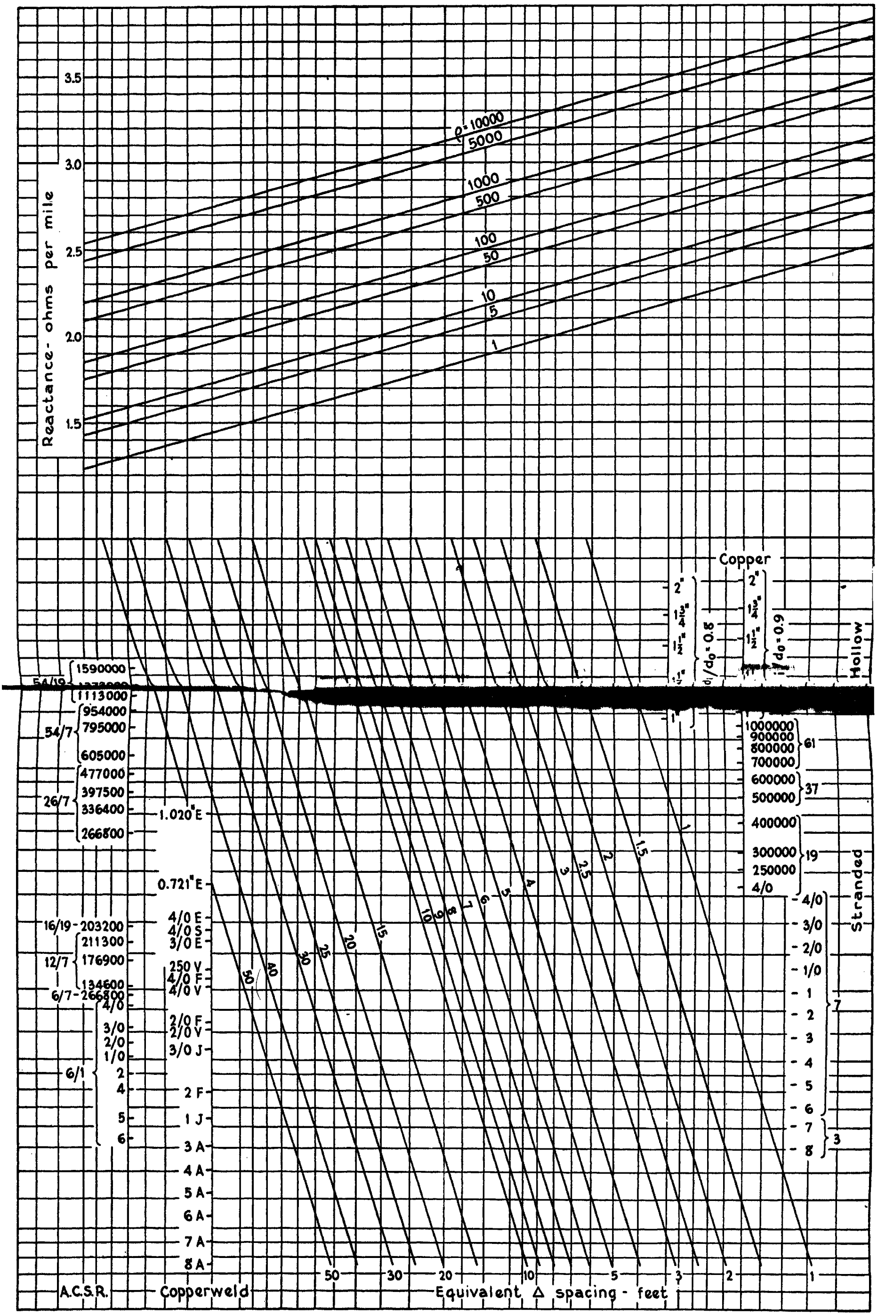


Fig. 9(b). 50-cycle, zero-sequence reactance of overhead transmission lines without ground wires.

ρ = earth resistivity in ohms per meter cube

Average height of conductors above ground, 50 ft.

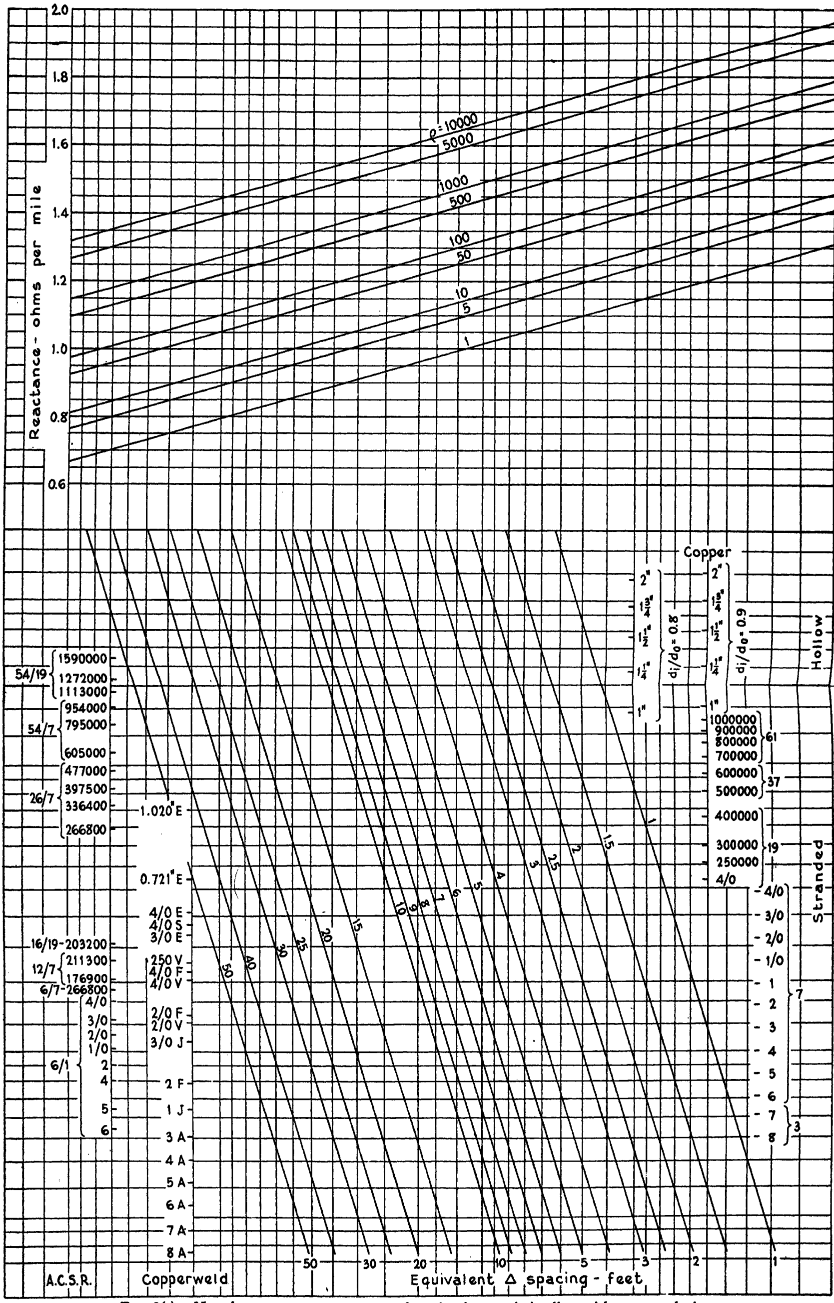


FIG. 9(c). 25-cycle, zero-sequence reactance of overhead transmission lines without ground wires.

ρ = earth resistivity in ohms per meter cube

Average height of conductors above ground, 50 ft.

with indexes given. For a slight change in *gmr* there is but a slight change in x_{00}.

Zero-Sequence Reactance of a Three-Phase Circuit without Ground Wires Read from Figs. 9(*a*), (*b*), and (*c*). On the chart corresponding to the given frequency, first locate the index corresponding to the type of line conductor used; follow horizontally across from this index to the diagonal line giving the equivalent Δ spacing of the conductors (the cube root of the product of the three spacings between conductors); from this point, move vertically upwards to the line corresponding to the average earth resistivity; then move horizontally to the left to read the zero-sequence reactance. (Note that the indexes for A.C.S.R. correspond to a current density of 1200 amperes per square inch; lower current densities give slightly smaller reactances.)

Problem 2. Find the zero-sequence self-impedance per mile of a three-phase 60-cycle transmission line consisting of three 4/0, 19-strand copper conductors spaced horizontally 10 feet apart, height above ground 30 feet, earth resistivity 1000 ohms per meter cube.

Solution. The average height above ground is 30 feet; the average horizontal spacing is $\frac{1}{3}(10 + 10 + 20) = 13.3$ feet; the equivalent Δ spacing is $\sqrt[3]{10 \times 10 \times 20} = 12.6$ feet. From Appendix B, Table II, $r_c = 0.276$ ohm per mile. From Fig. 3(*a*), $R_{\overline{aa}-g} = R_{\overline{ab}-g} = 0.0947$ ohm per mile.

$$r_{00} = r_c + R_{\overline{aa}-g} + 2R_{\overline{ab}-g} = 0.276 + 0.284 = 0.560 \text{ ohm per mile}$$

From Fig. 9(*a*), the zero-sequence reactance read directly is $x_{00} = 3.19$ ohms per mile. The zero-sequence self-impedance $Z_{00} = 0.56 + j3.19$ ohms per mile.

Three-Phase Circuits with Ground Wires

The self-impedance Z_{ww-g} of a ground wire w with earth return and the mutual impedance Z_{aw-g} between ground wire w and conductor a with common earth return are determined just as the self- and mutual impedance of conductors with earth return. Outside diameters of ground wires are, in general, smaller than conductor diameters and their resistances and internal reactances are higher. However, in some circuits ground wires of the same material as the conductor are used with diameters of the same size or only slightly smaller. The wire tables of Appendix B give resistances and internal reactances of some of the ground wires used in present-day overhead transmission circuits. In ground wires of magnetic materials, the resistances and internal reactances vary with the current in the wires. In such cases it is necessary to estimate the ground wire current before selecting the resistance and internal reactance to be used in the self-impedance of the ground wire with earth return.

In developing equations for the sequence impedances of transmission circuits with ground wires, it will be assumed that the ground wires and earth are connected in parallel at the two ends of the section of circuit under consideration and that the ground wire current is uniform along the wire. Actually, ground wires are grounded at every tower, but the connection is through the tower footing resistance. There are many of these tower footing resistances in parallel between the ground wire and earth in a section of several miles. Measurements[11] made of currents in ground wires between towers show different values within the first few tower lengths from a ground fault; but at greater distances from the fault, the ground wire current is substantially uniform along the line. When the fault is a flashover between conductor and tower, there is only the arc impedance between the faulted conductor and the ground wire which is at tower potential, but the path to ground is through the tower footing resistance. The ground wire therefore carries a larger proportion of fault current in the first tower length and also in the next few adjacent tower lengths than its average value. Since the equations given here are based on the assumption of uniform ground wire current, the higher values of ground wire currents measured near the fault will not be obtained from them. If there are two or more ground wires connected to a common ground at the ends of the line, the grounding resistance should be multiplied by the number of ground wires before being included in the self-impedances of the individual ground wires.

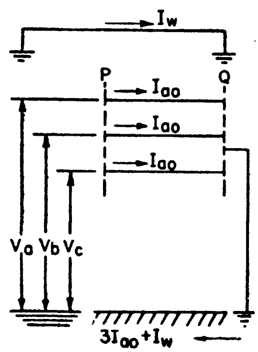


FIG. 10. Zero-sequence currents flowing from P to Q and returning in the ground and ground wire w.

One Ground Wire. With zero-sequence currents only flowing in the three phases of the circuit shown in Fig. 10, the voltage drops in the four-loop circuits consisting of the three conductors and the ground wire, each with ground return, are

$$\begin{aligned} V_a &= I_{a0}(Z_{aa-g} + Z_{ab-g} + Z_{ac-g}) + I_w Z_{aw-g} \\ V_b &= I_{a0}(Z_{bb-g} + Z_{ab-g} + Z_{bc-g}) + I_w Z_{bw-g} \\ V_c &= I_{a0}(Z_{cc-g} + Z_{ac-g} + Z_{bc-g}) + I_w Z_{cw-g} \\ 0 &= I_w Z_{ww-g} + I_{a0}(Z_{aw-g} + Z_{bw-g} + Z_{cw-g}) \end{aligned} \qquad [53]$$

From the last of the above equations,

$$I_w = -I_{a0}\frac{Z_{aw-g} + Z_{bw-g} + Z_{cw-g}}{Z_{ww-g}} = -3I_{a0}\frac{\overline{Z_{aw-g}}}{Z_{ww-g}} \qquad [54]$$

$Z_{\overline{aw}-g}$ in [54] is the average value of the mutual impedances with earth return between ground wire and conductors and is determined for a spacing $s_{\overline{aw}}$, where

$s_{\overline{aw}} = \sqrt[3]{s_{aw}s_{bw}s_{cw}}$ = geometric mean distance between ground wire and conductors

The proportion of total zero-sequence current returning in the ground wire is

$$\frac{-I_w}{3I_{a0}} = \frac{Z_{\overline{aw}-g}}{Z_{ww-g}} \tag{55}$$

Replacing I_w in [53] by its value from [54] and solving for Z_{a0}, Z_{b0}, and Z_{c0},

$$\begin{aligned} Z_{a0} &= \frac{V_a}{I_{a0}} = Z_{aa-g} + Z_{ab-g} + Z_{ac-g} - 3\frac{Z_{\overline{aw}-g}}{Z_{ww-g}} Z_{aw-g} \\ Z_{b0} &= \frac{V_b}{I_{a0}} = Z_{bb-g} + Z_{ab-g} + Z_{bc-g} - 3\frac{Z_{\overline{aw}-g}}{Z_{ww-g}} Z_{bw-g} \\ Z_{c0} &= \frac{V_c}{I_{a0}} = Z_{cc-g} + Z_{ac-g} + Z_{bc-g} - 3\frac{Z_{\overline{aw}-g}}{Z_{ww-g}} Z_{cw-g} \end{aligned} \tag{56}$$

Substituting [56] in [7] of Chapter VIII, in ohms per mile,

$$Z_{00-w} = Z_{00} - 3\frac{(Z_{\overline{aw}-g})^2}{Z_{ww-g}} \tag{57}$$

$$Z_{10-w} = Z_{02-w} = Z_{10} - \frac{Z_{\overline{aw}-g}}{Z_{ww-g}}(Z_{aw-g} + aZ_{bw-g} + a^2Z_{cw-g}) \tag{58}$$

$$Z_{20-w} = Z_{01-w} = Z_{20} - \frac{Z_{\overline{aw}-g}}{Z_{ww-g}}(Z_{aw-g} + a^2Z_{bw-g} + aZ_{cw-g}) \tag{59}$$

where the additional subscript w indicates one ground wire w and Z_{00}, Z_{10}, and Z_{20} are given by [44], [45], and [46], respectively, with no ground wire. It is shown in [96] that $Z_{02-w} = Z_{10-w}$ and $Z_{01-w} = Z_{20-w}$.

It will be noted from [57]–[59] that the zero-sequence self-impedance Z_{00-w} and the mutual impedances Z_{10-w} and Z_{20-w} of a three-phase circuit with one ground wire can be determined from Z_{00}, Z_{10}, and Z_{20}, respectively, the corresponding impedances without ground wire, by applying corrections which depend only upon the self-impedance of the ground wire with earth return and the mutual impedances between ground wires and conductors with common earth return.

Problem 3. Find the zero sequence self-impedance in ohms per mile of the circuit given in Problem 1 with one ground wire 8 feet above the center conductor. (*a*) The ground wire is $\frac{3}{8}$ inch high strength steel. Assume the current in the ground wires to be such that its resistance is 6.03 ohms per mile and internal reactance 0.61 ohm per mile. (*b*) The ground wire is 00 copper.

Solution.

$$Z_{00} = 0.55 + j2.78 \text{ ohms per mile (from Problem 1)}$$
$$\rho = 100 \text{ ohms per meter cube}$$
$$s_{\overline{aw}} = \sqrt[3]{8(10^2 + 8^2)} = 10.9 \text{ feet; } h = 48 \text{ feet}$$

(*a*) $\frac{3}{8}$ inch high strength steel ground wire: $d_w = 0.375$ inch; $z = 6.03 + j0.61$. From Figs. 3(*a*), (*b*), and (*c*), respectively, in ohms per mile,

$$R_{ww-g} = R_{\overline{aw}-g} = 0.092$$
$$X_{ww-g} = 1.47$$
$$X_{\overline{aw}-g} = 0.675$$
$$Z_{ww-g} = (6.03 + 0.09) + j(1.47 + 0.61) = 6.12 + j2.08 = 6.45 \underline{/18.8^\circ}$$
$$Z_{\overline{aw}-g} = 0.092 + j0.675 = 0.682 \underline{/82.2^\circ}$$

Substituting Z_{ww-g} and $Z_{\overline{aw}-g}$ in [55],

$$\frac{-I_w}{3I_{a0}} = \frac{0.682 \underline{/82.2^\circ}}{6.45 \underline{/18.8^\circ}} = 0.106 \underline{/63.4^\circ}$$

The second term of [57] is

$$-\frac{3(Z_{\overline{aw}-g})^2}{Z_{\overline{ww}-g}} = -3\frac{(0.682 \underline{/82.2^\circ})^2}{6.45 \underline{/18.8^\circ}} = -0.217 \underline{/145.6^\circ}$$
$$= -(-0.18 + j0.12) = 0.18 - j0.12$$

Adding the correction $0.18 - j0.12$ to Z_{00},

$$Z_{00-w} = (0.55 + j2.78) + (0.18 - j0.12) = 0.73 + j2.66 \text{ ohms per mile}$$

With the given steel ground wire, approximately 10% of the zero-sequence current flowing in the three conductors returns in the ground wire; the effect upon the zero-sequence self-impedance calculated without ground wire is to increase the resistance component and to decrease the reactance component. Both effects are relatively small, and the error in neglecting the presence of a steel ground wire will not be serious in the usual system problem.

(*b*) 00 copper ground wire: $d = 0.419$; $z = 0.440 + j0.039$.

$$Z_{ww-g} = (0.440 + 0.092) + j(1.45 + j0.039) = 0.532 + j1.49 = 1.58 \underline{/70.3^\circ}$$

$Z_{\overline{aw}-g}$ which does not depend upon ground wire characteristics is the same as for part (*a*).

$$\frac{-I_w}{3I_{a0}} = \frac{0.682 \underline{/82.2^\circ}}{1.58 \underline{/70.3}} = 0.43 \underline{/11.9^\circ}$$

$$-3\frac{(Z_{\overline{aw}-g})^2}{Z_{ww-g}} = -3\frac{(0.682 \underline{/82.2^\circ})^2}{1.58 \underline{/70.3^\circ}} = -0.882 \underline{/94.1^\circ} = 0.06 - j0.88$$

$$Z_{00-w} = 0.55 + j2.78 + (0.06 - j0.88) = 0.61 + j1.90 \text{ ohms per mile}$$

With a 00 copper ground wire instead of a $\frac{3}{8}$ inch steel ground wire, the ground wire current is 43% of the total zero-sequence current in the three conductors; the zero-sequence reactance of the circuit is appreciably reduced from that calculated without ground wire, while the zero sequence resistance is increased. The lower the self-impedance of the ground wire, the greater the ground wire current and the lower the zero-sequence impedance. A ground wire always reduces the reactance component of the zero-sequence self-impedance calculated without a ground wire. It usually increases the resistance component, but a very low resistance ground wire may decrease this component.

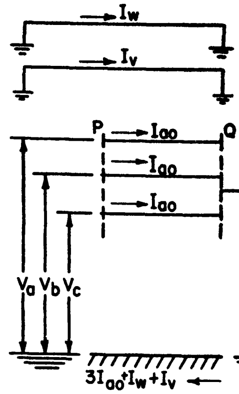


FIG. 11. Zero-sequence currents flowing from P to Q and returning in the ground and ground wires w and v.

Two Ground Wires. Figure 11 shows zero sequence current only flowing in a three-phase circuit with two ground wires, w and v. The voltage drops in the five-loop circuits consisting of the three conductors and the two ground wires, each with earth return, are

$$\begin{aligned}
V_a &= I_{a0}(Z_{aa-g} + Z_{ab-g} + Z_{ac-g}) + I_w Z_{aw-g} + I_v Z_{av-g} \\
V_b &= I_{a0}(Z_{bb-g} + Z_{ab-g} + Z_{bc-g}) + I_w Z_{bw-g} + I_v Z_{bv-g} \\
V_c &= I_{a0}(Z_{cc-g} + Z_{ac-g} + Z_{bc-g}) + I_w Z_{cw-g} + I_v Z_{cv-g} \\
0 &= I_w Z_{ww-g} + I_v Z_{wv-g} + I_{a0}(Z_{aw-g} + Z_{bw-g} + Z_{cw-g}) \\
0 &= I_w Z_{wv-g} + I_v Z_{vv-g} + I_{a0}(Z_{av-g} + Z_{bv-g} + Z_{cv-2})
\end{aligned} \qquad [60]$$

Solving the last two equations of [60], I_w and I_v in terms of I_{a0} are

$$I_w = -3I_{a0}\frac{Z_{vv-g}Z_{\overline{aw}-g} - Z_{wv-g}Z_{\overline{av}-g}}{Z_{ww-g}Z_{vv-g} - Z_{wv-g}^2} \qquad [61]$$

$$I_v = -3I_{a0}\frac{Z_{ww-g}Z_{\overline{av}-g} - Z_{wv-g}Z_{\overline{aw}-g}}{Z_{ww-g}Z_{vv-g} - Z_{wv-g}^2} \qquad [62]$$

where $Z_{\overline{aw}-g}$ and $Z_{\overline{av}-g}$ are average values of the mutual impedances with earth return between ground wires w and v, respectively, and the three conductors, and are determined by the distances $s_{aw} = \sqrt[3]{s_{aw}s_{bw}s_{cw}}$ and $s_{av} = \sqrt[3]{s_{av}s_{bv}s_{cv}}$, respectively.

Substituting I_w and I_v from [61] and [62] in the first three equations of [60], solving these equations for Z_{a0}, Z_{b0}, and Z_{c0}, and then substituting Z_{a0}, Z_{b0}, and Z_{c0} in [7] of Chapter VIII, the zero-sequence self-impedance and the mutual impedances between the zero-sequence network and the positive- and negative-sequence networks are determined. The zero-sequence self-impedance is

$$Z_{00-wv} = Z_{00} - 3\,\frac{(Z_{\overline{aw}-g})^2 Z_{vv-g} + (Z_{\overline{av}-g})^2 Z_{ww-g} - 2Z_{\overline{aw}-g}Z_{\overline{av}-g}Z_{wv-g}}{Z_{ww-g}Z_{vv-g} - (Z_{wv-g})^2} \qquad [63]$$

where Z_{00-wv} is the zero-sequence self-impedance with two ground wires w and v, and Z_{00} without ground wires is given by [44] or [47].

Unlike Ground Wires. When the two ground wires have unequal diameters, are of different materials, or are unsymmetrically spaced with respect to the circuit, the zero-sequence self-impedance is given by [63].

Identical Ground Wires, Symmetrically Spaced. When the two ground wires are identical, symmetrically spaced with respect to the conductors and equidistant from ground, their self-impedances with ground return and average mutual impedances with the conductors with ground return are equal:

$$Z_{ww-g} = Z_{vv-g} \quad \text{and} \quad Z_{\overline{aw}-g} = Z_{\overline{av}-g}$$

Substituting these values in [61], [62], and [63],

$$I_w = I_v = -3I_{a0}\,\frac{Z_{\overline{aw}-g}(Z_{ww-g} - Z_{wv-g})}{Z^2_{ww-g} - Z^2_{wv-g}}$$

$$= -3I_{a0}\,\frac{Z_{\overline{aw}-g}}{Z_{ww-g} + Z_{wv-g}} \qquad [64]$$

$$I(\text{total ground wire current}) = I_w + I_v = -3I_{a0}\,\frac{2Z_{\overline{aw}-g}}{Z_{ww-g} + Z_{wv-g}}$$

$$= -3I_{a0}\,\frac{Z_{\overline{aw}-g}}{\frac{1}{2}(Z_{ww-g} + Z_{wv-g})} \qquad [65]$$

$$Z_{00-wv} = Z_{00} - 3\,\frac{(Z_{\overline{aw}-g})^2(2Z_{ww-g} - 2Z_{wv-g})}{(Z_{ww-g})^2 - (Z_{wv-g})^2}$$

$$= Z_{00} - \frac{6(Z_{\overline{aw}-g})^2}{Z_{ww-g} + Z_{wv-g}}$$

$$= Z_{00} - \frac{3(Z_{\overline{aw}-g})^2}{\frac{1}{2}(Z_{ww-g} + Z_{wv-g})} \qquad [66]$$

where the mutual impedance $Z_{\overline{aw}-g}$ is determined by the distance

$$s_{\overline{aw}} = s_{\overline{av}} = \sqrt[3]{s_{aw}s_{bw}s_{cw}}$$

Unsymmetrically Spaced Ground Wires on the Same Towers. With two circuits and two identical ground wires on the same towers and one of the circuits out of service, the ground wires may not be symmetrically located with respect to the remaining circuit. If the assumption is made that the total ground wire current divides equally between the two ground wires, w and v, the zero-sequence impedance is given by [66] if $Z_{\overline{aw}-g}$ corresponds to the geometric mean distance between ground wires and conductors, i.e., to s, where $s = \sqrt[6]{s_{aw}s_{bw}s_{cw}s_{av}s_{bv}s_{cv}}$. Since the ground wires are at the same potential at every tower, this assumption leads to but slight error.

With two circuits, each with one ground wire, on the same right of way but not on the same towers and one of the circuits out of service, the zero-sequence self-impedance of the circuit remaining in service is given by [63].

More Than Two Ground Wires. The equations developed for two symmetrical ground wires can be extended by analogy to three or more identical ground wires. Under the assumption of equal division of total ground wire current among the ground wires, the following equations for ground wire current and zero-sequence self-impedance in ohms per mile are obtained:

For three identical ground wires,

$$Z_{00-3} = Z_{00} - \frac{9(Z_{\overline{aw}-g})^2}{Z_{\overline{ww}-g} + 2Z_{\overline{wv}-g}} \qquad [67]$$

$$I \text{ (total ground wire current)} = 3I_{a0}\frac{3Z_{\overline{aw}-g}}{Z_{\overline{ww}-g} + 2Z_{\overline{wv}-g}} \qquad [68]$$

For n identical ground wires,

$$Z_{00-n} = Z_{00} - \frac{3n(Z_{\overline{aw}-g})^2}{Z_{\overline{ww}-g} + (n-1)Z_{\overline{wv}-g}} \qquad [69]$$

$$I \text{ (total ground wire current)} = 3I_{a0}\frac{nZ_{\overline{aw}-g}}{Z_{\overline{ww}-g} + (n-1)Z_{\overline{wv}-g}} \qquad [70]$$

where Z_{00} is the zero-sequence self-impedance without ground wires, $Z_{\overline{ww}-g}$ is the average self-impedance with earth return of the ground wires, $Z_{\overline{aw}-g}$ the average mutual impedance with earth return between ground wires and conductors, and $Z_{\overline{wv}-g}$ the average mutual impedance with earth return between ground wires, the average mutual impedances corresponding to the geometric mean spacings.

Two or More Ground Wires Replaced by One Equivalent Ground Wire. Comparing [65] and [66] for two ground wires with [54] and [57] for one ground wire, it can be concluded that the total ground wire current and the zero-sequence self-impedance of a three-phase circuit with two identical ground wires symmetrical with respect to the circuit are the same as with a single ground wire of equivalent self-impedance with earth return equal to $\frac{1}{2}(Z_{ww-g} + Z_{wv-g})$ and the same average mutual impedance $Z_{\overline{aw}-g}$. This applies also to n ground wires.

Replacing n in [69] by 2, 3, 4, etc., the equivalent self-impedances Z_{ww-g} of 2, 3, 4, etc., identical ground wires carrying equal current are

$\frac{1}{2}(Z_{xx-g} + Z_{\overline{xy}-g})$ for two ground wires x and y

$\frac{1}{3}(Z_{xx-g} + 2Z_{\overline{xy}-g})$ for three ground wires x, y, and z [71]

$\frac{1}{4}(Z_{xx-g} + 3Z_{\overline{xy}-g})$ for four ground wires x, y, z, and s

where Z_{xx-g} is the self-impedance with earth return of one ground wire and $Z_{\overline{xy}-g}$ is the average mutual impedance with common earth return between ground wires and corresponds to the geometric mean spacing between them. The average mutual impedance $Z_{\overline{aw}-g}$ between ground wires and conductors corresponds to the geometric mean spacing between ground wires and conductors.

When there are more than two ground wires, and the assumption of equal ground wire currents in all ground wires cannot be made, although it can be for each of two groups of wires, it is convenient to replace each of the two groups by one equivalent ground wire, and use equations [61], [62], and [63] to obtain the total ground wire current in each of the groups and the zero-sequence self-impedance of the circuit. With two or more ground wires in one group, there may be one or more ground wires in the other group. The method may be applied to ground wire and counterpoises or ground wires and rails, symmetrical with respect to the circuit.

Counterpoises. To reduce tower footing resistances, counterpoises are buried in the earth one to three feet below the earth's surface. Continuous counterpoises are connected to the tower feet at every tower. They may be treated approximately as ground wires of zero height above ground. With two ground wires, x and y, and two counterpoises, p and q, the two ground wires may be replaced by a single ground wire w and the two counterpoises by a single counterpoise v. The self-impedance with earth return of the equivalent ground wire from [71] is

$$Z_{ww-g} = \frac{1}{2}(Z_{xx-g} + Z_{xy-g})$$

and of the equivalent counterpoise

$$Z_{vv-g} = \frac{1}{2}(Z_{pp-g} + Z_{pq-g})$$

The mutual impedance with earth return between equivalent counterpoise and equivalent ground wire is Z_{wv-g}, corresponding to the geometric mean spacing between ground wires and counterpoises, i.e., the fourth root of the product of the four distances. $Z_{\overline{aw}-g}$ and $Z_{\overline{av}-g}$ are the mutual impedances corresponding to the geometric mean distances between conductors and ground wires and conductors and counterpoises, respectively.

The division of current between the two ground wires replaced by a single equivalent ground wire and the two counterpoises replaced by a single equivalent counterpoise can be determined from [61] and [62]; and the zero-sequence self-impedance of the circuit, from [63]. In these equations w and v then represent equivalent ground wire and equivalent counterpoise, respectively.

Rails. Bonded rails, symmetrically located with respect to a circuit, can be grouped into one equivalent rail as described above and treated as an equivalent ground wire. (See Appendix B for the resistances and internal reactances of various types of rails, bonded to permit signaling.)

Three Ground Wires. When it is necessary to consider three separate ground wires, or three ground wire groups, the method used for two ground wires can be extended to three ground wires, w, v, and u. The zero sequence voltage drop V_{a0} in any one of the three phase conductors in the direction of I_{a0} and the voltage drops in the ground wires w, v, and u are

$$\begin{aligned} V_{a0} &= I_{a0}Z_{00} + I_wZ_{\overline{aw}-g} + I_vZ_{\overline{av}-g} + I_uZ_{\overline{au}-g} \\ V_w &= 0 = 3I_{a0}Z_{\overline{aw}-g} + I_wZ_{ww-g} + I_vZ_{wv-g} + I_uZ_{wu-g} \\ V_v &= 0 = 3I_{a0}Z_{\overline{av}-g} + I_wZ_{wv-g} + I_vZ_{vv-g} + I_uZ_{vu-g} \\ V_u &= 0 = 3I_{a0}Z_{\overline{au}-g} + I_wZ_{wu-g} + I_vZ_{vu-g} + I_uZ_{uu-g} \end{aligned} \qquad [72]$$

where Z_{00} is the zero-sequence impedance without ground wires; and bars over the subscripts $Z_{\overline{aw}-g}$, $Z_{\overline{av}-g}$, and $Z_{\overline{au}-g}$ indicate average mutual impedances between the conductors and ground wires w, v, and u, respectively.

Writing the equations for I_w, I_v, and I_u in terms of $3I_{a0}$ directly from [72], using determinants (see Appendix A),

$$\begin{aligned} I_w = \frac{-3I_{a0}}{\Delta} [&Z_{\overline{aw}-g}(Z_{vv-g}Z_{uu-g} - Z^2_{vu-g}) \\ &- Z_{\overline{av}-g}(Z_{wv-g}Z_{uu-g} - Z_{vu-g}Z_{wu-g}) \\ &+ Z_{\overline{au}-g}(Z_{wv-g}Z_{vu-g} - Z_{vv-g}Z_{wu-g})] \\ I_v = -\frac{3I_{a0}}{\Delta} [&-Z_{aw-g}(Z_{wv-g}Z_{uu-g} - Z_{wu-g}Z_{vu-g}) \\ &+ Z_{av-g}(Z_{ww-g}Z_{uu-g} - Z^2_{wu-g}) \\ &- Z_{au-g}(Z_{ww-g}Z_{vu-g} - Z_{wv-g}Z_{wu-g})] \end{aligned} \qquad [73]$$

$$I_u = -\frac{3I_{a0}}{\Delta}[Z_{aw-g}(Z_{wv-g}Z_{vu-g} - Z_{vv-g}Z_{wu-g})$$
$$- Z_{av-g}(Z_{ww-g}Z_{vu-g} - Z_{wu-g}Z_{wv-g})$$
$$+ Z_{au-g}(Z_{ww-g}Z_{vv-g} - Z^2_{wv-g})]$$

$$\Delta =$$
$$Z_{ww-g}(Z_{vv-g}Z_{uu-g} - Z^2_{vu-g}) - Z_{wv-g}(Z_{wv-g}Z_{uu-g} - Z_{vu-g}Z_{wu-g})$$
$$+ Z_{wu-g}(Z_{wv-g}Z_{vu-g} - Z_{vv-g}Z_{wu-g})$$

$$I_g = -(3I_{a0} + I_w + I_v + I_u) = \text{ground current in the direction of } I_{a0}$$

Substituting the equations for I_w, I_v, and I_u in the first equation of [72], and dividing by I_{a0}, $V_{a0}/I_{a0} = Z_{00-wvu}$ is obtained. As this substitution is simpler after numerical values of I_w, I_v, and I_u in terms of $3I_{a0}$ have been calculated, a general equation for Z_{00-wvu} will not be given here.

The additional precision obtained by the division of ground wires into three ground wire groups instead of two is seldom justified because of the uncertainty of the earth resistivity, temperature, and other variables which may not be definitely known. The above equations are given primarily for determining the portion of zero sequence current which returns in each of the ground wires. These equations will be further discussed in Volume II in connection with underground insulated cable, where the grounded lead sheaths are treated as ground wires.

60-Cycle Ground Wire Correction Chart. The zero-sequence self-impedances of a three-phase circuit with one ground wire and with two identical symmetrical ground wires, given by [57] and [66], respectively, have as their first term Z_{00}, the zero sequence self-impedance without ground wires. The second terms in these equations may be considered a ground wire correction $K(\Delta Z)$, where

$$K(\Delta Z) = K(\Delta R) + jK(\Delta X)$$

Figure 12 gives curves[10] from which the corrections ΔR and ΔX for one and two ground wires at a frequency of 60 cycles can be obtained. The multiplier K is also given in Fig. 12. For both one and two ground wires, ΔR and ΔX were calculated with an earth resistivity of $\rho = 100$, and $Z_{\overline{aw}-g}$ in [57] and [66] corresponding to a geometric mean spacing between conductors and ground wire or wires of $s_{\overline{aw}} = 20$ feet. Any values of ΔR and ΔX as read must therefore be multiplied by K taken from the auxiliary multiplier curves corresponding to the actual earth resistivity and the actual geometric mean spacing between conductors and ground wire or wires. The multiplier K has a small phase angle

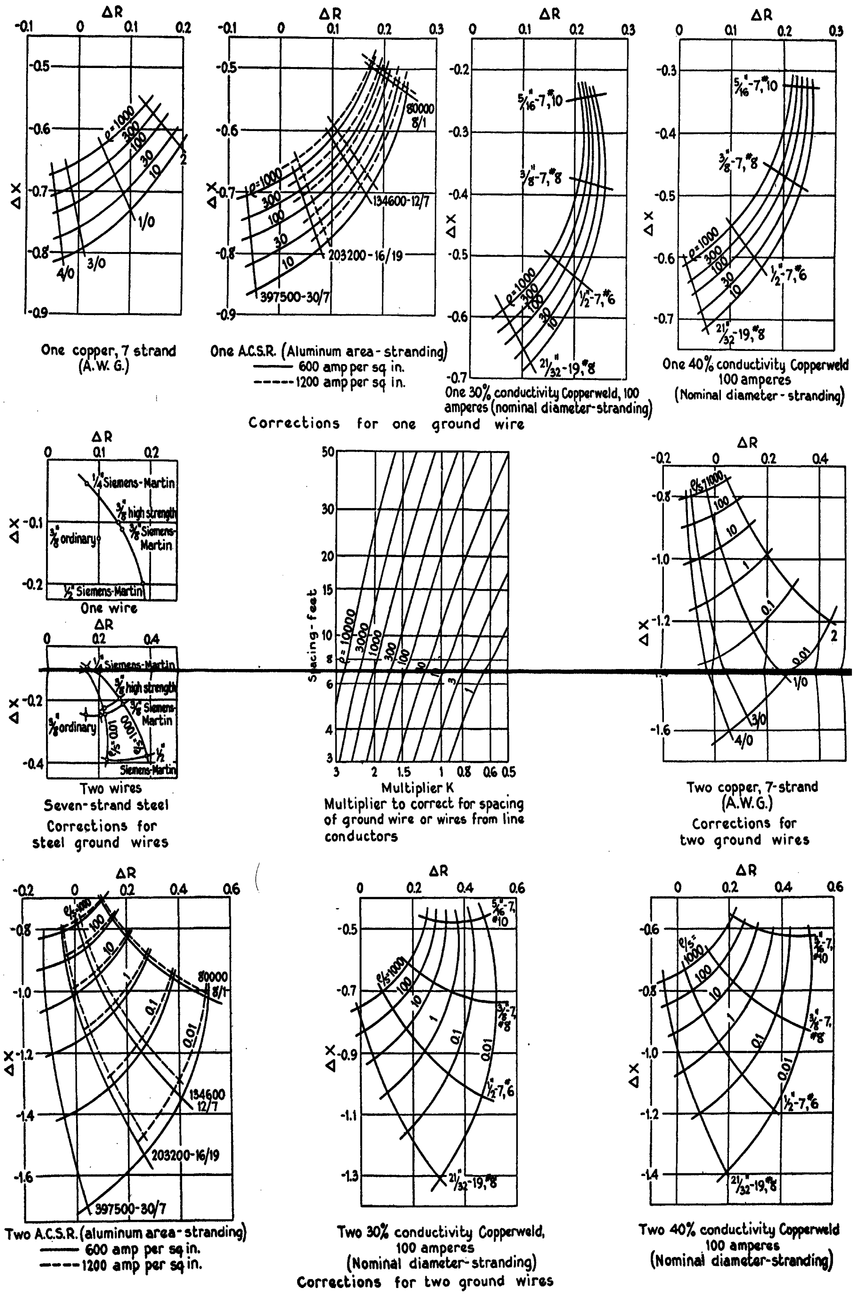


FIG. 12. Ground wire corrections for 60-cycle, zero-sequence impedance of transmission lines.

(here neglected) which has but little effect on zero-sequence impedances in the range given. An average height above ground of 50 feet was assumed for ground wires.

With one ground wire, ΔR and ΔX are determined by the type of ground wire and the earth resistivity ρ. With two ground wires, ΔR and ΔX are determined by the type of ground wire and ρ/s_{wv}, the ratio of the earth resistivity in ohms per meter cube to the spacing between the ground wires w and v in feet.

Problem 4. Find the zero-sequence self-impedance of the circuit described in Problem 2 with

(*a*) one ground wire of No. 2 seven-strand copper, 8 feet above the center conductor.

(*b*) two ground wires of No. 2 seven-strand copper, 8 feet above the two outside conductors.

Solution. (*a*) $s_{\overline{aw}} = \sqrt[3]{8(10^2 + 8^2)} = 10.9$ feet. Read from the auxiliary multiplier chart with $\rho = 1000$ and $s_{\overline{aw}} = 10.9$; $K = 1.78$. Entering the copper chart for one ground wire and locating the intersection of curve 2 with $\rho = 1000$,

$$\begin{aligned} \Delta R + j\Delta X &= 0.13 - j0.56 \text{ ohm per mile} \\ K(\Delta R + j\Delta X) &= 1.78(0.13 - j0.56) = 0.23 - j1.00 \text{ ohm per mile} \\ Z_{00-w} &= 0.56 + j3.20 + (0.23 - j1.00) = 0.79 + j2.20 \text{ ohms per mile} \end{aligned}$$

(*b*) $s_{\overline{aw}} = \sqrt[6]{8 \times 8 \times (10^2 + 8^2)(20^2 + 8^2)} = 13.0$ feet, $s_{wv} = 20$ feet. With $s_{\overline{aw}} = 13.0$ and $\rho = 1000$ and $K = 1.70$,

$$\frac{\rho}{s_{wv}} = \frac{1000}{20} = 50$$

Entering the copper chart for two ground wires and locating the intersection of curve 2 with $\rho/s_{wv} = 50$,

$$\begin{aligned} \Delta R + j\Delta X &= 0.10 - j0.84 \text{ ohm per mile} \\ K(\Delta R + j\Delta X) &= 1.70(0.10 - j0.84) = 0.17 - j1.43 \text{ ohms per mile} \\ Z_{00-wv} &= 0.56 + j3.20 + (0.17 - j1.43) = 0.73 + j1.77 \text{ ohms per mile} \end{aligned}$$

ZERO-SEQUENCE EQUIVALENT CIRCUITS FOR PARALLEL TRANSMISSION LINES

Zero-Sequence Self-Impedances of Parallel Transmission Lines and Zero-Sequence Mutual Impedances between Them. In Chapter VI, equivalent circuits are developed for parallel transmission circuits in terms of their zero-sequence self- and mutual impedances. In a one-line diagram of the zero-sequence system, parallel transmission lines can be represented by an equivalent circuit when the zero-sequence self-impedances of each circuit alone and the mutual impedances between them taken two at a time are known. See Chapter VI, Figs. 5 and 6, for circuits with negligible capacitance and Fig. 8 for circuits with appreciable capacitance.

Figure 13 shows two parallel transmission circuits on the same towers. a, b, c are the three conductors of one circuit and A, B, C the conductors of the other circuit, a and A, b and B, c and C being conductors of the same phase. One ground wire is indicated by w, two ground wires by w and v. The ground wire, or wires, in parallel circuits on the same towers will be assumed symmetrically located with respect to the two circuits.

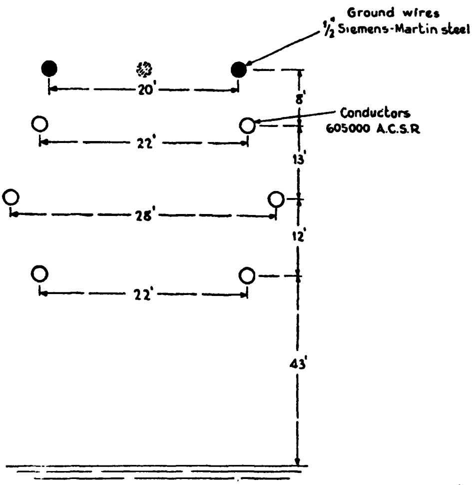


FIG. 13. Configuration of a double-circuit transmission line.

The *zero-sequence self-impedance* of either of the two circuits is its self-impedance with the other circuit open. Equations [47], [57], [63], or [66], and [69] give the zero-sequence self-impedance of a circuit with no ground wires, one ground wire, two ground wires, and n ground wires, respectively. It should be remembered that when a circuit with ground wires is opened, the effect of its ground wires on the parallel circuit cannot be neglected.

The *zero-sequence mutual impedance* between two parallel transmission circuits is influenced by the spacing between the circuits and the characteristics, number, and location of the ground wires. The effect of ground wires is to reduce zero-sequence mutual impedance between circuits. The magnitude of the zero-sequence mutual impedance between two circuits on the same towers is of the order of 50% (plus or minus) of the self-impedance of either circuit. In the usual system problem it cannot therefore be neglected.

Zero-sequence currents flowing in one of two parallel three-phase circuits induce voltages in the three phases of the other circuit which can be separated into their sequence components of induced voltage by substituting the induced phase voltages in [10]–[12] of Chapter II.

The positive- and negative-sequence components of induced voltage will be small. Considering only zero-sequence components of voltage induced by zero-sequence components of current, zero-sequence mutual impedance between two parallel transmission circuits will be defined as the ratio of the zero-sequence voltage induced in one circuit to the zero-sequence current per phase flowing in the other circuit which induces it. Zero-sequence mutual impedance between two parallel circuits can be determined with zero-sequence currents only flowing in one circuit and the other circuit open at one end but the three phase connected and grounded at the other. The average of the three voltages induced in the three phases of the open circuit by zero-sequence currents in the closed circuit and by the ground wire currents is the zero-sequence induced voltage. The ratio of this voltage to the zero-sequence current per phase in the excited circuit is the zero-sequence mutual impedance between the circuits.

The average voltage V_{A0} induced in phases A, B, C of the open circuit because of zero-sequence currents I_{a0} in phases a, b, c of the excited circuit and currents I_w, $I_v \cdots I_n$ in the ground wires w, v, $\cdots n$ is

$$V_{A0} = 3I_{a0}Z_{\overline{aA}-g} + I_wZ_{\overline{Aw}-g} + I_vZ_{\overline{Av}-g} + \cdots + I_nZ_{\overline{An}-g} \quad [74]$$

If the zero-sequence mutual impedance is Z_{0m}, then, by definition,

$$Z_{0m} = \frac{V_{A0}}{I_{a0}} = 3Z_{\overline{aA}-g} + \frac{I_w}{I_{a0}} Z_{\overline{Aw}-g} + \frac{I_v}{I_{a0}} Z_{\overline{Av}-g} + \cdots \frac{I_n}{I_{a0}} Z_{\overline{An}-g} \quad [75]$$

$Z_{\overline{aA}-g}$ is the average mutual impedance with ground return between the three conductors of the open circuit and those of the excited circuit and is determined from $s_{\overline{aA}}$, the geometric mean distance between conductors of the two circuits, where $s_{\overline{aA}} = \sqrt[9]{s_{aA}s_{aB}s_{aC}s_{bA}s_{bB}s_{bC}s_{cA}s_{cB}s_{cC}}$. $s_{\overline{Aa}}$ can also be determined by logarithms. It is the number whose logarithm is one-ninth of the sum of the logarithms of the nine distances between the conductors of the two circuits. $Z_{\overline{Aw}-g}$, $Z_{\overline{Av}-g}$, $\cdots Z_{\overline{An}-g}$ are the average mutual impedances with ground return between the conductor A, B, C of the open circuit and ground wires w, v, $\cdots n$, respectively, and correspond to geometric mean distances $s_{\overline{Aw}}$, $s_{\overline{Av}}$, $s_{\overline{An}}$. With no ground wires, $I_w = I_v = \cdots = I_n = 0$. I_w is given in terms of I_{a0} in [54] for one ground wire, I_w and I_v in [61] and [62] for the general case of two ground wires; and, under the assumption of equal currents in the ground wires, total ground wire current is given by [65], [68], and [70] for two, three, and n ground wires, respectively. Substituting ground wire currents in [75], Z_{0m} is obtained.

No Ground Wires. With $I_w = I_v = \cdots I_n = 0$ in [75],

$$Z_{0m} = 3Z_{\overline{aA}-g} = 3(R_{aA-g} + jX_{\overline{aA}-g}) \quad [76]$$

R_{aA-g} and X_{aA-g} can be read from parts (*a*) and (*c*), respectively, of Figs. 3, 4, or 5 at 60, 50 or 25 cycles.

One Ground Wire. Substituting [54] in [75],

$$Z_{0m-w} = Z_{0m} - 3\frac{Z_{\overline{Aw}-g}Z_{\overline{aw}-g}}{Z_{ww-g}} \quad [77]$$

If the ground wire is at the same distance from both circuits, $Z_{\overline{Aw}-g} = Z_{\overline{aw}-g}$ and

$$Z_{0m-w} = Z_{0m} - 3\frac{(Z_{\overline{aw}-g})^2}{Z_{ww-g}} \quad [78]$$

Two Ground Wires (General Case). Substituting [61] and [62] in [75],

$$Z_{0m-wv} = Z_{0m} - 3\frac{[Z_{\overline{aw}-g}Z_{\overline{Aw}-g}Z_{vv-g} + Z_{\overline{av}-g}Z_{\overline{Av}-g}Z_{ww-g} - (Z_{\overline{aw}-g}Z_{\overline{Av}-g} + Z_{\overline{av}-g}Z_{\overline{Aw}-g})Z_{wv-g}]}{Z_{ww-g}Z_{vv-g} - (Z_{wv-g})^2} \quad [79]$$

Two Ground Wires, Assuming Identical Ground Wires and Equal Division of Ground Wire Current.

$$Z_{m0-wv} = Z_{0m} - 3\frac{2(Z_{\overline{aw}-g})^2}{Z_{ww-g} + Z_{wv-g}} \quad [80]$$

n Ground Wires, Assuming Identical Ground Wires and Equal Division of Ground Wire Current.

$$Z_{0m-n} = Z_{0m} - 3\frac{n(Z_{\overline{aw}-g})^2}{Z_{ww-g} + (n-1)Z_{\overline{wv}-g}} \quad [81]$$

where Z_{0m} is given by [76].

Comparing [78] and [80] with [57] and [66], respectively, it will be seen that the terms involving ground wires are the same in the mutual impedance equations for these cases as in the self-impedance equations. The ground wire corrections given in Fig. 12 for frequencies of 60 cycles can be applied to the zero-sequence mutual impedance Z_{0m} without ground wires to obtain the mutual impedance with one ground wire equidistant from the two circuits and two identical ground wires carrying equal currents.

Problem 5. Find the zero-sequence mutual impedance per mile at 60 cycles between the two circuits shown in Fig. 13, assuming an earth resistivity of $\rho = 500$ ohms per meter cube, (*a*) with no ground wires, (*b*) with one ground wire on the tower center line, and (*c*) with two ground wires spaced symmetrically 20 feet apart.

Solution. (*a*) The average height above ground of the six conductors is 55 feet; the average horizontal spacing between the conductors of one circuit and those of the other circuit is 24 feet; the geometric mean spacing between the conductors of one circuit and those of the other circuit is

$$s_{\overline{aA}} = \sqrt[9]{22^2 \times 28(25^2 + 13^2)(25^2 + 12^2)(25^2 + 22^2)} = 27.6 \text{ feet}$$

From Fig. 3(*a*), $R_{\overline{aA}-g} = 0.093$ ohm per mile. From Fig. 3(*b*) and Fig. 3(*e*), $X_{\overline{aA}-g} = 0.65$ ohm per mile. Hence the zero-sequence mutual impedance is

$$Z_{0m} = 3(0.093 + j0.65) = 0.28 + j1.95 \text{ ohms per mile}$$

(*b*) With one ground wire on the tower center line, the geometric mean distance from the ground wire to the six conductors is

$$s_{\overline{aw}} = \sqrt[6]{(8^2 + 11^2)(21^2 + 14^2)(33^2 + 11^2)} = 22.9 \text{ feet}$$

From Fig. 12, the correction for a $\frac{1}{2}$-inch Siemens-Martin steel ground wire is $(0.18 - j0.20)$ with a multiplier of 1.25, giving

$$Z_{0m-w} = (0.28 + j1.95) + 1.25(0.18 - j0.20) = 0.51 + j1.70 \text{ ohms per mile}$$

(*c*) With two ground wires spaced symmetrically 20 feet apart, the geometric mean distance from the two ground wires to the six conductors is

$$\begin{aligned} s_{\overline{aw}} &= \sqrt[12]{(1^2 + 8^2)(4^2 + 21^2)(1^2 + 33^2)(21^2 + 8^2)(24^2 + 21^2)(21^2 + 33^2)} \\ &= 23.3 \text{ feet} \end{aligned}$$

The ratio of resistivity to ground wire spacing is $500/20 = 25$. From Fig. 12, the correction is $(0.31 - j0.39)$ and the multiplier is 1.25, giving

$$Z_{0m-wv} = (0.28 + j1.95) + 1.25(0.31 - j0.39) = 0.67 + j1.46 \text{ ohms per mile}$$

Faults on Parallel Transmission Lines. When a fault occurs at any point on one of two parallel transmission lines, the lines may be considered to be divided into two sections by the fault. Each of these sections can then be replaced by an equivalent circuit. If the lines are bussed at their end points, the equivalent circuit for each section is an equivalent Y; if not bussed, the equivalent circuits are four-terminal circuits. Instead of using a four-terminal circuit to replace two parallel lines supplied through separate transformers, a three-terminal circuit to replace the two lines and their transformers is sometimes more convenient. This is illustrated in Fig. 14. Part (*a*) of this figure shows transformer and transmission circuit connections; the circuits *AB* and *A'B'* are parallel, the former extending beyond *B* to *C*; a fault is indicated at *F* on the section *AB*; *A* and *A'*, *F* and *F'*, *B* and *B'* are corresponding points on the two parallel circuits. Part (*b*) of Fig. 14 is a one-line zero-sequence impedance diagram with mutual impedances between line sections indicated. In Fig. 14(*b*), let

Z_x = total zero-sequence self-impedance between F and ground in the direction FA.

Z_y = total zero-sequence self-impedance between F' and ground in the direction $F'A'$.

Z_x' = total zero-sequence self-impedance between F and ground in the direction FB.

Z_y' = total zero-sequence self-impedance between F' and ground in the direction $F'B'$.

Z_{0m} = zero-sequence mutual impedance between lines AF and $A'F'$.

Z_{0m}' = zero-sequence mutual impedance between lines FB and $F'B'$.

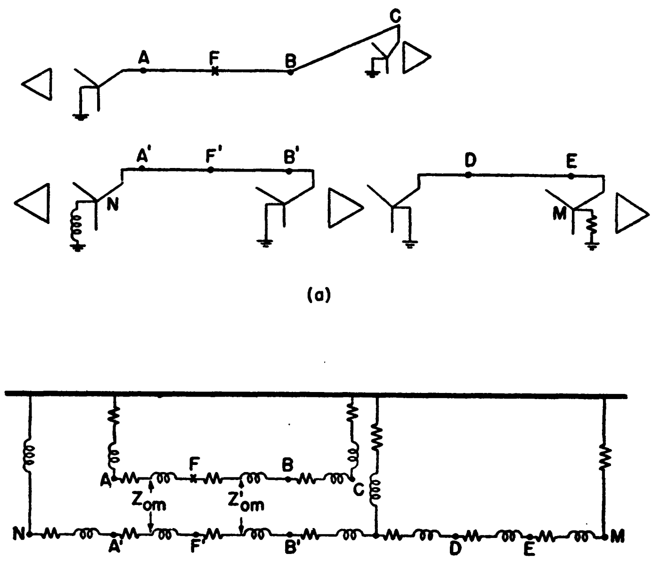
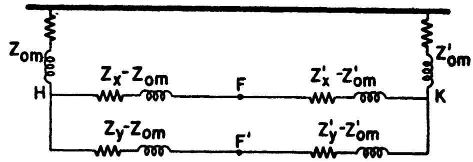
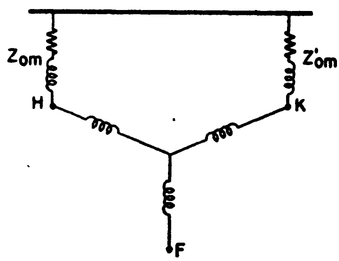


FIG. 14. (*a*) One-line zero-sequence system diagram showing transformer connections and ground fault at F on one of two transmission lines, parallel between points A and B. (*b*) Zero-sequence diagram with mutual impedances between parallel section of transmission line indicated by Z_{0m} and Z_{0m}'.

Making use of these definitions, the zero-sequence equivalent network, suitable for analytical calculations with a ground fault at F or F', is given by Fig. 14(*c*). In this equivalent circuit, the identity of all system points except F and F' has been lost. For faults at locations other than F or F', a different zero-sequence equivalent circuit is required. With a fault at F, the system of Fig. 14(*c*) can be further simplified by replacing the Δ circuit between points H, K, and F by an equivalent Y between these points, as indicated in Fig. 14(*d*), from which the zero-sequence impedance viewed from the fault is determined. After the zero-sequence currents flowing into the fault and in Z_{0m} and Z'_{0m} of Fig. 14(d) have been calculated, the zero-



(c)



(d)

FIG. 14. (c) Zero-sequence equivalent circuit for ground fault at F or F'. (d) At F only.

sequence currents in the line sections FA, FB, $F'A'$, and $F'B'$ can be obtained from them and Fig. 14(c); and knowing the zero-sequence currents in these line sections, the zero-sequence system voltages and currents can be determined from Fig. 14(b).

Two Parallel Three-Phase Circuits Operated at Different Voltages. When base voltage in two parallel three-phase circuits is the same, per unit mutual impedance is based on system base kva per phase and the base line-to-neutral voltage of the circuits. Consider two parallel circuits operated at different voltages — for example, a 110-kv circuit and a 66-kv circuit on the same towers. Let the two parallel circuits be a and A, with base system kva per phase indicated by kva, and base line-to-neutral voltage in kv in circuits a and A by kv_a and kv_A, respectively. With I_{A0} in amperes in circuit A, the zero sequence voltage induced in circuit a is

$$V_{a0} \text{ (volts)} = I_{A0} \text{ (amperes)} Z_{0m} \text{ (ohms)}$$

$$V_{a0} \text{ (per unit)} = \frac{I_{A0} \text{ (amperes)} Z_{0m} \text{ (ohms)}}{\text{kv}_a \times 10^3}$$

But I_{A0} (amperes) $= I_{A0}$ (per unit)(kva/kv$_A$), so that

$$V_{a0} \text{ (per-unit)} = I_{A0} \text{ (per unit)} \frac{Z_{0m} \text{ (ohms)kva}}{\text{kv}_A\text{kv}_a \times 10^3}$$

$$= I_{A0} \text{ (per unit)} \frac{Z_{0m} \text{ (ohms)kva}}{(\sqrt{\text{kv}_A\text{kv}_a})^2 \times 10^3} \quad [82]$$

The mutual impedance in per unit is based on system kva per phase and a base line-to-neutral voltage equal to the geometric mean of the base line-to-neutral voltages in the two parallel circuits. In Fig. 14 the two parallel transmission circuits may be operated at the same or at different voltages.

With impedances in per unit, the mutual impedance may be greater than one of the self-impedances. This will offer no difficulty in an analytic solution; on an a-c network analyzer, if the mutual impedance is greater than the self-impedance of circuit a, $Z_{aa} - Z_{ab}$ in Fig. 6(a) of Chapter VI (either the resistance or reactance component, or both) is negative. Negative resistances can often be avoided by combining them with series impedances at the circuit terminals.

Effect of Zero-Sequence Mutual Impedance between Parallel Circuits. Consider two parallel, three-phase, 60-cycle, transmission circuits of 19 strand, 4/0 copper, 25 miles long on the same towers. The geometric mean spacing between the conductors of each circuit is 12 feet, that between conductors of the two circuits is 20 feet. Average conductor height above ground is 55 feet. There are no ground wires. Average earth resistivity is 100 ohms per meter cube. From Appendix B, Table II and Figs. 3(a) and 9(a),

$$Z_{00} = [0.278 + 3(0.091)] + j2.79 = 0.55 + j2.79 \text{ ohms per mile}$$

From Figs. 3(a), 3(c), and 3(d),

$$Z_{m0} = 3(0.091 + j0.600 + j0.004) = 0.27 + j1.81 \text{ ohms per mile}$$

These impedances will be used in the problem which follows.

Parallel Circuits Operated at the Same Voltage. The two circuits are operated at 115 kv, bussed at both ends A and B and connected to 75,000 kva transformer banks of 10% reactance at each end of the line. The transformers are connected Δ–Y with the Y's on the line side solidly grounded. A fault to ground occurs on one circuit near bus A and is followed by the opening of the breakers between the fault and bus A. This leaves the fault on the system and the circuits bussed at B only, as shown in Fig. 15(a). The equivalent circuit for the zero-sequence system of Fig. 15(a) is shown in Fig. 15(b). In per unit based on 75,000 kva and 115 kv,

$$Z_{00} = 25(0.55 + j2.79) \times \frac{75{,}000}{(115)^2 \times 10^3} = 0.078 + j0.396$$

$$Z_{0m} = 25(0.27 + j1.81) \times \frac{75{,}000}{(115)^2 \times 10^3} = 0.038 + j0.257$$

$$Z_{00} - Z_{0m} = 25(0.28 + j0.98) = 0.040 + j0.139$$

The transformer impedances with resistance neglected are each $j0.10$ in per unit based on 75,000 kva and 115 kv. The transmission circuits, which are bussed at B, are replaced by an equivalent Y with the mutual impedance Z_{0m} connected to the bus B.

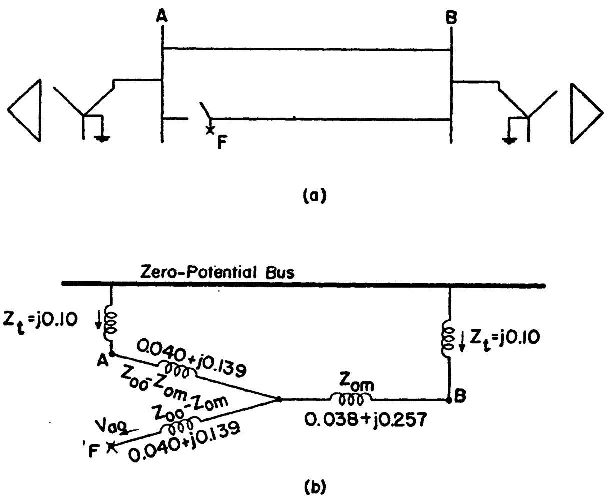


FIG. 15. (*a*) One-line system diagram showing ground fault F on one of two parallel transmission lines and the breakers open between F and bus A. (*b*) Zero-sequence equivalent circuit for (*a*) with impedances in per unit based on 75,000 kva and 115 kv (25,000 kva per phase and $115/\sqrt{3}$ kv line-to-neutral voltage).

The problem is to determine the relative magnitudes of the ground currents in the transformer banks at A and B.

It can be seen that the transfer impedance in Fig. 15(*b*) between A and F is less than that between B and F; and therefore the zero-sequence current from the bank at A is greater than that from the bank at B. If there were no mutual impedance, the ground current in the transformer bank at B would be larger than that in the bank at A. For the circuit given, the zero-sequence mutual reactance between circuits is 65% of the zero-sequence self-reactance of either circuit. If the mutual reactance were 50% of the self-reactance, neglecting resistance, the ground currents in the two transformer banks which have equal reactances would be equal; if less than 50%, the ground current in the transformer bank at B would be greater than that in the bank at A.

Ground wires reduce both the zero-sequence self-impedance of transmission circuits and the mutual impedance between two parallel circuits. With identical parallel circuits, the self-impedance Z_{00} and the mutual impedance Z_{0m} calculated without ground wire are reduced by the same correcting factor $K(\Delta R + j\Delta X)$. In Fig. 15(b), $Z_{00} - Z_{0m}$ is unaffected by ground wires but Z_{0m} is reduced. If the circuits in Fig. 15(a) have two $\frac{1}{2}$-inch ground wires of 40% conductivity copperweld, 20 feet apart, and at a geometric mean distance of 20 feet from the conductors, the correction to be added to the zero-sequence self- and mutual impedances without ground wires to obtain self- and mutual impedances with ground wires is obtained from Fig. 12. With $\rho = 100$ and $s_{\overline{aw}} = 20$, the multiplier K is 1.0; with $\rho = 100$ and $s_{wv} = 20$ feet, $\rho/s = 100/20 = 5$. The approximate correction $K(\Delta Z) = (0.12 - j0.92)$ ohms per mile. Applying this correction,

$$\begin{aligned} Z_{00-wv} &= (0.55 + j2.79) + (0.12 - j0.92) \\ &= 0.67 + j1.87 \text{ ohms per mile} \\ Z_{0m-wv} &= (0.27 + j1.81) + (0.12 - j0.92) \\ &= 0.39 + j0.89 \text{ ohm per mile} \\ Z_{00-wv} - Z_{0m-wv} &= 0.28 + j0.98 \end{aligned}$$

If these impedance values for 25 miles of line are expressed in per unit and substituted in Fig. 15(b), it will be found that the transfer impedance between A and F is now greater than that between B and F, and there will be more ground current in the transformer bank at B than at A.

With the two given ground wires, the zero-sequence mutual reactance is approximately one-half of its value without ground wires. With ground wires, the mutual reactance is 47% of the self-reactance; without ground wires, it is 65% of the self-reactance. The positive-sequence reactance of either circuit from Appendix B is 0.79 ohm per mile; without ground wires, the zero-sequence self-reactance of either circuit is approximately 3.5 times the positive-sequence self-reactance of either circuit; with the two given ground wires, the zero-sequence self-reactance is approximately 2.4 times the positive-sequence self-reactance. These figures will vary with earth resistivity, conductor and ground wire characteristics, and circuit configuration. As approximations for 60-cycle overhead transmission circuits, where a high degree of precision is not required, the following figures may be useful to have in mind: 0.8 ohm per mile for positive-sequence self-reactance; 3.5 times the positive-sequence reactance for zero-sequence self-reactance without ground wires, and 60% of the zero-sequence self-reactance for mutual reactance between two parallel lines without ground wires. From Fig. 9(a), Fig. 3(c), and the charts of Appendix B, it will be seen that these are approximate figures only, but useful when the configuration of a circuit and the earth resistivity are not given; or, if known, they would influence calculations but slightly.

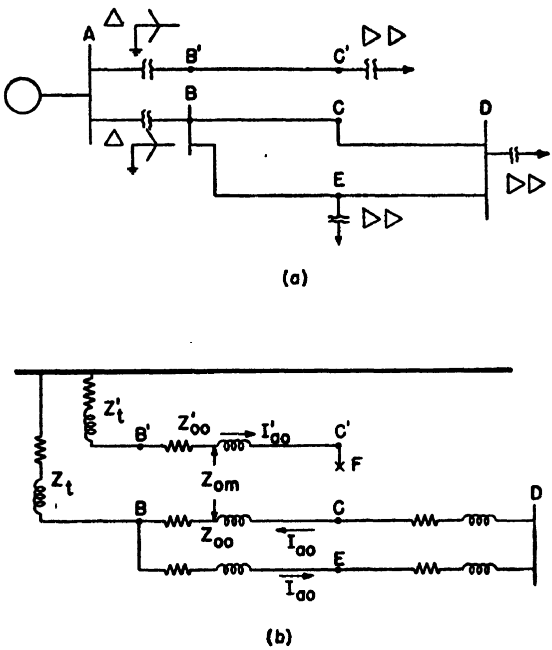


FIG. 16. (a) Circuit BC of transmission loop paralleled by second circuit $B'C'$. (b) Zero-sequence network for (a) with mutual impedance Z_{0m} between BC and $B'C'$ indicated.

The effect of one and two ground wires on the zero-sequence self- and mutual reactances can be determined from the charts of Fig. 12, not forgetting the multiplier K. Light steel ground wires affect zero-sequence self- and mutual reactances but slightly, while low-resistance ground wires reduce them appreciably.

Zero-Sequence Currents in Ungrounded Transmission Loops. Figure 16(a) is a one-line diagram of a three-phase loop $BCDEB$ supplied at B through a grounded transformer bank; the sections of line BC and $B'C'$ are on the same towers, the section of line BE may be on the same or on a different right-of-way; the only grounds on the system are those at the neutrals of the supply transformer banks. The zero-sequence self-impedance of section BC is Z_{00} and that of $B'C'$ is Z'_{00}, the mutual impedance between BC and $B'C'$ is Z_{0m}. A line-to-ground fault occurs at C', causing the zero-sequence current I'_{a0} to flow in the line $B'C'$, which induces a zero-sequence voltage in the line BC. If the line BE is out of service, there will not be a complete path for zero-sequence currents, and the voltage drop $I'_{a0}Z_{0m}$ induced in BC by I'_{a0} flowing from B' to C' will exist as a zero-sequence voltage drop between B and C. B will be at zero potential in the zero-sequence network, but the zero-sequence potential at C will be $-I'_{a0}Z_{0m}$. If the circuit BE is closed, completing the loop, the voltage drop $I'_{a0}Z_{0m}$ in the direction of I'_{a0}, or the voltage rise $I'_{a0}Z_{0m}$ in the direction CB, tends to send current around the closed loop in the direction $CBED$. If the line BE is on a different right-of-way from $B'C'$, no voltage will be induced in it; if on the same right-of-way but not on the same towers, a voltage smaller than that induced in CB will be induced in BE; if BE is on the same towers with $B'C'$ and BC, the voltages induced in BE and BC will be of the same order of magnitude. In any case, the vector *difference* between the zero-sequence voltages induced in BC and BE will tend to circulate current in the loop. Zero-sequence currents flowing in the loop will induce voltages in the ground which will circulate ground currents in closed loops beneath the conductors. If the assumption is made that the total ground current is equal in magnitude and opposite in direction to the sum of the currents in the overhead conductors, the impedance met by zero-sequence currents flowing in the closed loop will be the sum of the zero-sequence self-impedances of the circuits of the loop, with zero-sequence mutual impedances between circuits or parts of circuits which make up the loop taken into account. An example of mutual impedance between parts of circuits would be provided if the circuit CD paralleled part of the circuit BED. In that case, the zero-sequence impedance of the loop would consist of the sum of the zero-sequence self-impedances of the circuits making up the loop minus twice the zero-sequence mutual impedance between any circuits or parts of circuits of the loop.

The current I_{a0} flowing in the loop through the circuit BC will in turn induce a voltage in the circuit $B'C'$ which reduces the zero-sequence impedance viewed from C' used to calculate the symmetrical components of fault current $I'_{a1} = I'_{a2} = I'_{a0}$. In other words, there is a closed loop mutually coupled with circuit $B'C'$ which reduces its zero-sequence impedance.

Let Z_{00} (loop) = impedance of loop $CBEDC$ to zero-sequence currents

Z'_0 = zero-sequence impedance viewed from C' with loop closed.

V'_{a0} = zero-sequence voltage at C'.

I'_{a0} = zero-sequence current in circuit $B'C'$ with direction $B'C'$.

I_{a0} = zero-sequence current in loop with direction CB.

Z'_t = impedance of transformer between A and B'.

V'_{a0} at C' and the voltage drop in the loop are

$$\begin{aligned} V'_{a0} &= -I'_{a0}Z'_0 = -I'_{a0}(Z'_t + Z'_{00}) + I_{a0}Z_{0m} \\ 0 &= I'_{a0}Z_{0m} - I_{a0}Z_{00}\ \text{(loop)} \end{aligned} \qquad [83]$$

From [83],

$$I_{a0} = I'_{a0}\frac{Z_{0m}}{Z_{00}\ \text{(loop)}} \qquad [84]$$

and

$$V'_{a0} = -I'_{a0}Z'_0 = -I'_{a0}\left(Z'_t + Z'_{00} - \frac{(Z_{0m})^2}{Z_{00}\ \text{(loop)}}\right) \qquad [85]$$

From [85], the equivalent zero-sequence impedance viewed from C' is

$$Z'_0 = Z' + Z'_{00} - \frac{(Z_{0m})^2}{Z_{00}\ \text{(loop)}} \qquad [86]$$

The zero-sequence impedance viewed from C' with the loop open is $Z_t + Z'_{00}$. Closing the loop reduces this impedance by $(Z_{0m})^2/Z_{00}$ (loop). Using the zero-sequence impedance given by [86], and the positive- and negative-sequence impedances viewed from C', the symmetrical components of current flowing into the fault are determined. Knowing I'_{a0}, the zero-sequence current in $B'C'$, the current in the loop is obtained from [84].

In Fig. 16(a), the line-to-ground fault is taken on a circuit supplied at one end only; the part of the system in which the fault occurs could be a network as well. In that case, the current I_{a0} in the loop $BCDEB$ is given by [84] in terms of the current I'_{a0} in the circuit $B'C'$, but I'_{a0} is not the fault current at C.

In Fig. 16(a) the transformer bank between A and B is shown as a Δ–Y grounded bank. No zero-sequence currents flow in this transformer bank with a line-to-ground fault at C'; however, if the fault occurs at C, there will be zero-sequence current in this bank and also zero-sequence voltage induced in line $B'C'$, and, if the line $B'C'$ is part of a closed loop, zero-sequence currents will flow in the loop.

POSITIVE- AND NEGATIVE-SEQUENCE SELF-IMPEDANCES AND MUTUAL IMPEDANCES BETWEEN THE SEQUENCE NETWORKS

No Ground Wires. Equations [6] give the positive-sequence self-impedance and the mutual impedances associated with positive-sequence currents in a three-phase circuit without ground wires. Equations [6*a*] give the corresponding equations for negative-sequence self- and mutual impedances. In the development of [6] and [6*a*], the presence of the earth is neglected. With positive- or negative-sequence currents only flowing in the three conductors, the sum of the currents is zero and no currents can flow from the conductors into the ground. Voltages will be induced in the ground, however, at all points where the sum of the fluxes is not zero, and eddy currents will flow in the ground. If the assumption is made that each of the currents flowing in the conductors induces an equal and opposite current in the ground, the effect of the presence of the earth can be taken into account by replacing the self- and mutual conductor impedances in [6] and [6*a*] by the corresponding impedances with ground return. If this is done, the self- and mutual impedances become

$$Z_{11} = Z_{22} = \tfrac{1}{3}(Z_{aa-g} + Z_{bb-g} + Z_{cc-g}) - \tfrac{1}{3}(Z_{ab-g} + Z_{ac-g} + Z_{bc-g}) \qquad [87]$$

$$Z_{21} = \tfrac{1}{3}(Z_{aa-g} + aZ_{bb-g} + a^2Z_{cc-g}) + \tfrac{2}{3}(a^2Z_{ab-g} + aZ_{ac-g} + Z_{bc-g}) \qquad [88]$$

$$Z_{12} = \tfrac{1}{3}(Z_{aa-g} + a^2Z_{bb-g} + aZ_{cc-g}) + \tfrac{2}{3}(aZ_{ab-g} + a^2Z_{ac-g} + Z_{bc-g}) \qquad [89]$$

$$Z_{01} = Z_{20} = \tfrac{1}{3}(Z_{aa-g} + a^2Z_{bb-g} + aZ_{cc-g}) - \tfrac{1}{3}(aZ_{ab-g} + a^2Z_{ac-g} + Z_{bc-g}) \qquad [90]$$

$$Z_{02} = Z_{10} = \tfrac{1}{3}(Z_{aa-g} + aZ_{bb-g} + a^2Z_{cc-g}) - \tfrac{1}{3}(a^2Z_{ab-g} + aZ_{ac-g} + Z_{bc-g}) \qquad [91]$$

It will be noted that Z_{01} and Z_{02} are equal, respectively, to Z_{20} and Z_{10} given by [45] and [46]; Z_{20} and Z_{10} were determined with zero-sequence currents only flowing in the circuit, while Z_{01} and Z_{02} in [90] and [91] were determined with positive- and negative-sequence currents, respectively.

The *positive- and negative-sequence self-impedance* given by [87], with the effect of the earth included, differs from that given by [6] or [6a], with the presence of the earth neglected, only in small differences between terms involving conductor heights above ground and horizontal spacings between conductors. For any given case, the difference in the resistance components can be determined from Figs. 3(a), 4(a), and 5(a), for 60, 50, and 25 cycles, respectively. The difference in the reactance components at 60 cycles can be obtained from Fig. 3(d) at 60 cycles. From these curves, it will be seen that the effect of the earth is to increase the positive- or negative-sequence resistance component of the self-impedance given by [7], and to decrease the reactance component. These changes for conventional overhead circuits are negligible for frequencies of 60 cycles or less. Neglecting these small resistance and reactance components, [87] reduces to [7], in which the presence of the earth is neglected.

Mutual Impedances between Sequence Networks. Remembering that $(1 + a + a^2) = 0$, the first terms on the right-hand sides of equations [88]–[91] disappear for conductors of equal diameters and equal heights above ground (flat horizontal spacing), and even for conductors of unequal heights above ground they are relatively too small to be considered, because of the multipliers 1, a, and a^2. This is true also of the terms in R_{ab-g} and X_{ab-g}, involving conductor heights and horizontal spacings between conductors in the second terms on the right-hand sides of these equations; the resistance components of these terms therefore are practically zero, and the reactance terms are functions of s_{ab}, s_{ac}, and s_{bc}, the spacings between conductors. Equations [88]–[91] therefore reduced to [8]–[11], in which the presence of the earth is neglected.

Since the presence of the earth has but little effect upon the mutual impedances between the sequence networks in a three-phase circuit at a frequency of 60 cycles or less, it can be concluded that sinusoidal currents of a given sequence flowing in unsymmetrical untransposed transmission circuits without ground wires induce voltages of the other two sequences in the conductors, but voltages induced in the ground by positive- and negative-sequence currents and positive- and negative-sequence voltages induced in the conductors by zero-sequence currents in the ground are negligible.

Untransposed Circuit with Inside Conductor Equidistant from the Other Two. In the flat horizontal or vertical arrangement of conductors, the inside conductor in high voltage circuits is usually equidistant from the other two. This is frequently true also in circuits where two conductors are vertically arranged with the inside conductor offset. Selecting phase a as the inside phase, if $s_{ab} = s_{ac}$, $\log_{10} s_{ac}/s_{ab} = \log_{10} 1 = 0$; and the j terms in the reactances given by equations [8]–[11] (which in effect are resistance terms) become zero, and in ohms per mile,

$$Z_{12} = Z_{21} = j3.105f \times 10^{-3} \log_{10} \frac{s_{ab}}{s_{bc}} \qquad [92]$$

$$Z_{10} = Z_{02} = Z_{20} = Z_{01} = -\tfrac{1}{2}Z_{12} = -j1.552f \times 10^{-3} \log_{10} \frac{s_{ab}}{s_{bc}} \qquad [93]$$

In flat horizontal or vertical arrangements of conductors, with phase a equidistant from phases b and c, $s_{bc} = 2s_{ab} = 2s_{ac}$, and, in ohms per mile,

$$\begin{aligned} Z_{12} = Z_{21} &= j3.105f \times 10^{-3} \log_{10} \tfrac{1}{2} = -j0.935f \times 10^{-3} \\ &= -j0.0561 \left(\frac{f}{60}\right) \end{aligned} \qquad [94]$$

$$\begin{aligned} Z_{10} = Z_{02} = Z_{20} = Z_{01} &= -\tfrac{1}{2}Z_{12} = j0.467f \times 10^{-3} \\ &= j0.0281 \left(\frac{f}{60}\right) \end{aligned} \qquad [95]$$

The mutual impedances per mile between the sequence networks given by [94] and [95] for a single circuit without ground wires are independent of the spacing between conductors, and are constant at a given frequency.

Relative Values of Sequence Self- and Mutual Impedances. The following problem will serve to show the relative importance of the sequence self- and mutual impedances of a three-phase transmission line without ground wires, short enough for the effects of capacitance to be neglected.

Problem 6. (*a*) Find the positive-, negative-, and zero-sequence self-impedances and the mutual impedances between the sequence networks in ohms per mile of a three-phase, 60-cycle, overhead transmission circuit of 4/0 copper, 19 strands, 10-foot horizontal spacing between adjacent conductors, height above ground of all conductors 40 feet, earth resistivity 100 ohms per meter cube.

(*b*) Assume a circuit 25 miles long, operating voltage 115 kv (line-to-line), line loading approximately 40,000 kva; the transformer banks at the ends of the line consist of three single-phase units connected Δ–Y with the Y's on the line (high) side solidly grounded; each single-phase unit is rated 13,333 kva and has a reactance of 9% on its rating. Assume the negative-sequence impedances of the system viewed from the low voltage sides of the transformer banks to be 10% at the supply end and 15% at the receiving end based on 40,000 kva and base line-to-line voltages of 115 kv referred to the line side of the transformer banks.

Determine the negative-sequence line currents and the ground currents in the transformer neutrals. To simplify calculations, neglect resistance and assume a value for the positive-sequence current.

Solution. (a) From Problem 1, $Z_{00} = 0.55 + j2.78$ ohms per mile. From Appendix B, $Z_{11} = Z_{22} = 0.278 + j0.805$ ohm per mile. From [94], $Z_{12} = Z_{21} = 0 - j0.0561$ ohm per mile. From [95], $Z_{10} = Z_{02} = Z_{01} = Z_{20} = j0.02805$ ohm per mile.

(b) Expressed in per unit, with resistance neglected, based on 40,000 kva (base three-phase kva) and 115 kv (base line-to-line voltage), the multiplier is $25 \times \dfrac{40{,}000}{(115)^2 \times 10^3}$, and

$$\begin{aligned} Z_{00} &= j0.210 \\ Z_{22} &= j0.061 \\ Z_{21} &= Z_{12} = -j0.00425 \\ Z_{01} &= Z_{02} = Z_{20} = Z_{10} = j0.00212 \end{aligned}$$

$$\text{Base line current} = \frac{40{,}000}{\sqrt{3}\,115} = 200 \text{ amperes}$$

Assume the positive-sequence current equal to base line current; then

$$I_{a1} = 200 \text{ amperes} = 1.0 \text{ in per unit} = 100\%$$

The series negative- and zero-sequence impedances include the transformer banks at both ends of the line, and the negative-sequence impedance also includes the negative system impedances viewed from the transformer banks. In per unit, the total impedances z_{22} and z_{00} to negative- and zero-sequence currents are

$$\begin{aligned} z_{22} &= j(0.18 + 0.25) + j0.061 = j0.49 \\ z_{00} &= j(0.18 + 0.21) = j0.39 \end{aligned}$$

Neglecting the effect of zero- and negative-sequence currents on each other, and substituting in equations [40a] of Chapter VIII,

$$I_{a2}' = -\frac{I_{a1}Z_{21}}{z_{22}} = \left(\frac{0.00425}{0.49}\right) \times 100\% = 0.87\%$$

$$3I_{a0} = -3\left(\frac{0.00212}{0.39}\right) \times 100\% = -1.63\%$$

The negative-sequence voltage on the low voltage side of the receiving end transformers is $I_{a2} \times$ (negative-sequence load impedance) $= 0.87\% \times 0.15 = 0.13\%$, or 0.0013 per unit of base line-to-neutral voltage.

With 200 amperes line current, the negative-sequence line current will be less than 2 amperes; the current in the transformer neutral will be approximately 3 amperes. The negative-sequence voltage at the load is 87 volts.

Although the high voltage overhead transmission line is an unsymmetrical circuit, the dissymmetry is but slight, and the error made by neglecting mutual impedance between the sequence networks is not serious unless a high degree of precision is required. See Problem 7 for unsymmetrical distribution circuits with ungrounded neutral conductor.

The *effect of ground wires* on the positive- and negative-sequence self-impedance and the mutual impedances between the sequence networks is but slight. A ground wire equidistant from the three conductors will have no voltage induced in it by positive- or negative-sequence currents in the three phases, and therefore will not affect these sequence self- and mutual impedances.

One Ground Wire. When the three conductors are at unequal distances from the ground wire and ground, positive- or negative-sequence currents flowing in the conductors will induce voltages in the ground wire and ground, and a current will flow in the loop consisting of ground wire with earth return of magnitude determined by the resultant voltage induced in the loop and its impedance. As the induced voltage is small and the loop impedance high, the current will be small. This current in turn induces voltages in the conductors which will influence the positive- and negative-sequence self-impedance of the circuit and the mutual impedances between the sequence networks.

The current flowing in the loop consisting of ground wire and ground, in series, caused by positive- or negative-sequence currents in the conductors, is not a zero-sequence current. Zero-sequence currents are equal and in phase in the three conductors; they flow from a grounded neutral and return by way of the ground wire and ground in parallel. Ground wire and ground currents resulting from positive- or negative-sequence currents do not flow from the neutral, but only in the loop consisting of ground wire and ground in series. The effect of this closed loop on the positive- and negative-sequence self-impedances and the mutual impedances between the sequence network can be determined by assuming only positive-sequence currents in the three phases; then, following a procedure analogous to that used in equations [53]–[59] to determine zero-sequence self- and mutual impedances, the following equations are obtained:

$$
\begin{aligned}
Z_{11-w} &= Z_{22-w} = Z_{11} - \frac{1}{3Z_{ww-g}}(Z_{aw-g} + aZ_{bw-g} + a^2Z_{cw-g}) \times \\
&\qquad\qquad (Z_{aw-g} + a^2Z_{bw-g} + aZ_{cw-g}) \\
Z_{21-w} &= Z_{21} - \frac{1}{3Z_{ww-g}}(Z_{aw-g} + a^2Z_{bw-g} + aZ_{cw-g})^2 \\
Z_{12-w} &= Z_{12} - \frac{1}{3Z_{ww-g}}(Z_{aw-g} + aZ_{bw-g} + a^2Z_{cw-g})^2 \qquad [96] \\
Z_{01-w} &= Z_{20-w} = Z_{01} - \frac{Z_{\overline{aw-g}}}{Z_{ww-g}}(Z_{aw-g} + a^2Z_{bw-g} + aZ_{cw-g}) \\
Z_{02-w} &= Z_{10-w} = Z_{02} - \frac{Z_{\overline{aw-g}}}{Z_{ww-g}}(Z_{aw-g} + aZ_{bw-g} + a^2Z_{cw-g})
\end{aligned}
$$

where the impedances $Z_{11} = Z_{22}$, Z_{21}, Z_{12}, $Z_{01} = Z_{20}$, and $Z_{02} = Z_{10}$ (without ground wire) are given by [87]–[91], respectively.

Because of the multipliers 1, a, and a^2, the second terms on the right-hand sides of equations [96] which give the changes in the impedances because of the ground wire will have small numerators relative to the denominator Z_{ww-g}. Therefore, unless a very high degree of precision is required, these terms are insignificant. For any given problem the self-impedance Z_{ww-g} and the mutual impedances Z_{aw-g}, Z_{bw-g}, and Z_{cw-g} for substitution in [96] can be determined to any desired degree of precision from [15] and [16], or from the charts prepared from equations [30]–[37]. See Fig. 3 for a frequency of 60 cycles.

For *two or more ground wires*, the change in positive-sequence self-impedance resulting from the ground wires can be determined by a procedure analogous to that used for determining the zero-sequence self-impedances with two or more ground wires.

For a *completely transposed circuit* there would be no ground wire current resulting from positive- or negative-sequence currents in the conductors if the ground wire or wires were grounded at their terminals only; but, since ground wires are grounded through tower footing impedance at every tower, there will be ground wire currents in each section of line between transposition which are displaced 120° from each other in phase. The positive- or negative-sequence impedance of the transposed circuit is therefore the same as the positive- or negative-sequence self-impedance of the untransposed circuit. The high-resistance loop or loops consisting of ground wire or wires with earth return tend to decrease slightly the positive-sequence self-impedance of the circuit calculated without ground wires but to increase the resistance component of this impedance. Theoretically, the increase in positive-sequence resistance would increase losses under normal operation. This increase in resistance, however, is usually much too small to be taken into consideration for conventional overhead transmission circuits.

Positive-Sequence Mutual Impedance. Positive-sequence currents flowing in one of two parallel three-phase circuits will induce voltages in the other circuit. Let the conductors of circuits 1 and 2 be a, b, c and A, B, C, respectively, a and A, b and B, c and C indicating conductors of the same phase. Positive-sequence mutual impedance between two parallel circuits will be defined as the ratio of the positive-sequence voltage induced in one circuit to the positive-sequence current flowing in the other circuit which produces it. The positive-sequence voltage induced in the reference phase of circuit 2 by positive-sequence currents in circuit 1, in the general case, will not be exactly the same as the positive-sequence voltage induced in the reference phase of circuit 1 by positive-sequence currents of the same magnitude flowing in circuit 2. The difference between either and the average of the two, however, will be small. In a static network, the mutual impedances between two conductors is reciprocal; in the sequence networks, the mutual impedances are not, in general, reciprocal.

With only positive-sequence currents I_{A1}, a^2I_{A1}, aI_{A1} flowing in conductors A, B, C of circuit 2, the voltages induced in conductors a, b, c of circuit 1 are

$$\begin{aligned} V_a &= j(X_{aA} + a^2X_{aB} + aX_{aC})I_{A1} \\ V_b &= j(X_{bB} + a^2X_{bC} + aX_{bA})a^2I_{A1} \\ V_c &= j(X_{cC} + a^2X_{cA} + aX_{cB})aI_{A1} \end{aligned} \qquad [97]$$

where X with two subscripts indicates the mutual reactance between the two conductors indicated by the subscripts.

The positive-sequence voltage induced in phase a of circuit 1 is $V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_c)$ and the positive mutual impedance Z_{aA1} by definition is V_{a1}/I_{A1}. Therefore,

$$\begin{aligned} Z_{aA1} = \frac{V_{a1}}{I_{A1}} = \tfrac{1}{3}j[X_{aA} + X_{bB} + X_{cC} &+ a^2(X_{aB} + X_{bC} + X_{cA}) \\ &+ a(X_{aC} + X_{bA} + X_{cB})] \end{aligned}$$

Likewise, the ratio of the positive-sequence voltage V_{A1} induced in circuit 2 by the positive-sequence currents in conductors a, b, and c of circuit 1 to the current I_{a1} is

$$\begin{aligned} Z_{Aa1} = \frac{V_{A1}}{I_{a1}} = \tfrac{1}{3}j[X_{aA} + X_{bB} + X_{cC} &+ a^2(X_{aC} + X_{bA} + X_{cB}) \\ &+ a(X_{aB} + X_{bC} + X_{cA})] \end{aligned}$$

Replacing the mutual impedances between conductors indicated by X's in the above equations by $2\pi fM$, where M is defined in [2], the average positive-sequence mutual impedance between two parallel circuits in ohms per mile is

$$Z_{m1} = j0.0466\left(\frac{f}{60}\right)\log_{10}\frac{s_{aB}s_{aC}s_{bA}s_{bC}s_{cA}s_{cB}}{s_{aA}^2s_{bB}^2s_{cC}^2} \qquad [98]$$

The average positive-sequence mutual impedance given by [98] may be positive or negative, depending upon whether the numerator or the denominator of the fraction following $\log_{10}$ is the larger. In any case, the positive-sequence mutual impedance is small — usually not more than $\pm 5\%$ of the positive-sequence self-impedance of either circuit, and it may be zero.

If V_a, V_b, and V_c in [97] are resolved in their *negative- and zero-sequence components*, the components of negative- and zero-sequence voltages induced in circuit 1 by positive-sequence currents in circuit 2 will be obtained. These induced components may be larger or smaller than the positive-sequence induced voltage, depending upon the configuration of the two parallel circuits.

With *negative-sequence currents* flowing in one of two parallel circuits, positive-, negative-, and zero-sequence components of voltage will be induced in the other. These induced voltages can be calculated in a manner similar to that given above for positive-sequence applied currents.

The question sometimes arises as to the arrangement of the phases in the six conductor positions on double-circuit towers which will give the lowest impedance during normal operation. The positive-sequence impedance of two parallel three-phase circuits with equal self-impedances Z_{11} and mutual impedance Z_{m1} between them is

$$Z_1 = \tfrac{1}{2}(Z_{11} + Z_{m1}) \qquad [99]$$

Using the average mutual impedance given by [98] for Z_{m1} and neglecting the mutual coupling with the negative- and zero-sequence networks, Z_1 in [99] is a minimum when Z_{m1} has its maximum negative value. With phases a, b, c occupying tower positions 1, 2, 3, respectively, in circuit 1, the phases A, B, C of circuit 2 can be assigned the tower positions 3, 4, 5 in six different arrangements. From [98], it is evident that the distances s_{aA}, s_{bB}, s_{cC} between conductors of the same phase should be large relative to the distances between conductors of different phases to give Z_{m1} its maximum negative value. By the "cut-and-try method," the arrangement which gives Z_{m1} its maximum negative value can be readily obtained.

Three-Phase Circuits with Neutral Conductors. Distribution circuits, as distinguished from high-voltage main transmission circuits, are frequently provided with a neutral conductor. If the neutral conductor is multigrounded, it can be treated as a ground wire in determining the sequence self- and mutual impedances of the circuit.

Ungrounded or Unigrounded Neutral Conductor. It has been shown that the presence of the earth has but slight influence on the positive- and negative-sequence self- and mutual impedances. With an ungrounded neutral conductor, or a unigrounded one and no ground fault on the circuit, zero-sequence currents flowing in the three phases return by way of the neutral conductor; as the vector sum of the currents in the same direction in the four conductors is zero, the voltages induced in the earth by zero-sequence currents is small, and the effect of the presence of the earth on zero-sequence impedances is comparable to that on positive- and negative-sequence impedances. The sequence self- and mutual impedances are given by [7], [8], [9], and [12], [13], and [14] with the presence of the earth neglected.

The location of the conductors in a 4-kv, three-phase distribution circuit with neutral conductor is indicated in Fig. 17(*a*); Fig. 17(*b*) gives a diagram of the supply transformer and the transmission circuit with single-phase loads of equivalent impedances Z_a, Z_b, and Z_c taken off between phases *a*, *b*, and *c* and the neutral conductor at the terminals of the circuit. Because of circuit dissymmetry, positive-sequence currents flowing in the circuit produce negative- and zero-sequence voltages which cause negative- and zero-sequence currents to flow during normal operation. This is one cause of unbalanced currents and voltages. Another is provided by the unequal loading of the three phases.

Although care is taken in assigning loads to the phases to obtain substantially balanced currents, variations in demand make it difficult to maintain this balance at all times. If the currents in the three phases are substantially balanced at full load, it is probable that they will not remain balanced at light load. The following problem illustrates the effect of an unsymmetrical transmission circuit on the voltages at the load when the equivalent load impedances between phase conductors and neutral conductor can be assumed equal with all loads concentrated at the circuit terminals.

Problem 7. In Fig. 17(*b*), the transformer bank of three single-phase units, each rated 200 kva, supplies a three-phase load of approximately 600 kva, 2400 volts (line-to-neutral), 0.707 power factor, through a 60-cycle distribution circuit 4 miles in length, with conductor and phase arrangement shown in Fig. 17(*a*). The neutral and phase conductors are of 1/0 copper. The voltages at R on the Δ side of the transformer bank can be assumed balanced and of such magnitude, or equivalent magnitude referred to the Y side, that approximately 2400 kv (line-to-neutral voltage) is maintained at the load. Each unit of the bank has a reactance of 5% based on 200 kva and 2400 volts on the Y side.

Determine the phase voltages at the load referred to the neutral conductor.

Solution. Rated voltage on the Δ side of the transformer bank and the transformer turn ratio are not required for the solution of the problem, as all voltages will be referred to the Y side of the bank.

Assume positive direction of current flow for phase currents and their symmetrical components from P to Q as indicated by arrows. Let base line-to-neutral voltage = 2400 volts, and base kva = 200 kva. Base line current is then 200/2.4 = 83.3 amperes, and base ohms = 28.8 ohms.

As a first approximation, the load impedances Z_a, Z_b, and Z_c in Fig. 17(*b*) will be assumed equal and of such values that, with balanced positive-sequence voltages of 2400 volts applied to them, the kva of each phase at the load will be 200 kva and the power factor 0.707. Then

$$Z_L = Z_a = Z_b = Z_c = \frac{(2400)^2}{200} \underline{/45°} \text{ ohms} = 28.8 \underline{/45°} = 20.36 + j20.36 \text{ ohms}$$
$$= 1 \underline{/45°} = 0.707 + j0.707 \text{ per unit}$$

Equations [40] of Chapter VIII give the positive-, negative-, and zero-sequence currents flowing in an unsymmetrical series circuit between points P and Q of an otherwise symmetrical system in which the two symmetrical parts of the system

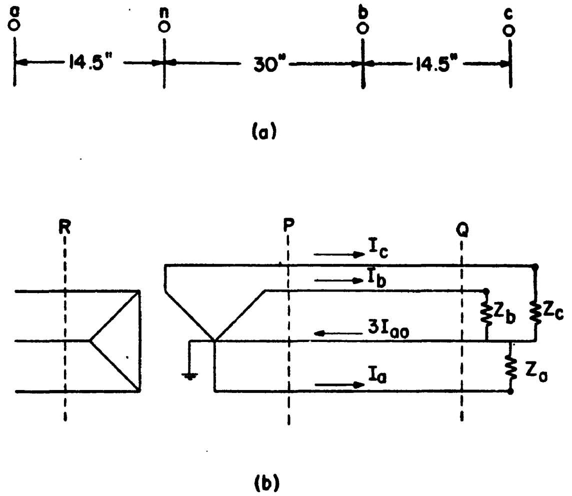


FIG. 17. Three-phase with neutral conductor n. (a) Arrangement of conductors on tower. (b) Three-line diagram of system.

viewed from P and Q are replaced by equivalent synchronous machines, as shown in Fig. 4 of Chapter VIII. Applying these equations to the system of Fig. 17(b) with the neutral conductor providing the return path for zero-sequence currents, Z_1, Z_2, Z_0 are the positive-, negative-, and zero-sequence impedances of the transformer bank; $Z_1 = Z_2 = Z_0 = 0.05$ per unit $= 1.44$ ohms. E_a is the voltage of phase a on the Δ side of the bank referred to the Y side. Z_1', Z_2', and Z_0' are the positive-, negative-, and zero-sequence impedances of the load.

$$Z_1' = Z_2' = Z_0' = 28.8 \underline{/45°} \text{ ohms}$$

As equal constant load impedances have been assumed, $E_a' = 0$. The Z's, with double subscripts which are the sequence self- and mutual impedances of the unsymmetrical transmission circuit, are determined as follows:

From Fig. 17(a),

$$s_{\overline{ab}} = \sqrt[3]{14.5 \times 44.5 \times 59.0} = 33.6 \text{ inches} = 2.80 \text{ feet}$$
$$s_{\overline{an}} = \sqrt[3]{14.5 \times 30 \times 44.5} = 26.8 \text{ inches}$$

From Appendix B, Table II, for 1/0 copper at 25°C, $r = 0.554$; $d = 0.368$ inch; x_i (at 60 cycles) $= 0.039$ ohm per mile; x_{11} (from Fig. 5, Appendix B) $= 0.67$ ohm per mile. For 4 miles of line,

$$Z_{11} = Z_{22} = 4(0.554 + j0.67) = 2.22 + j2.68 \text{ ohms}$$

From [8] and [9], for 4 miles of line,

$$\begin{aligned} Z_{12} &= 4\left[j0.1863\left(\log_{10}\frac{\sqrt{44.5 \times 59}}{14.5} + j\frac{\sqrt{3}}{2}\log_{10}\frac{59}{44.5}\right)\right] \\ &= -0.079 + j0.408 \text{ ohm} \end{aligned}$$

$$Z_{21} = 0.079 + j0.408 = 0.416 \underline{/79.0°} \text{ ohms}$$

From Z_{11}, Z_{12}, and Z_{21} and equations [12]–[14], for 4 miles of line,

$$\begin{aligned} Z_{00} &= 2.22 + j2.68 + 4\left[3(0.554 + j0.039) + j0.838 \log_{10}\frac{2(26.8)^2}{(33.6)(0.368)}\right] \\ &= 8.86 + j10.08 \text{ ohms} \end{aligned}$$

$$\begin{aligned} Z_{10} = Z_{02} &= -0.040 - j0.204 \\ &\quad - j4(0.2794)\left(\log_{10}\frac{\sqrt{30 \times 44.5}}{14.5} + j\frac{\sqrt{3}}{2}\log_{10}\frac{44.5}{30}\right) \\ &= 0.126 - j0.653 \text{ ohm} \end{aligned}$$

$$Z_{20} = Z_{01} = -0.126 - j0.653 = -0.665 \underline{/79.1°} \text{ ohms}$$

These impedances are expressed in per unit if divided by 28.8.

Knowing the impedances of the system under consideration, the only unknown in [40] of Chapter VIII is E_a, the equivalent applied voltage. Instead of assuming a reasonable value for E_a (possibly 10% above 2400 volts), a simpler procedure is to assume I_{a1} and to calculate I_{a2}, I_{a0}, and E_a from I_{a1}. The sequence voltages at the load can then be determined from I_{a1}, I_{a2}, I_{a0}, and the load impedances. As these voltages vary directly with E_a, they will be increased or decreased directly with the applied voltage.

In the given problem, the negative- and zero-sequence voltage drops $Z_{21}I_{a1}$ and $Z_{01}I_{a1}$ in the direction PQ caused by positive-sequence currents during normal operating conditions are small relative to E_a, and the resulting negative- and zero-sequence currents are likewise small relative to I_{a1}. If the effect of I_{a2} and I_{a0} on I_{a1} is neglected, equations [40*a*] of Chapter VIII can be used to determine I_{a2} and I_{a0}. The error in using [40*a*] instead of [40] will be negligible for this problem. It should be noted that there is no error in assuming I_{a1} instead of E_a when [40] is used to determine I_{a2} and I_{a0}, but that [40*a*] introduces an error which is negligible only when the mutual impedances in the unsymmetrical circuit are small relative to the total negative- and zero-sequence series self-impedances z_2 and z_0.

Expressed in ohms, z_2 and z_0 are

$$\begin{aligned} z_2 = Z_2 + Z_{22} + Z_2' &= (j1.44 + 2.22 + j2.68 + 20.36 + j20.36) \\ &= 22.58 + j24.48 = 33.3 \underline{/47.3°} \text{ ohms} \end{aligned}$$

$$\begin{aligned} z_0 = Z_0 + Z_{00} + Z_0 &= (j1.44 + 8.86 + j10.08 + 20.36 + j20.36) \\ &= 29.22 + j31.88 = 43.2 \underline{/47.5°} \text{ ohms} \end{aligned}$$

From [40a] of Chapter VIII,

$$I_{a2} = -I_{a1}\frac{0.416\,/79.0^\circ}{33.3\,/47.3^\circ} = (-0.0125\,/31.7^\circ)I_{a1}$$

$$I_{a0} = -I_{a1}\frac{-0.665\,/79.1^\circ}{43.2\,/47.5^\circ} = (0.0154\,/31.6^\circ)I_{a1}$$

E_a in terms of I_{a1} can be obtained from [37] of Chapter VIII with $E_a' = 0$. Neglecting the terms $I_{a2}Z_{12}$ and $I_{a0}Z_{10}$, which are small relative to $I_{a1}(Z_{11} + Z_1 + Z_1')$, E_a in per unit of base voltage with $I_{a1} = 1\,/0^\circ$ is

$$E_a = I_{a1}\left(\frac{33.3\,/47.3^\circ}{28.8}\right) = 1.155\,/47.3^\circ$$

Let the per unit positive-, negative-, and zero-sequence voltages at the load be V_{a1}, V_{a2}, and V_{a0}. Then, with $I_{a1} = 1\,/0^\circ$ as reference vector,

$$V_{a1} = I_{a1}Z_L = 1.00\,/45^\circ = 0.7071 + j0.7071$$

$$V_{a2} = I_{a2}Z_L = (-0.0125\,/31.7^\circ \times 1.00\,/45^\circ) = -0.0125\,/76.7^\circ$$
$$= -0.0029 - j0.0122$$

$$V_{a0} = I_{a0}Z_L = (0.0154\,/31.6^\circ \times 1.00\,/45^\circ) = 0.0154\,/76.6^\circ$$
$$= 0.0036 + j0.0149$$

The per unit phase voltages V_a, V_b, and V_c at the load referred to the neutral conductor are

$$V_a = 0.708 + j0.710 = 1.003\,/45.1^\circ$$

$$V_b = 0.274 - j0.947 = 0.986\,\overline{/73.9^\circ}$$

$$V_c = -0.971 + j0.282 = 1.011\,/163.8^\circ$$

For the arrangement of the conductors considered and loads of equal impedances, the voltage of phase b at the load is 1.4% below the average voltage at the load and that of phase c 1.1% above this average. For the equal load impedances assumed, the power and kva at the load in phases a, b, and c will vary as the squares of the phase voltages. The load kva of phase a is 201.2 kva; of phase b, it is 194.4 kva; and of phase c, it is 204.4. The total load kva is 600 kva, and therefore the given conditions of the problem are satisfied, although the loads in the three phases are only approximately equal. If the total calculated load kva had been different from 600 kva, this value could readily be modified by assuming a higher or lower value for I_{a1} (or for E_a).

Loads of Unequal Impedances. When the loads on the three phases at the terminals of the transmission line in Fig. 17(b) are unequal, they will usually be given in kva (or kw) at a specified power factor. As the voltages across the loads will be unbalanced, it may be necessary to assume several sets of values for the load impedances before the correct values are obtained. For any assumed load impedances, the system voltages and currents can be expressed in terms of I_{a1} or E_a, as explained in Problem 7. With unequal load impedances, the mutual impedances between the sequence networks, because of the unsymmetrical loads, may be much larger relative to the self-impedances than in Problem 7, and equations [40] of Chapter VIII rather than equations [40*a*] will be required.

Ground Fault on Single-Phase Circuit Tapped from Three-Phase Four-Wire Circuit. In three-phase four-wire distribution circuits where loads are taken off between the phases and a neutral conductor, one or two phases and the neutral conductor n may extend beyond the terminals T of the three-phase circuit. If a ground fault occurs on a phase conductor beyond T, the conditions at T are similar to those for a fault to ground at T through a fault impedance Z_f, where Z_f is the impedance met by the fault current in the single-phase circuit.

Neutral Conductor Grounded. Let Z_{a0} be the impedance of conductor a in series with the neutral conductor n and ground in parallel. The voltage drops in conductors a and n, each with ground return (considering a second phase conductor b open, if extended beyond T), are

$$V_a = I_a Z_{aa-g} + I_n Z_{an-g}$$

$$0 = I_a Z_{an-g} + I_n Z_{nn-g}$$

From these equations,

$$Z_{a0} = \frac{V_a}{I_a} = Z_{aa-g} - \frac{(Z_{an-g})^2}{Z_{nn-g}} \qquad [100]$$

where Z_{aa-g} and Z_{nn-g} are the self-impedances with earth return of conductors a and n, respectively, and Z_{an-g} is the mutual impedance between a and n with common earth return.

With a fault to ground on the single-phase circuit, the symmetrical components of fault current in the three-phase part of the circuit, obtained by replacing Z_f in [28] of Chapter IV by Z_{a0}, are

$$I_{a1} = I_{a2} = I_{a0} = \frac{V_f}{Z_1 + Z_2 + Z_0 + 3Z_{a0}} \qquad [101]$$

where V_f is the voltage of phase a at T before the fault and Z_1, Z_2, and Z_0 are the positive-, negative-, and zero-sequence impedances of the three-phase part of the system viewed from T.

If the neutral conductor n is ungrounded in the single-phase circuit but grounded in the three-phase circuit, the impedance met by the current in a ground fault is Z_{aa-g}, the impedance of the faulted conductor with earth return. Z_{aa-g} then replaces Z_{a0} in [101].

Impedances of Single-Phase Circuits. In a *two-wire single-phase circuit* supplied through a transformer with the midpoint of the secondary winding grounded, the voltage drops in the two loop circuits consisting of conductors a and b, each with earth return, are

$$\left.\begin{aligned} V_a &= I_a Z_{aa-g} + I_b Z_{ab-g} \\ V_b &= I_a Z_{ab-g} + I_b Z_{bb-g} \end{aligned}\right\} \quad [102]$$

Replacing I_a and I_b in these equations by their values in terms of their positive- and zero-sequence symmetrical components given by [7] of Chapter IX, resolving V_a and V_b into their positive- and zero-sequence components by [6] of Chapter IX, and equating the coefficients of I_{a1} and I_{a0} in the resulting equations to the corresponding coefficients in [21] of Chapter IX,

$$\begin{aligned} Z_{11} &= \tfrac{1}{2}(Z_{aa-g} + Z_{bb-g}) - Z_{ab-g} = Z_{\overline{aa}-g} - Z_{ab-g} \\ Z_{00} &= \tfrac{1}{2}(Z_{aa-g} + Z_{bb-g}) + Z_{ab-g} = Z_{\overline{aa}-g} + Z_{ab-g} \\ Z_{10} &= Z_{01} = \tfrac{1}{2}(Z_{aa-g} - Z_{bb-g}) \end{aligned} \quad [103]$$

For identical conductors at equal heights above ground, $Z_{aa-g} = Z_{bb-g}$. Neglecting the terms in Z_{aa-g}, Z_{bb-g}, and Z_{ab-g} involving height above ground, Z_{11} in [103] reduces to Z_{11} given by [4] and $Z_{10} = Z_{01} = 0$.

Three-Wire Single-Phase Circuit. The impedances of the three-wire single-phase circuit with ungrounded neutral conductor are given by [4] and [5]. With grounded neutral conductor or ground wire n, zero-sequence currents are equal in phases a and b and return through the ground in parallel with n. With zero-sequence currents only flowing in the phases a and b, the voltage drops in the three loop circuits consisting of conductors a, b, and n, each with earth return, are

$$\begin{aligned} V_a &= I_{a0}(Z_{aa-g} + Z_{ab-g}) + I_n Z_{an-g} \\ V_b &= I_{a0}(Z_{bb-g} + Z_{ab-g}) + I_n Z_{bn-g} \\ 0 &= I_{a0}(Z_{an-g} + Z_{bn-g}) + I_n Z_{nn-g} \end{aligned} \quad [104]$$

Solving the last equation for I_n,

$$I_n = -I_{a0}\frac{2Z_{\overline{an}-g}}{Z_{nn-g}} \quad [105]$$

Substituting I_n from [105] in the equations for V_a and V_b, resolving V_a and V_b into their positive- and zero-sequence components of voltage by [6] of Chapter IX, and equating the coefficients of I_{a0} in the resulting equations for V_{a0} and V_{a1} to the corresponding coefficient in [21] of

Chapter IX,

$$Z_{00} = (Z_{\overline{aa}-g} + Z_{ab-g}) - \frac{2Z^2_{\overline{an}-g}}{Z_{nn-g}}$$

$$Z_{10} = Z_{01} = \tfrac{1}{2}(Z_{aa-g} - Z_{bb-g}) - \frac{Z_{\overline{an}-g}}{Z_{nn-g}}(Z_{an-g} - Z_{bn-g}) \qquad [106]$$

For identical conductors, the first term in mutual impedance equation is zero; if $s_{an} = s_{bn}$, the second term is also zero.

Two-Phase, Three-Wire Circuits. In Chapter IX, two-phase, three-wire circuits are discussed, and the phase voltages and currents resolved into two different sets of components. If positive- and zero-sequence symmetrical components are used, Z_{11} is given by [4]; Z_{00} and $Z_{01} = Z_{10}$ are given by [5] if the neutral conductor is ungrounded, and by [106] if grounded.

If positive- and negative-sequence right-angle components $Z_{11}(90°)$, Z_{22}, Z_{12}, and Z_{21} are required, they can be determined by substituting the positive- and zero-sequence self- and mutual impedances in [40] of Chapter IX.

If the phase quantities are used, Z_{AA}, Z_{BB}, and Z_{AB} can be determined by substituting the positive- and zero-sequence self and mutual impedances in [42] of Chapter IX. The equivalent self-impedances of phases A and B and the neutral conductor n are

$$\begin{aligned} Z_{AA} - Z_{AB} &= Z_{11} + Z_{10} = \text{equivalent impedance of phase } A \\ Z_{BB} - Z_{AB} &= Z_{11} - Z_{10} = \text{equivalent impedance of phase } B \\ Z_{AB} &= \tfrac{1}{2}(Z_{00} - Z_{11}) = \text{equivalent impedance of neutral conductor} \end{aligned} \qquad [107]$$

For identical phase conductors equidistant from the neutral conductor,

$$\begin{aligned} Z_{AA} - Z_{AB} &= Z_{BB} - Z_{AB} = Z_{11} \\ Z_{AB} &= \tfrac{1}{2}(Z_{00} - Z_{11}) \end{aligned} \qquad [108]$$

where Z_{11} is defined in [4], Z_{00} in [5] for an ungrounded neutral conductor, and in [106] for a multigrounded neutral conductor.

ZERO-SEQUENCE EQUIVALENT CIRCUITS WITH IDENTITY OF GROUND-RETURN PATH RETAINED

The zero-sequence equivalent circuits developed in the preceding pages are based on the assumption that phase voltages at any system point are referred to ground at that particular point. Fault impedance may be included in these equivalent circuits, as explained in Chapter IV, but impedance grounds between ground wires or multigrounded neutral conductors and true ground are neglected. Zero-sequence equivalent circuits in which the identity of the ground-return path is retained may be required in special problems.

Rüdenberg's Equations. In calculating the impedance of a conductor with earth return, Dr. Reinhold Rüdenberg[12] assumes that both an insulated underground conductor of diameter d at a distance h from the surrounding earth and an overhead conductor of diameter d at a distance h above ground can be replaced approximately by a conductor of diameter d at the center of a semicircular trough in the earth of radius h. Under this assumption, with uniform earth resistivity ρ in abohms per centimeter cube and the frequencies encountered in power systems, the impedance corresponding to the flux in the earth in abohms per centimeter is shown to be approximately

$$Z_g = R_g + jX_g = \pi^2 f + j2\omega \log_e \frac{0.178}{h}\sqrt{\frac{\rho}{f}} \qquad [109]$$

where $\omega = 2\pi f$ and h is the radius of the semicircular trough; h corresponds to the radius of the sheath of an underground cable buried directly in the earth or the height above ground of an overhead wire.

The self-impedance of any conductor a with earth return includes, in addition to Z_g, the impedance Z_{aa-h} corresponding to the flux within the trough of radius h:

$$Z_{aa-h} = (r_a + jx_i) + j2\omega \log_e \frac{2h}{d_a} \text{ abohms per centimeter} \qquad [110]$$

where r, d, x_i are the resistance, diameter, and internal reactance, respectively, of the conductor indicated by double subscripts of Z. The term conductor here includes ground wires and sheaths.

Adding [109] and [110], and indicating the self-impedance of the conductor a with ground return by an additional subscript g, in absolute units,

$$\begin{aligned} Z_{aa-g} &= (r_a + R_g) + j(X_{aa-g} + x_i) \\ &= (r_a + \pi^2 f) + j\left(2\omega \log_e \frac{0.178}{\frac{d_a}{2}}\sqrt{\frac{\rho}{f}} + x_i\right) \end{aligned} \qquad [111]$$

Z_{ab-g}, the mutual impedance with common earth return between two parallel conductors a and b within the trough of radius h with spacing s_{ab} between their centers in absolute units, is shown to be

$$Z_{ab-g} = R_g + jX_{ab-g} = \pi^2 f + j2\omega \log_e \frac{0.178}{s_{ab}}\sqrt{\frac{\rho}{f}} \qquad [112]$$

Z_{ab-g} can also be written as the sum of Z_g and Z_{ab-h}, where

$$Z_{ab-h} = j2\omega \log_e \frac{h}{s_{ab}} \text{ abohms per centimeter} \qquad [113]$$

Rüdenberg's equations for Z_{aa-g} and Z_{ab-g} at power system frequencies given in [111] and [112] are independent of h, the height of the overhead conductor above ground or the distance of the underground insulated conductor from the surrounding earth; they are therefore the same for underground as for the overhead conductors.

Comparing equations [111] and [112] with Carson's equations for R_{aa-g}, R_{ab-g}, X_{aa-g}, and X_{ab-g} in absolute units given by [26]–[29], with the terms involving conductor height above ground omitted and λ replaced by $1/\rho$, the earth resistance component $R_g = R_{aa-g} = R_{ab-g} = \pi^2 f$ is the same. The reactance terms of Carson's equations are higher than Rüdenberg's by the difference $2\omega \log_e 0.208/0.178$ abohms per centimeter. Expressed in ohms per mile, the difference is

$$2 \times 377 \left(\frac{f}{60}\right) \times 0.1609 \times 10^{-3} \times 2.303 \log_{10} \frac{0.208}{0.178} = 0.019 \left(\frac{f}{60}\right)$$

This difference is unimportant in circuits in which the effects of tower height can be neglected. It has already been pointed out that the effect of tower height on self- and mutual impedances with earth return calculated by Carson's equations is insignificant for frequencies of 60 cycles or less with earth resistivities of $\rho = 100$ ohms per meter cube or more. (See Figs. 3(a) and 3(d).)

With a three-conductor cable in a lead sheath buried in the ground, the division of flux into flux within the sheath and flux in the earth is helpful in calculating zero-sequence impedances of cables. This method is used in the chapter on insulated power cables in Vol. II. The conductors of the three-conductor cable are completely transposed so that the effective distance of each from the earth is the same, as is also the distance between any two conductors.

With a completely transposed three-phase overhead transmission line, where the distance between conductors is less than the height of the conductors above ground, the earth return components of self- and mutual impedances based on Rüdenberg's equations will be the same. It is possible, therefore, to separate the impedance of the earth return path $Z_g = R_g + jX_g$ from the self- and mutual impedances with earth return.

Determination of Impedance of Earth-Return Path. The resistance component R_g of the impedance Z_g of the earth-return path is independent of h, the conductor height above ground, but the reactance component X_g is a function of h.

$$R_g = 0.095 \frac{f}{60} \text{ ohms per mile} \qquad [114]$$

X_g, at frequencies of 60, 50, or 25 cycles per second, may be read from Figs. 3(*c*), 4(*c*), or 5(*c*), respectively, with *average height in feet* as abscissa instead of "distance between conductors." Thus, with $f = 60$ cycles, $\rho = 3000$ ohms per meter cube, and $h = 40$ feet, Z_g is approximately

$$Z_g = 0.095 + j0.72 \text{ ohm per mile}$$

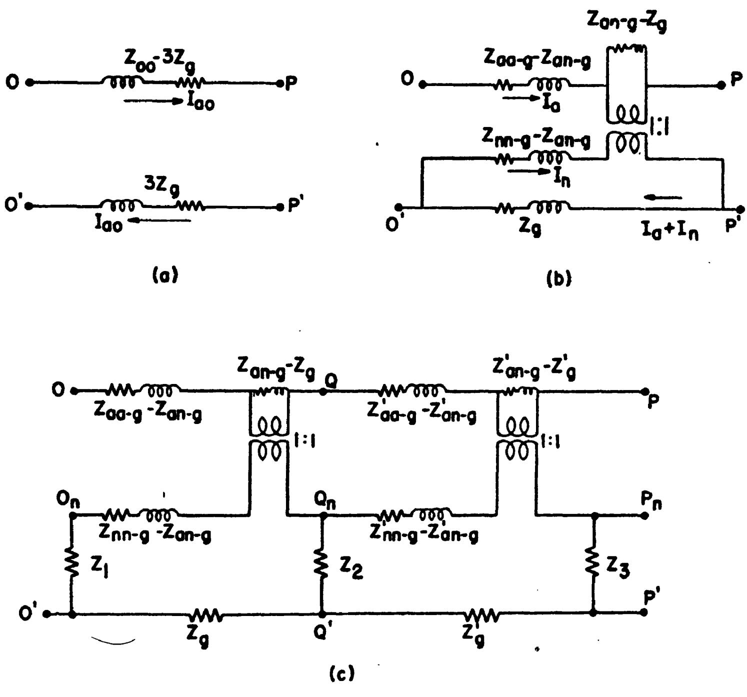


FIG. 18. Approximate zero-sequence equivalent circuits with identity of earth return path retained. (*a*) Three-phase circuit with or without ground wires, no impedance between ground wires and ground. (*b*) Phase conductor *a* and multigrounded neutral conductor *n*. No impedance between neutral conductor and ground. (*c*) Two sections of conductor *a* and multigrounded neutral conductor *n* with unequal impedances Z_1, Z_2, Z_3, · · · , between *n* and ground for use on an a-c network analyzer.

Zero-Sequence Equivalent Circuits with Impedance in Earth-Return Path. Figure 18(*a*) shows an equivalent circuit on a per phase basis for a three-phase transmission circuit between *O* and *P*, in which $3Z_g$

is subtracted from Z_{00}, the zero-sequence self-impedance of the circuit (calculated as previously explained), and $3Z_g$ placed between the ground points O' and P' at O and P, respectively. The points O and P are to be connected into the zero-sequence equivalent circuit for the system, the zero-potential bus of the equivalent circuit being opened between O' and P' and $3Z_g$ inserted in the opening. In the equivalent circuit of Fig. 18(a), voltages at O and P may be referred to ground at either O or P. For a single-phase circuit treated as a two-vector system (see Chapter IX) or for a two-phase circuit, $2Z_g$ is placed between O' and P' and $2Z_g$ subtracted from the zero-sequence self-impedance of the circuit. For the case of a single conductor a, with return in a multigrounded neutral conductor n in parallel with the ground, Z_g is placed between O' and P' and Z_g subtracted from Z_{a0} given by [100]. If it is desired to retain the identity of the multigrounded neutral conductor n, Fig. 18(b) may be used in which Z_g is subtracted from Z_{aa-g}, Z_{nn-g}, and Z_{an-g}, and Z_g placed in the ground return path. In this equivalent circuit impedances between n and true ground are neglected.

To include *resistance grounds* between ground wires or multigrounded neutral conductor and ground, the equivalent circuit can be represented on an a-c network analyzer in sections between the resistance grounds, the identity of each circuit and of each ground wire or grounded neutral conductor being retained. This is illustrated in Fig. 18(c) for the case of one conductor a and a multigrounded neutral conductor n with impedance grounds Z_1, Z_2, Z_3, etc., between n and ground; two sections only are shown, one between O and Q and one between Q and P. On an a-c network analyzer, the number of sections which can be set up is limited only by the number of available mutual coupling transformers. In the general case of more sections than available coupling transformers, the zero-sequence equivalent circuit can be represented by several equivalent circuits in series. For example, if eight coupling transformers are available, eight sections are set up between O and P, then these eight sections can be replaced by one equivalent circuit with the identity of the six terminals O, O_n, O', P, P_n, and P' retained. The next group of eight sections is then replaced by a second six-terminal equivalent circuit, and so on, until all sections are included. These equivalent circuits can then be reduced to one equivalent circuit, if desired.

A six-terminal equivalent circuit, in which each terminal is directly connected to every other terminal through impedance, is readily obtained on the a-c network analyzer; a voltage is applied between a reference bus and each of the terminals in turn, all other terminals being connected by leads to the reference bus. The ratio of the applied voltage to the current in any leads will give one of the fifteen branch impedances of the six-terminal equivalent circuit. For example, with a voltage E applied between any reference bus B and terminal P_n of Fig. 18(c), with terminals O, O_n, O', P, and P' shorted by leads to bus B, the ratio of E to the current in the lead from O will give the impedance to be inserted in the equivalent circuit between terminals O and P_n. This is the second method of determining an equivalent circuit given in Chapter I.

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CHAPTER XII

CAPACITANCES OF OVERHEAD TRANSMISSION LINES

The importance of capacitance in overhead transmission lines depends not only upon the length of the line, but also upon the manner in which capacitance influences a given problem. In short-circuit calculations or in determining voltage regulation in a transmission circuit, the error in neglecting the capacitance of lines less than approximately 50 miles in length at 60 cycles is relatively unimportant. On the other hand (as illustrated in Vol. II), the capacitance of a line only a few miles in length supplying an unloaded or lightly loaded ungrounded transformer bank may be of importance if one of the conductors should be open, or open and grounded, and the capacitive reactance of the circuit approximately the same as the magnetizing reactance of the transformer bank.

Assumptions. The surface of the earth will be assumed to be an equipotential plane of zero potential in calculating capacitances of overhead transmission lines. The electric field above ground and at the surface of the ground resulting from the charge on a conductor and the presence of the zero-potential ground plane is the same as that which would be produced by the charge on the conductor and an equal and opposite charge on the image of the conductor, the image being as far below ground as the conductor is above ground.[1] Making use of this principle, calculations of capacitance can be simplified by considering the charges on the conductors and on their images instead of the charges on the conductors and on the earth.

It will also be assumed that the charge on a conductor is uniformly distributed, and the potential of a conductor is the same throughout the length considered. Under the latter assumption, the effects of leakance and impedance upon capacitance are neglected. Transmission lines operated below corona starting voltage are effectively insulated so that leakance is negligible. Impedance is taken into account in developing equivalent circuits for transmission lines with distributed constants, but in *calculating* capacitance, the effect of impedance is neglected. As contrasted with this, the effect of capacitance upon impedance is taken into account in the fundamental equations[2] for ground return circuits used in calculating zero-sequence impedances of overhead transmission lines.

The conductors are assumed to be at constant distances from the surface of the earth. Sag in conductors can be taken into account by using equivalent constant distances between the earth's surface and the conductors. The effect of supporting structures, trees, buildings, and the character of the soil and its covering are neglected. These effects, in that they effectively bring the conductors nearer to the earth, tend to increase capacitance, especially zero-sequence capacitance.

Zero-sequence capacitances calculated from equations based on the assumptions given here, in general, are lower than zero-sequence capacitances obtained from tests[3] on actual transmission lines. The equations, however, are judged adequate unless a high degree of precision is required; as, for example, in the design of a ground-fault neutralizer, where the reactance used depends upon the zero-sequence capacitances of the circuits involved. (See Chapter VI.) When capacitance formulas are developed which include effects now neglected, they can probably be expressed in terms of equivalent height above ground of conductors; if this is done, the equations and charts given in this chapter and elsewhere in the literature can still be applied.

Capacitance is defined as the ratio of charge to potential. In absolute units,

$$C = \frac{Q}{V} \tag{1}$$

Equation [1] can be used in the calculation of capacitance of overhead transmission lines if Q is the charge in statcoulombs per centimeter length of conductor, V the potential of the conductor in statvolts, and C the capacitance in statfarads per centimeter. To express C in farads per mile, C in statfarads per centimeter is multiplied by $10^{-11}/9 \times 1.6093 \times 10^5 = 0.17881 \times 10^{-6}$.

Potential of a Conductor Due to Its Own Charge. The potential of a conductor a above ground in statvolts due to its charge Q_a and the charge $-Q_a$ on its image a in statcoulombs is

$$V_a = 2Q_a \log_e \frac{h + \sqrt{h^2 - r^2}}{r} \tag{2}$$

where h is the height of the center of the conductor above ground and r is the radius of the conductor.[1] When h is large relative to r, as is the case with overhead transmission lines, [2] becomes

$$V_a = 2Q_a \log_e \frac{2h}{r} = 2Q_a \log_e \frac{S_{aa}}{r_a} \tag{3}$$

where S_{aa} is the distance between conductor a and its image a. (See Fig. 1 (a).)

Single Conductor. The capacitance to ground of a single conductor parallel to the ground from [1] and [3] is

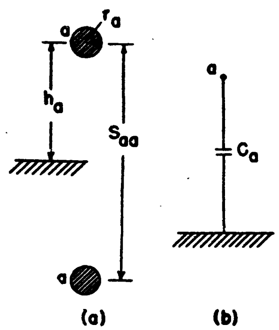


FIG. 1. (a) One conductor a parallel to ground. (b) Equivalent capacitance circuit for (a) where C_a is defined in [4].

$$C = \frac{Q}{V} = \frac{1}{2 \log_e \dfrac{2h}{r}} \text{ statfarads per centimeter}$$

Replacing $\log_e$ by 2.3026 $\log_{10}$ and multiplying by 0.17881×10^{-6},

$$C = \frac{0.03883}{\log_{10} \dfrac{2h}{r}} 10^{-6} \text{ farads per mile} \qquad [4]$$

The capacitance to ground of a single conductor parallel to the ground with leakance and impedance neglected is indicated in Fig. 1 (b). The capacitive susceptance b in mhos per mile is

$$b = 2\pi f C = \left(\frac{f}{60}\right) \frac{14.64}{\log_{10} \dfrac{2h}{r}} \times 10^{-6} \qquad [4a]$$

The admittance to ground of a conductor ℓ miles in length, with leakance and impedance neglected, is

$$Y = 0 + jb\ell = j2\pi f C\ell \text{ mhos} \qquad [4b]$$

Example 1. The capacitance to ground of a single conductor with diameter of 0.5 inch, 40 feet above the ground, in farads per mile, from [4], is

$$C = \frac{0.03883}{\log_{10} \dfrac{80 \times 12}{0.25}} \times 10^{-6} = \frac{0.03883}{3.584} 10^{-6} = 0.0108 \times 10^{-6}$$

The capacitive susceptance per mile at 60 cycles from [4a] is

$$b = 2\pi f C = 4.09 \times 10^{-6} \text{ mho per mile}$$

The admittance of ℓ miles of line is

$$Y = j4.09 \times 10^{-6}\, \ell \text{ mhos}$$

Potential of a Conductor Due to the Charge on a Neighboring Conductor. The potential of a conductor a in statvolts due to the charge Q_b in statcoulombs per centimeter on a neighboring conductor b and the charge $-Q_b$ on the image of b is

$$V_a = 2Q_b \log_e \frac{S_{ab}}{s_{ab}} \tag{5}$$

where s_{ab} is the spacing between the centers of conductors a and b, S_{ab} that between conductor a and the image of conductor b (or between conductor b and the image of a), and the distortion of the field because of the presence of the second conductor is neglected.[1] (See Fig. 2(a.))

Potential Coefficients. With more than one conductor the capacitance of each conductor is influenced by the presence of the other conductors. Consider the n conductors a, b, $\cdots$ n charged with quantities of electricity Q_a, Q_b, $\cdots$ Q_n to potentials V_a, V_b, $\cdots$ V_n. The potential above ground of a conductor due to its own charge can be expressed in terms of that charge by [3], and the potential due to the charge on a neighboring conductor by [5]. By superposition, the potential of a conductor above ground due to its own charge and the charges on all neighboring conductors can be expressed in terms of these charges. The potentials to ground of the conductors in terms of the charges are

$$\begin{aligned} V_a &= P_{aa}Q_a + P_{ab}Q_b + \cdots + P_{an}Q_n \\ V_b &= P_{ba}Q_a + P_{bb}Q_b + \cdots + P_{bn}Q_n \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ V_n &= P_{na}Q_a + P_{nb}Q_b + \cdots + P_{nn}Q_n \end{aligned} \tag{6}$$

where the *potential coefficients* P_{aa}, P_{bb}, $\cdots$ P_{nn}, and P_{ab}, P_{ac}, $\cdots$ P_{an} depend upon physical dimensions and can be obtained from the following equations:

$$\begin{aligned} P_{aa} &= 2 \log_e \frac{S_{aa}}{r_a} \\ &\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots \\ P_{ab} &= P_{ba} = 2 \log_e \frac{S_{ab}}{s_{ab}} \end{aligned} \tag{7}$$

where r denotes radius of conductor, s spacing between the two conductors indicated by its subscripts, and S spacing between a conductor and its own image or the image of another conductor as indicated by its subscripts.[1] Figures 2(a) and 3(a) show s and S with subscripts for two and three conductors, respectively.

Sequence Capacitances. When the method of symmetrical components is applied to the solution of problems which involve the capacitances of transmission circuits, the *positive-*, *negative-*, and *zero-sequence capacitances* of the circuits, as distinguished from the capacitances between conductors or between conductors and ground, are required. Two methods of determining sequence capacitances will be given: one by means of Maxwell's coefficients and the other by the use of potential coefficients only, the two methods giving identical results. The differences between the two methods are summarized briefly as follows:

When *Maxwell's coefficients* are used, an equivalent capacitance circuit is developed of the same number of terminals as there are conductors (including ground wires) in the given circuit. In this equivalent circuit, the direct capacitances between conductors and the direct capacitances between conductors and ground are given. If one of the conductors is a ground wire, that conductor is grounded in the equivalent circuit by shorting out its direct capacitance to ground, giving an equivalent circuit of one less terminal but with its capacitances determined with all conductors (including the ground wire) considered. The sequence capacitances are not given directly by Maxwell's coefficients but they can be calculated from them, as explained later.

When *potential coefficients* only are used, the charges on the ground wires (or grounded neutral conductor) are eliminated in the equations of [6], and these equations reduced in number to the number of the phases. For the three-phase circuit with two ground wires, for example, there are five initial equations in [6], but the potentials of the ground wires are zero, and therefore the five equations can be reduced to three by eliminating the charges on the ground wires. From the resultant equations (three in number for a three-phase system), the sequence potential coefficients are determined as explained in detail later; and from the sequence potential coefficients, the sequence *capacitive impedances*. From the capacitive impedances, the sequence capacitive admittances or capacitances are obtained if required.

For the two-wire single-phase circuit and the three-phase circuit without ground wires, there is but little choice between the two methods. For circuits with ground wires or with a grounded neutral conductor, the potential coefficient method involves considerably less work. Moreover, it is possible to prepare charts based on this method from which the zero-sequence capacitive impedances of three-phase circuits and the mutual capacitive impedances between parallel three-phase circuits, with and without ground wires, can be quickly determined. Zero-sequence capacitive impedances are used in the equivalent circuits of Chapter VI.

Circuit and Sequence Capacitances Determined from Maxwell's Coefficients

Maxwell's Coefficients. The charges on the conductors can be expressed in terms of the potentials of the conductors and arbitrary coefficients by the following equations:

$$\begin{aligned} Q_a &= C_{aa}V_a - C_{ab}V_b - C_{ac}V_c - \cdots - C_{an}V_n \\ Q_b &= -C_{ba}V_a + C_{bb}V_b - C_{bc}V_c - \cdots - C_{bn}V_n \\ Q_c &= -C_{ca}V_a - C_{cb}V_b + C_{cc}V_c - \cdots - C_{cn}V_n \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Q_n &= -C_{na}V_a - C_{nb}V_b - C_{nc}V_c - \cdots + C_{nn}V_n \end{aligned} \qquad [8]$$

The significance of the coefficients, called Maxwell's coefficients, becomes apparent if all conductors except conductor a are grounded. Then $V_b = V_c = \cdots = V_n = 0$, and [8] becomes

$$\begin{aligned} Q_a &= C_{aa}V_a \\ Q_b &= -C_{ba}V_a \\ Q_c &= -C_{ca}V_a \\ &\cdots\cdots\cdots \\ Q_n &= -C_{na}V_a \end{aligned} \qquad [9]$$

From [9], $C_{aa} = Q_a/V_a$ is the capacitance to ground of conductor a when all other conductors are at zero potential. Similarly, C_{bb}, C_{cc}, $\cdots C_{nn}$ are the capacitances to ground of conductors $b, c, \cdots n$ when all conductors except $b, c, \cdots n$, respectively, are at zero potential. The C's with two identical subscripts (C_{aa}, C_{bb}, $\cdots C_{nn}$) are called the coefficients of self-induction of electrostatic charge.

With the charge Q_a on conductor a and all the other conductors grounded, each of the grounded conductors has a part of the induced negative charge. From [9], these induced negative charges (Q_b, Q_c, $\cdots Q_n$) are proportional to the potential V_a, and therefore the C's with two different subscripts (C_{ba}, C_{ca}, $\cdots C_{na}$) represent direct capacitances between the conductors indicated by the subscripts. That $C_{ba} = C_{ab}$, $C_{ca} = C_{ac}$, etc., will be shown in the following development. (See [14] and [31].)

Equations [8] are frequently written with all positive signs; when this is the case, the C's with two different subscripts are called the coefficients of mutual induction of electrostatic charge and are negative. With negative signs before these C's, the C's themselves are positive and represent direct capacitances between conductors. Maxwell's coefficients can be expressed in terms of the potential coefficients by solving [6] for the charges Q_a, Q_b, $\cdots Q_n$ and then equating the coefficients of V_a, V_b, $\cdots$ V_n in the resultant equations to the coefficients of V_a, V_b, $\cdots$ V_n in corresponding equations in [8]. This will be illustrated for two- and three-conductor circuits.

Maxwell's Coefficients for Two-Conductor Circuit. With two conductors, a and b, [6] becomes

$$\begin{aligned} V_a &= P_{aa}Q_a + P_{ab}Q_b \\ V_b &= P_{ab}Q_a + P_{bb}Q_b \end{aligned} \qquad [10]$$

Solving [10] for Q_a and Q_b,

$$\begin{aligned} Q_a &= \frac{P_{bb}V_a - P_{ab}V_b}{P_{aa}P_{bb} - P_{ab}^2} \\ Q_b &= \frac{-P_{ab}V_a + P_{aa}V_b}{P_{aa}P_{bb} - P_{ab}^2} \end{aligned} \qquad [11]$$

With two conductors, a and b, [8] becomes

$$\begin{aligned} Q_a &= C_{aa}V_a - C_{ab}V_b \\ Q_b &= -C_{ba}V_a + C_{bb}V_b \end{aligned} \qquad [12]$$

Equating the coefficients of V_a and V_b in the corresponding equations of [11] and [12],

$$\begin{aligned} C_{aa} &= \frac{P_{bb}}{P_{aa}P_{bb} - P_{ab}^2} \text{ statfarads per centimeter} \\ C_{bb} &= \frac{P_{aa}}{P_{aa}P_{bb} - P_{ab}^2} \text{ statfarads per centimeter} \\ C_{ab} = C_{ba} &= \frac{P_{ab}}{P_{aa}P_{bb} - P_{ab}^2} \text{ statfarads per centimeter} \end{aligned} \qquad [13]$$

where the potential coefficients P_{aa}, P_{bb}, and P_{ab} are defined in [7].

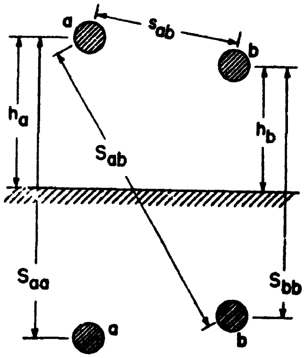
Expressed in farads per mile,

$$\begin{aligned} C_{aa} &= \frac{0.03883}{\Delta}\left(\log_{10}\frac{S_{bb}}{r_b}\right) \times 10^{-6} \\ C_{bb} &= \frac{0.03883}{\Delta}\left(\log_{10}\frac{S_{aa}}{r_a}\right) \times 10^{-6} \\ C_{ab} = C_{ba} &= \frac{0.03883}{\Delta}\left(\log_{10}\frac{S_{ab}}{s_{ab}}\right) \times 10^{-6} \end{aligned} \qquad [14]$$

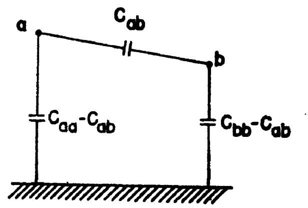
where

$$\Delta = \left(\log_{10}\frac{S_{aa}}{r_a}\right)\left(\log_{10}\frac{S_{bb}}{r_b}\right) - \left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)^2$$

The equivalent circuit for the capacitances associated with two conductors is given in Fig. 2(*b*). This circuit satisfies the conditions that (1) the total capacitance to ground of conductor a with b grounded is C_{aa}, (2) the total capacitance of b with a grounded is C_{bb}, and (3) the



(*a*)



(*b*)

FIG. 2. (*a*) Two conductors a and b parallel to ground. (*b*) Equivalent capacitance circuit for (*a*) in terms of Maxwell's coefficients defined in [14].

direct capacitance between a and b is C_{ab}. From Fig. 2(*b*), the direct capacitance to ground of each of the two conductors, their total capacitance to ground in parallel, and the total capacitance between the conductors can be obtained.

The direct capacitances to ground of conductors a and b from Fig. 2(*b*) are

$$\begin{aligned} C_{a0} &= C_{aa} - C_{ab} \\ C_{b0} &= C_{bb} - C_{ab} \end{aligned} \tag{15}$$

The total capacitance to ground C_g of the two conductors in parallel, obtained by applying equal voltages to ground at a and b, is the sum of their direct capacitances to ground.

$$C_g = C_{a0} + C_{b0} = C_{aa} + C_{bb} - 2C_{ab} \tag{16}$$

The *capacitance between the conductors*, when they form an unsymmetrical single-phase circuit, will depend upon whether the midpoint of the applied single-phase voltage is grounded or isolated.

The total capacitance C between conductors, obtained by applying a single-phase ungrounded voltage between a and b in Fig. 2(*b*), is C_{ab}, the direct capacitance between conductors a and b paralleled by the capacitance between a and b by way of the ground, which is

$$C = C_{ab} + \frac{(C_{aa} - C_{ab})(C_{bb} - C_{ab})}{C_{aa} + C_{bb} - 2C_{ab}} = \frac{C_{aa}C_{bb} - C_{ab}^2}{C_{aa} + C_{bb} - 2C_{ab}} \tag{17}$$

The total capacitance between conductors with the midpoint of the applied voltages grounded can also be obtained from Fig. 2(*b*); the direct capacitance C_{ab} between a and b can be replaced by $2C_{ab}$ from a to neutral in series with $2C_{ab}$ from b to neutral. The total capacitance between a and b is the parallel value of $(C_{aa} - C_{ab})$ and $2C_{ab}$ in series with the parallel value of $2C_{ab}$ and $(C_{bb} - C_{ab})$, giving

$$C = \frac{(C_{aa} + C_{ab})(C_{bb} + C_{ab})}{C_{aa} + C_{bb} + 2C_{ab}} \qquad [18]$$

The total capacitance between conductors given by [17] and [18] are calculated under different assumptions. In [17] there is no ground on the system other than that resulting from the capacitance to ground; in [18] the applied voltages are grounded. In [17] the phase charges are equal and opposite; in [18] the phase voltages are equal and opposite.

If the conductors are identical and at equal heights above ground, $C_{aa} = C_{bb}$, and the total capacitance C between conductors by either [17] or [18] is

$$C = \tfrac{1}{2}(C_{aa} + C_{ab}) = \frac{0.03883 \times 10^{-6}}{2 \log_{10}\left(\dfrac{S_{aa}}{r}\right)\left(\dfrac{s_{ab}}{S_{ab}}\right)} \text{ farads per mile} \qquad [19]$$

Single-Phase Two-Wire Circuit, One Conductor Grounded. Let the conductors be a and b, with b grounded at frequent intervals. The capacitance to ground of conductor a is C_{aa}, given by [14].

Single-Phase Two-Wire Circuit, without Ground Wires. Figure 2(*b*) is an equivalent *two-conductor circuit*, suitable for use where calculations are made with phase quantities rather than by means of their positive- and zero-sequence symmetrical components defined in Chapter IX. However, when symmetrical components are to be used, the sequence capacitances can be determined from Fig. 2(*b*) or equations [12], in terms of Maxwell's coefficients, defined in [14].

Positive- and Zero-Sequence Capacitances of Two-Vector Circuits. Following the definitions and notation of Chapter XI, the positive-sequence capacitances of phases a and b will be indicated by C_{a1} and C_{b1} and defined as the ratios of the phase charges Q_a and Q_b to the phase voltages V_a and V_b, respectively, with positive-sequence voltages only applied to the system. Similarly, the zero-sequence capacitances of phases a and b will be indicated by C_{a0} and C_{b0} and defined as the ratios of the phase charges to the corresponding phase voltages with zero-sequence voltages only applied to the system.

With positive-sequence voltages only applied to the two-vector circuit, $V_b = -V_a$. From [12], by definition,

$$\begin{aligned} C_{a1} &= \frac{Q_a}{V_a} = C_{aa} + C_{ab} \\ C_{b1} &= \frac{Q_b}{V_b} = C_{bb} + C_{ab} \end{aligned} \qquad [20]$$

C_{a0} and C_{b0} are the direct capacitances to ground of conductors a and b given by [15].

Since capacitive admittance with no leakance is directly proportional to capacitance, [53] of Chapter IX can be used to determine sequence capacitances from the positive- and zero-sequence capacitances of the two phases. Substituting [15] and [20] in [53] of Chapter IX, in which the Y's have been replaced by C's with the same subscripts,

$$C_{11} = \tfrac{1}{2}(C_{a1} + C_{b1}) = \tfrac{1}{2}(C_{aa} + C_{bb} + 2C_{ab}) \qquad [21]$$

$$C_{00} = \tfrac{1}{2}(C_{a0} + C_{b0}) = \tfrac{1}{2}(C_{aa} + C_{bb} - 2C_{ab}) \qquad [22]$$

$$C_{10} = C_{01} = \tfrac{1}{2}(C_{a1} - C_{b1}) = \tfrac{1}{2}(C_{a0} - C_{b0}) = \tfrac{1}{2}(C_{aa} - C_{bb}) \qquad [23]$$

where C_{aa}, C_{bb}, and C_{ab} are given in farads per mile by [14] in terms of circuit dimensions.

For identical conductors at equal heights above ground, $C_{aa} = C_{bb}$, the mutual capacitance between the sequence networks disappears, and

$$C_{11} = C_{aa} + C_{ab} = \frac{0.03883}{\log_{10}\left(\dfrac{S_{aa}}{r}\right)\left(\dfrac{s_{ab}}{S_{ab}}\right)} \times 10^{-6} \text{ farads per mile} \qquad [24]$$

$$C_{00} = C_{aa} - C_{ab} = \frac{0.03883}{\log_{10}\left(\dfrac{S_{aa}}{r}\right)\left(\dfrac{S_{ab}}{s_{ab}}\right)} \times 10^{-6} \text{ farads per mile} \qquad [25]$$

Equation [24] gives the capacitance to neutral of a symmetrical single-phase circuit with the presence of the earth considered. For overhead transmission lines, S_{aa} and S_{ab} are of the same order of magnitude. (See Fig. 2(a).) If the earth is assumed infinitely distant, which is equivalent to neglecting its presence, S_{aa} and S_{ab} become equal, and the capacitance to neutral is

$$C_{11} = \frac{0.03883}{\log_{10} \dfrac{s_{ab}}{r}} \times 10^{-6} \text{ farads per mile} \qquad [26]$$

The capacitance C between conductors is one-half that given by [26], which checks [19] with $S_{aa} = S_{ab}$. Equation [26], which neglects the presence of the earth, is generally used for calculating capacitance to neutral in a single-phase circuit; and capacitance between conductors is one-half of this amount. It is shown in Example 2 that the difference between [24] and [26] is negligible for conventional overhead transmission circuits.

Two Wires Alone in Space. Of interest is the capacitance C between two parallel round wires alone in space, taking into account the *non-uniform distribution*[4] *of the charge on each wire:*

$$C = \frac{8.467 \times 10^{-9}}{\cosh^{-1} \dfrac{s_{ab}}{2r}} \text{ farads per 1000 feet}$$

$$\cosh^{-1} x = \log_e(x + \sqrt{x^2 - 1})^* = 2.3025 \log_{10} (x + \sqrt{x^2 - 1})$$

When s_{ab} is large relative to r ($s/2r = 10$ or more), the capacitance between wires becomes, with negligible error,

$$C = \frac{3.677 \times 10^{-9}}{\log_{10} \dfrac{s_{ab}}{r}} \text{ farads per 1000 feet}$$

For overhead transmission lines, $C_{11} = 2C$ in farads per mile is correctly given by [26], with the presence of the earth neglected.

Example 2. Given two conductors each 40 feet above ground with 0.5 inch diameters, spaced 10 feet apart. From [14],

$$\begin{aligned} \Delta &= \left(\log_{10} \frac{80 \times 12}{0.25}\right)^2 - \left(\log_{10} \frac{\sqrt{80^2 + 10^2}}{10}\right)^2 \\ &= (3.5843)^2 - (0.9064)^2 = 12.026 \end{aligned}$$

$$C_{aa} = C_{bb} = \frac{0.03883}{12.026} \times 3.5843 \times 10^{-6} = 0.01157 \times 10^{-6} \text{ farad per mile}$$

$$C_{ab} = \frac{0.03883}{12.026} \times 0.9064 \times 10^{-6} = 0.00293 \times 10^{-6} \text{ farad per mile}$$

The total capacitance to ground of either conductor with the other conductor grounded is

$$C_{aa} = C_{bb} = 0.01157 \times 10^{-6} \text{ farad per mile}$$

The direct capacitance to ground of either conductor is

$$C_{a0} = C_{b0} = C_{aa} - C_{ab} = 0.00864 \times 10^{-6} \text{ farad per mile}$$

The capacitance to ground of the two conductors in parallel is

$$C_{a0} + C_{b0} = 0.0173 \times 10^{-6} \text{ farad per mile}$$

* Equation 680, *A Short Table of Integrals*, by B. O. Peirce, Second Revised Edition, Ginn and Co.

The zero-sequence self-capacitance from [22] is

$$C_{00} = \tfrac{1}{2}(C_{a0} + C_{b0}) = 0.00864 \times 10^{-6} \text{ farad per mile}$$

The positive-sequence self-capacitance of the single-phase circuit with the presence of the earth considered from [24] is

$$C_{11} = (C_{aa} + C_{ab}) = 0.01450 \times 10^{-6} \text{ farad per mile}$$

From [26], with the presence of the earth neglected, it is

$$C_{11} = \frac{0.03883}{\log_{10} \dfrac{10 \times 12}{0.25}} \times 10^{-6} = 0.01448 \times 10^{-6} \text{ farad per mile}$$

Comparing the results of Example 2 with the capacitance to ground of the single conductor given in Example 1, which has the same diameter and is located at the same height above ground, it can be seen that (1) the total capacitance to ground of a conductor is *increased* by the presence of a second conductor at ground potential, but (2) the direct capacitance to ground of a conductor is *reduced* by the presence of a second conductor at the same potential. The difference in positive-sequence capacitance calculated from [24] and [26] for a symmetrical single-phase circuit of the given dimensions is negligible.

Example 3. The effect of unequal conductor heights above ground upon the sequence capacitances will be determined for a single-phase circuit with 0.5 inch diameters. Conductors a and b are 40 and 50 feet, respectively, above ground and 10 feet apart. From [14],

$$\Delta = \left(\log_{10}\frac{80 \times 12}{0.25}\right)\left(\log_{10}\frac{100 \times 12}{0.25}\right) - \left(\log_{10}\frac{90}{10}\right)^2$$
$$= (3.5843)(3.6813) - (0.9542)^2 = 12.28$$

$$C_{aa} = \frac{0.03883}{12.28} \times 3.6813 \times 10^{-6} = 0.01164 \times 10^{-6} \text{ farad per mile}$$

$$C_{bb} = \frac{0.03883}{12.28} \times 3.5843 \times 10^{-6} = 0.01133 \times 10^{-6} \text{ farad per mile}$$

$$C_{ab} = \frac{0.03883}{12.28} \times 0.9542 \times 10^{-6} = 0.00302 \times 10^{-6} \text{ farad per mile}$$

From [21]–[23], the sequence capacitances in farads per mile are

$$C_{11} = \tfrac{1}{2}(C_{aa} + C_{bb} + 2C_{ab}) = 0.01450 \times 10^{-6}$$
$$C_{00} = \tfrac{1}{2}(C_{aa} + C_{bb} - 2C_{ab}) = 0.00847 \times 10^{-6}$$
$$C_{01} = C_{10} = \tfrac{1}{2}(C_{aa} - C_{bb}) = 0.00015 \times 10^{-6}$$

Comparing the sequence capacitances for the unsymmetrical circuit with those for the symmetrical circuit of Example 2 with the same conductor diameter and the same distance between conductors, there is no appreciable difference in C_{11} the positive-sequence self-capacitance; the zero-sequence self-capacitance C_{00} is decreased slightly because of increased average conductor height above ground. The mutual capacitance between the positive- and zero-sequence networks is approximately one per cent of the positive-sequence self-capacitance, and relatively too small to be considered.

Maxwell's Coefficients for Three-Conductor Circuit. With three conductors a, b, and c, [6] becomes

$$\begin{aligned} V_a &= P_{aa}Q_a + P_{ab}Q_b + P_{ac}Q_c \\ V_b &= P_{ab}Q_a + P_{bb}Q_b + P_{bc}Q_c \\ V_c &= P_{ac}Q_a + P_{bc}Q_b + P_{cc}Q_c \end{aligned} \tag{27}$$

Solving [27] by determinants (see Appendix A),

$$\begin{aligned} Q_a &= \frac{1}{\Delta}[V_a(P_{bb}P_{cc}-P_{bc}^2) - V_b(P_{ab}P_{cc}-P_{ac}P_{bc}) + V_c(P_{ab}P_{bc}-P_{ac}P_{bb})] \\ Q_b &= \frac{1}{\Delta}[-V_a(P_{ab}P_{cc}-P_{ac}P_{bc}) \\ &\qquad + V_b(P_{aa}P_{cc}-P_{ac}^2) - V_c(P_{aa}P_{bc}-P_{ab}P_{ac})] \\ Q_c &= \frac{1}{\Delta}[V_a(P_{ab}P_{bc}-P_{ac}P_{bb}) \\ &\qquad - V_b(P_{aa}P_{bc}-P_{ab}P_{ac}) + V_c(P_{aa}P_{bb}-P_{ab})] \end{aligned} \tag{28}$$

where

$$\Delta = P_{aa}(P_{bb}P_{cc}-P_{bc}^2) - P_{ab}(P_{ab}P_{cc}-P_{ac}P_{bc}) + P_{ac}(P_{ab}P_{bc}-P_{ac}P_{bb})$$

For three conductors, [8] becomes

$$\begin{aligned} Q_a &= C_{aa}V_a - C_{ab}V_b - C_{ac}V_c \\ Q_b &= -C_{ba}V_a + C_{bb}V_b - C_{bc}V_c \\ Q_c &= -C_{ca}V_a - C_{cb}V_b + C_{cc}V_c \end{aligned} \tag{29}$$

Equating the coefficients of V_a, V_b, and V_c in the corresponding equations of [28] and [29], the capacitance to ground of each of the three conductors with the other two grounded and the direct capacitances between the conductors in statfarads per centimeter are given by the following equations:

$$\begin{aligned} C_{aa} &= \frac{1}{\Delta}(P_{bb}P_{cc} - P_{bc}^2) \\ C_{bb} &= \frac{1}{\Delta}(P_{aa}P_{cc} - P_{ac}^2) \\ C_{cc} &= \frac{1}{\Delta}(P_{aa}P_{bb} - P_{ab}^2) \end{aligned} \tag{30}$$

$$C_{ab} = C_{ba} = \frac{1}{\Delta}(P_{ab}P_{cc} - P_{ac}P_{bc})$$

$$C_{ac} = C_{ca} = \frac{1}{\Delta}(P_{ac}P_{bb} - P_{ab}P_{bc})$$

$$C_{bc} = C_{cb} = \frac{1}{\Delta}(P_{bc}P_{aa} - P_{ab}P_{ac})$$

where

$$\Delta = P_{aa}(P_{bb}P_{cc} - P_{bc}^2) - P_{ab}(P_{ab}P_{cc} - P_{ac}P_{bc}) + P_{ac}(P_{ab}P_{bc} - P_{bb}P_{ac})$$

and the potential coefficients are given by [7].

Expressed in farads per mile,

$$C_{aa} = \frac{0.03883 \times 10^{-6}}{\Delta}\left[\left(\log_{10}\frac{S_{bb}}{r_b}\right)\left(\log_{10}\frac{S_{cc}}{r_c}\right) - \left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)^2\right]$$

$$C_{bb} = \frac{0.03883 \times 10^{-6}}{\Delta}\left[\left(\log_{10}\frac{S_{aa}}{r_a}\right)\left(\log_{10}\frac{S_{cc}}{r_c}\right) - \left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)^2\right]$$

$$C_{cc} = \frac{0.03883 \times 10^{-6}}{\Delta}\left[\left(\log_{10}\frac{S_{aa}}{r_a}\right)\left(\log_{10}\frac{S_{bb}}{r_b}\right) - \left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)^2\right]$$

$$C_{ab} = C_{ba} = \frac{0.03883 \times 10^{-6}}{\Delta} \times \left[\left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)\left(\log_{10}\frac{S_{cc}}{r_c}\right) - \left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)\left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)\right] \quad [31]$$

$$C_{ac} = C_{ca} = \frac{0.03883 \times 10^{-6}}{\Delta} \times \left[\left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)\left(\log_{10}\frac{S_{bb}}{r_b}\right) - \left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)\left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)\right]$$

$$C_{bc} = C_{cb} = \frac{0.03883 \times 10^{-6}}{\Delta} \times \left[\left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)\left(\log_{10}\frac{S_{aa}}{r_a}\right) - \left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)\left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)\right]$$

where

$$\Delta = \left\{\log_{10}\frac{S_{aa}}{r_a}\left[\left(\log_{10}\frac{S_{bb}}{r_b}\right)\left(\log_{10}\frac{S_{cc}}{r_c}\right) - \left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)^2\right] + \right.$$

$$-\log_{10}\frac{S_{ab}}{s_{ab}}\left[\left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)\left(\log_{10}\frac{S_{cc}}{r_c}\right)-\left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)\left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)\right]$$
$$-\log_{10}\frac{S_{ac}}{s_{ac}}\left[\left(\log_{10}\frac{S_{ac}}{s_{ac}}\right)\left(\log_{10}\frac{S_{bb}}{r_b}\right)-\left(\log_{10}\frac{S_{ab}}{s_{ab}}\right)\left(\log_{10}\frac{S_{bc}}{s_{bc}}\right)\right]\Bigg\}$$

Figure 3(a) shows the spacings between the conductors and between conductors and images indicated by s and S, respectively, with two subscripts. The equivalent three-conductor circuit for the capaci-

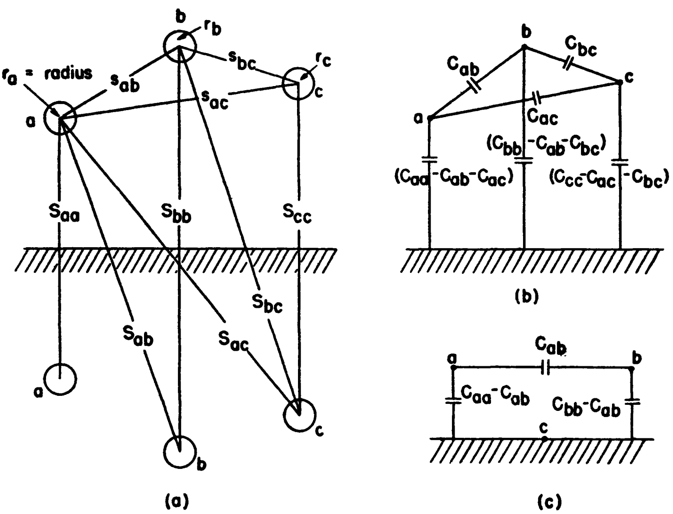


FIG. 3. (a) Three conductors parallel to ground. (b) Equivalent capacitance circuit for (a) in terms of Maxwell's coefficients defined in [31]. (c) Equivalent circuit for (a) with conductor c shorted to ground in terms of Maxwell's coefficients defined in [31].

tances associated with three conductors is given in Fig. 3(b). It satisfies the conditions for the total capacitance to ground of each of the conductors with the other two grounded, and also for the direct capacitances between conductors. From this equivalent circuit or from [29], the sequence capacitances of a single-phase circuit with one ground wire, a three-wire single-phase or two-phase circuit with grounded neutral conductor and a three-phase circuit without ground wires will be determined.

Single-Phase Circuit with One Ground Wire. Let the phases be a and b and the ground wire c. If conductor c is grounded, $V_c = 0$ in [29], and the first two equations reduce to those of [12]. In Fig. 3(b), with conductor c at ground potential, its direct capacitance to ground

$C_{c0} = C_{cc} - C_{ac} - C_{bc}$ is shorted, and the circuit becomes that shown in Fig. 3(*c*), which is similar to Fig. 2(*b*) with the *important distinction* that C_{aa}, C_{bb}, and $C_{ab} = C_{ba}$ are defined by [31] and *not* by [14]. Equations [21]–[23], already developed for the positive- and zero-sequence self-capacitances and the mutual capacitance between the positive- and zero-sequence networks for the two-conductor single-phase circuit without ground wires in terms of C_{aa}, C_{bb}, and C_{ab}, can be used for the single-phase circuit with one ground wire if C_{aa}, C_{bb}, and C_{ab} are understood to be defined by [31] instead of by [14] and the ground wire is indicated by *c*.

Three-Wire Single-Phase or Two-Phase Circuit with Grounded Neutral Conductor. When the components of current and voltage are positive- and zero-sequence symmetrical components, the positive- and zero-sequence self-capacitances and the mutual capacitance between the positive- and zero-sequence networks are the same for the three-wire single-phase or two-phase circuit with grounded neutral conductor as for the two-wire circuit with one ground wire.

Three-Phase Circuit without Ground Wires. Figure 3(*b*), which gives the equivalent circuit to replace the distributed capacitances of the transmission line, is a *three-conductor equivalent circuit*, suitable for studies in which phase quantities rather than symmetrical components are used; therefore, after Maxwell's coefficients have been calculated from [31] and Fig. 3(*b*) constructed, additional work is necessary to determine the sequence capacitances of the three-phase circuit.

In systems where the currents and voltages are sinusoidal, the electrostatic charges are also sinusoidal and the phase charges can be resolved into their positive-, negative-, and zero-sequence components of charge just as phase voltages and currents are resolved into their symmetrical components of voltage and current, respectively.

Current is defined as time rate of change of charge. If the charge Q is sinusoidal,—

$$Q = |Q| \sin \omega t$$

where $|Q|$ denotes crest value of the sinusoidal charge. The current I is sinusoidal if Q is sinusoidal.

$$I = \frac{dQ}{dt} = |Q|\omega \cos \omega t$$

or vectorially

$$I = j\omega Q = j2\pi f Q$$

where the subscripts of I and Q are identical, and I and Q may be rms values.

Equations [64], Chapter VIII, express the symmetrical components of phase currents flowing into an unsymmetrical three-phase shunt circuit in terms of the phase voltages to ground at the circuit terminals and the sequence admittances of the circuit. These equations are

$$\begin{aligned} I_{a1} &= Y_{11}V_{a1} + Y_{12}V_{a2} + Y_{10}V_{a0} \\ I_{a2} &= Y_{21}V_{a1} + Y_{22}V_{a2} + Y_{20}V_{a0} \\ I_{a0} &= Y_{01}V_{a1} + Y_{02}V_{a2} + Y_{00}V_{a0} \end{aligned} \qquad [32]$$

where the Y's with two subscripts represent the sequence admittances of the circuit, the first subscript referring to the sequence of the current given by the equation and the second to the sequence of the voltage associated with the admittance.

Since $I = j2\pi fQ$ and $Y = j2\pi fC$, dividing both sides of [32] by $j2\pi f$ with Q and C having the same subscripts as I and Y, respectively, the following equations are obtained:

$$\begin{aligned} Q_{a1} &= C_{11}V_{a1} + C_{12}V_{a2} + C_{10}V_{a0} \\ Q_{a2} &= C_{21}V_{a1} + C_{22}V_{a2} + C_{20}V_{a0} \\ Q_{a0} &= C_{01}V_{a1} + C_{02}V_{a2} + C_{00}V_{a0} \end{aligned} \qquad [33]$$

The C's with two subscripts in [33] represent the sequence capacitances of the circuit, the first subscript referring to the sequence of the charge given by the equation and the second to the sequence of the voltage associated with the capacitance. C_{11}, C_{22}, and C_{00} represent the positive-, negative-, and zero-sequence self-capacitances, respectively; the C's with two unlike subscripts represent mutual capacitances between the sequence networks indicated by the subscripts.

If V_a, V_b, and V_c in [29] are replaced by their symmetrical components of voltage and Q_a, Q_b, and Q_c then resolved into their symmetrical components of charge, the sequence self- and mutual capacitances can be determined by equating the coefficients of V_{a1}, V_{a2}, and V_{a0} in the resultant equations to the corresponding coefficients in [33], giving

$$\begin{aligned} C_{11} &= C_{22} = \tfrac{1}{3}[C_{aa} + C_{bb} + C_{cc} + C_{ab} + C_{ac} + C_{bc}] \\ C_{00} &= \tfrac{1}{3}[C_{aa} + C_{bb} + C_{cc} - 2(C_{ab} + C_{ac} + C_{bc})] \\ C_{12} &= \tfrac{1}{3}[C_{aa} + a^2C_{bb} + aC_{cc} - 2(aC_{ab} + a^2C_{ac} + C_{bc})] \\ C_{21} &= \tfrac{1}{3}[C_{aa} + aC_{bb} + a^2C_{cc} - 2(a^2C_{ab} + aC_{ac} + C_{bc})] \\ C_{10} &= C_{02} = \tfrac{1}{3}[C_{aa} + aC_{bb} + a^2C_{cc} + a^2C_{ab} + aC_{ac} + C_{bc}] \\ C_{20} &= C_{01} = \tfrac{1}{3}[C_{aa} + a^2C_{bb} + aC_{cc} + aC_{ab} + a^2C_{ac} + C_{bc}] \end{aligned} \qquad [34]$$

where Maxwell's coefficients C_{aa}, C_{bb}, C_{cc}, C_{ab}, C_{ac}, and C_{bc} are defined in [31].

The sequence self-capacitances and the mutual capacitances associated with the zero-sequence network can also be obtained from Fig. 3(*b*). The zero-sequence self-capacitance C_{00} and the mutual capacitances C_{10} and C_{20} (equal to C_{02} and C_{01}, respectively) can be obtained by substituting the direct capacitances to ground of the three phases in [72] of Chapter VIII, in which *Y*'s have been replaced by *C*'s. The positive- or negative-sequence capacitances are capacitances to neutral which are determined from the capacitances between conductors. In the equivalent circuit of Fig. 3(*b*), the point representing the ground is also a neutral point since it is common to the three phases. In this three-conductor equivalent circuit, the capacitances to ground represent part of the positive- and negative-sequence capacitances, the capacitances of the equivalent Δ between phases the other part. This does not mean that part of the positive- and negative-sequence capacitances are capacitances to ground. In fact, the ground can be neglected with but slight error in calculating them. But in an equivalent circuit, part of the positive- and negative-sequence capacitances can be represented by the same three-phase shunt circuit that represents the zero-sequence capacitances. In the usual equivalent circuit representing positive- or negative-sequence capacitances only, the capacitances are shown to neutral and there is no point representing ground.

The positive- or negative-sequence self-capacitances of the Δ, which are the average capacitances in the three phases of the Δ, are $\frac{1}{3}(C_{ab} + C_{ac} + C_{bc})$. Sequence self-capacitances are by definition equal in the three phases and form a symmetrical circuit. The symmetrical self-impedance Δ can be replaced by an equivalent symmetrical Y. The admittances of a symmetrical Y are three times the admittances of the equivalent symmetrical Δ; therefore the positive- and negative-sequence self-capacitances of the equivalent Y are $(C_{ab} + C_{ac} + C_{bc})$. The positive- or negative-sequence self-capacitance of the shunt circuit to ground is $\frac{1}{3}(C_{aa} + C_{bb} + C_{cc} - 2C_{ab} - 2C_{ac} - 2C_{bc})$. Adding the positive- or negative-sequence self-capacitance of the two circuits which together form the equivalent circuit of Fig. 3(*b*),

$$C_{11} = C_{22} = \tfrac{1}{3}(C_{aa} + C_{bb} + C_{cc} + C_{ab} + C_{ac} + C_{bc})$$

This checks the equations for $C_{11} = C_{22}$ given by [34].

Three-Phase Equivalent Capacitance Circuit from Calculated Positive- and Zero-Sequence Self-Capacitances. When the mutual capacitances between the sequence networks are neglected and positive- and zero-sequence self-capacitances have been calculated, an equivalent symmetrical three-phase capacitance circuit similar to Fig. 3(*b*) can be constructed, as shown in Figs. 4(*a*) and (*b*). In these equivalent three-phase circuits, the direct capacitances to ground in the three phases are C_{00}. Since C_{00} forms part of the positive-sequence capacitances, the remainder is $(C_{11} - C_{00})$, which is the capacitance to neutral of the symmetrical Y between phases shown in Fig. 4(*a*). If an equivalent Δ is used, the capacitance between phases is $\frac{1}{3}(C_{11} - C_{00})$, as shown in Fig. 4(*b*). These equivalent three-phase circuits can be used on the a-c network analyzer or the transient analyzer[5] in cases where it is desirable to represent the three phases of a system rather than the sequence networks.

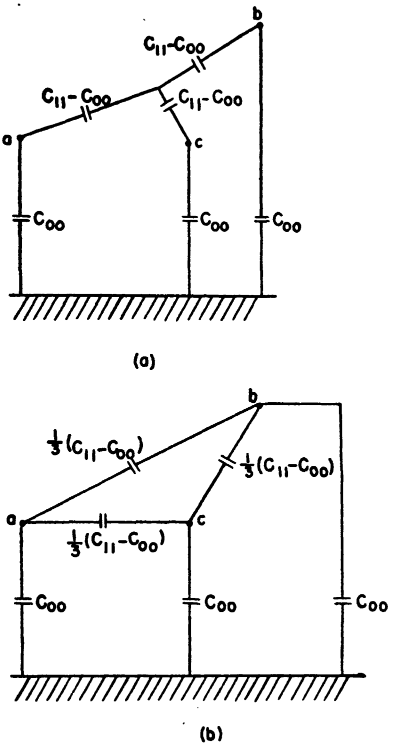


FIG. 4. Equivalent three-phase capacitance circuits in terms of positive- and zero-sequence self-capacitances C_{11} and C_{00}, with mutual capacitances between the sequence networks neglected.

The *use of Maxwell's coefficients* has been illustrated for two-conductor and three-conductor circuits. Maxwell's coefficients can be used to determine the equivalent capacitance circuits for transmission circuits of any number of conductors and ground wires, but the work involved becomes increasingly laborious as the number of wires is increased. For example, in a three-phase circuit with two ground wires there are five wires and therefore five equations in [6] and [8]. The number of potential coefficients is fifteen, and the formulas for Maxwell's coefficients in terms of the potential coefficients are lengthy. With two of the five wires grounded, only six of Maxwell's coefficients are required, but each of them is a function of all fifteen potential coefficients. Moreover, if the sequence capacitances are required, additional work is necessary to determine them from Maxwell's coefficients.

The sequence capacitances of a three-phase circuit without ground wires are calculated in Problem 1 from Maxwell's coefficients, using the equations of [34]. Before discussing these sequence capacitances, the potential coefficient method will be developed, so that Problem 1 may be solved by both methods.

Sequence Capacitances Determined from Potential Coefficients

Relations between Capacitances and Potential Coefficients. Capacitance is defined as the ratio of charge to potential: $C = Q/V$. From [6], potential V is a direct function of the potential coefficients. As V occurs in the denominator of the capacitance equation, capacitances and potential coefficients are inversely related.

Capacitances of transmission lines are normally expressed in farads per mile, the total capacitance of any sequence in farads varying *directly* with the length of line. The quantity which is dimensionally the reciprocal of capacitance is elastance.[6] The unit of elastance is the daraf (farad spelled backward). Potential coefficients are dimensionally the reciprocals of capacitances. With the capacitances of a transmission line expressed in farads per mile, the potential coefficients will here be expressed in daraf-miles, the potential coefficient of any sequence for the line in darafs varying *inversely* as the length of line. The symbol P with numerical subscripts will be used to indicate the sequence potential coefficients.

Given ℓ miles of line with capacitance of a specified sequence in farads per mile represented by C with appropriate subscripts (the zero-sequence self-capacitance is C_{00}), then the admittance Y corresponding to C is

$$Y = jb = j2\pi fC\ell \text{ mhos} \qquad [35]$$

where the subscripts of Y and b (the capacitive susceptance) are the same as those of C, and C is some function of the potential coefficients of the circuit.

The *capacitive impedance* of a specified sequence of ℓ miles of line will be indicated by Z with appropriate subscripts (the zero-sequence capacitive self-impedance is Z_{00}); then

$$Z = \frac{P}{j2\pi f\ell} = -j\frac{P}{2\pi f\ell} = -jx \text{ ohms} \qquad [36]$$

where Z, x (the capacitive reactance), and P in [36] have the same subscripts as Y, b, and C in [35] when they refer to the same sequence or sequences, but Z is not the reciprocal of Y, x is not the reciprocal of b, and P is not the reciprocal of C unless the circuit is a symmetrical one.

It is proposed to determine the sequence potential coefficients which correspond to sequence capacitive reactances, and from them, if required, sequence capacitances or capacitive admittances.

Three-Phase Circuit. If a, b, and c indicate the three conductors, equations [6] are given by [27] for the case of no ground wires. With ground wires, the number of equations in [6] is equal to three plus the number of ground wires but the voltages on the ground wires are zero. Eliminating the charges on the ground wires, the equations are reduced to three. If the charges Q_a, Q_b, and Q_c in the resultant equations are replaced by their symmetrical components of charge, and the phase voltages then resolved into their symmetrical components, the following equations in absolute units are obtained:

$$\begin{aligned} V_{a1} &= P_{11}Q_{a1} + P_{12}Q_{a2} + P_{10}Q_{a0} \\ V_{a2} &= P_{21}Q_{a1} + P_{22}Q_{a2} + P_{20}Q_{a0} \\ V_{a0} &= P_{01}Q_{a1} + P_{02}Q_{a2} + P_{00}Q_{a0} \end{aligned} \qquad [37]$$

where, for the case of no ground wires,

$$\begin{aligned} P_{11} = P_{22} &= \tfrac{1}{3}[P_{aa} + P_{bb} + P_{cc} - (P_{ab} + P_{ac} + P_{bc})] \\ P_{00} &= \tfrac{1}{3}[P_{aa} + P_{bb} + P_{cc} + 2(P_{ab} + P_{ac} + P_{bc})] \\ P_{12} &= \tfrac{1}{3}[P_{aa} + a^2P_{bb} + aP_{cc} + 2(aP_{ab} + a^2P_{ac} + P_{bc})] \\ P_{21} &= \tfrac{1}{3}[P_{aa} + aP_{bb} + a^2P_{cc} + 2(a^2P_{ab} + aP_{ac} + P_{bc})] \\ P_{10} = P_{02} &= \tfrac{1}{3}[P_{aa} + aP_{bb} + a^2P_{cc} - (a^2P_{ab} + aP_{ac} + P_{bc})] \\ P_{20} = P_{01} &= \tfrac{1}{3}[P_{aa} + a^2P_{bb} + aP_{cc} - (aP_{ab} + a^2P_{ac} + P_{bc})] \end{aligned} \qquad [38]$$

and the conductor potential coefficients P_{aa}, P_{ab}, etc., are defined in [7].

The P's in [37] with two numerical subscripts represent the sequence potential coefficients of the circuit, the first subscript referring to the sequence of the voltage given by the equation and the second to the sequence of the charge associated with the coefficient. P_{11}, P_{22}, and P_{00} represent the positive-, negative-, and zero-sequence potential coefficients, respectively; the P's with two unlike numerical subscripts represent mutual potential coefficients between the sequence networks indicated by the subscripts. With the subscripts of Z, x, and P identical in [36], the sequence capacitive impedances or reactances can be obtained directly from the corresponding sequence potential coefficients (given by [38] for no ground wires) in terms of conductor potential coefficients. Equations [38] are in absolute units, the P's with numerical subscripts being dimensionally reciprocals of capacitances in statfarads per centimeter. If $\log_{10}$ instead of $2 \log_e$ is used in determining the potential coefficients defined in [7] which appear in the original equations of [6], the multiplier to convert to the unit 1/(farads per mile) or daraf-miles is

$$\frac{2 \times 2.3026}{0.17881 \times 10^{-6}} = 25.76 \times 10^{6} = \frac{10^{6}}{0.03883}$$

Impedances versus Admittances in Calculations. In a three-phase unsymmetrical capacitive shunt circuit, just as in a three-phase unsymmetrical impedance shunt circuit, the symmetrical components of phase voltages at the circuit terminals are expressed in terms of the symmetrical components of phase currents flowing into the circuit and the sequence self- and mutual impedances of the circuit by equations [11] of Chapter VIII. These equations are

$$\begin{aligned} V_{a1} &= I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10} \\ V_{a2} &= I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \\ V_{a0} &= I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00} \end{aligned} \qquad [39]$$

Since the relations between the phase voltages to ground at the terminals of the capacitive shunt circuit and the phase currents flowing into the circuit can be expressed in terms of the sequence impedances by [39] with the same degree of precision as in terms of the sequence admittances by [32], the choice between admittances and impedances becomes a matter of the most convenient set of equations for the given problem. In analytic calculations and in the development of equivalent circuits for parallel transmission circuits discussed in Chapter VI, capacitive impedances will be found more convenient. By the potential coefficient method, the sequence potential coefficients are calculated from the conductor potential coefficients, and the sequence capacitive impedances from them by [36] for use in [39] or in equivalent circuits developed from [39] or from analogous equations.

If the sequence admittances are required for use in [32], they may be calculated from the capacitive impedances using [65] of Chapter VIII; or, after the *numerical values* of the sequence potential coefficients have been determined, the sequence capacitances can be obtained from them by the following equations, determined by solving [37] for Q_{a1}, Q_{a2}, and Q_{a0} and then equating the coefficients of V_{a1}, V_{a2}, and V_{a0} in the resultant equations to the coefficients in the corresponding equations of [33].

$$\begin{aligned} C_{11} = C_{22} &= \frac{1}{\Delta}(P_{11}P_{00} - P_{10}P_{01}) \\ C_{00} &= \frac{1}{\Delta}(P_{11}^2 - P_{12}P_{21}) \\ C_{12} &= \frac{1}{\Delta}(-P_{12}P_{00} + P_{10}^2) \end{aligned} \qquad [40]$$

$$C_{21} = \frac{1}{\Delta}(-P_{21}P_{00} + P_{01}^2)$$

$$C_{10} = C_{02} = \frac{1}{\Delta}(-P_{10}P_{11} + P_{12}P_{01})$$

$$C_{01} = C_{20} = \frac{1}{\Delta}(-P_{01}P_{11} + P_{21}P_{10})$$

where

$$\Delta = P_{11}^2P_{00} - 2P_{10}P_{01}P_{11} - P_{12}P_{21}P_{00} + P_{12}P_{01}^2 + P_{21}P_{10}^2$$

If the sequence potential coefficients are expressed in daraf-miles, the sequence capacitances in [40] will be in farads per mile; if they have been calculated using $\log_{10}$ and no conversion factor, the multiplier to convert to farads per mile is $(0.1788 \times 10^{-6})/(2 \times 2.3026) = 0.03883 \times 10^{-6}$.

Summary of Procedure by Potential Coefficient Method. The procedure in determining the sequence capacitive impedances or admittances of three-phase transmission circuits with and without ground wires at constant frequency may be summarized in the following steps:

1. Write the equations of [6]. The number of these equations is the same as the total number of wires, i.e., three phase conductors plus the number of ground wires. The voltages of the ground wires in these equations are zero.
2. Reduce the number of equations in [6] to three by eliminating the charges on the ground wires.
3. Replace the charges on the phases by their symmetrical components of charge and resolve the phase voltages into their symmetrical components of voltage; then equate the coefficients of the charges in the resultant equations to the corresponding coefficients in [37] to obtain the sequence potential coefficients in terms of the conductor potential coefficients defined in [7].
4. (*a*) If *sequence capacitive impedances* are required they are obtained by substituting the sequence potential coefficients in [36].

   (*b*) If the *sequence capacitive admittances* are required, the sequence capacitances are first calculated from the sequence potential coefficients, using [40]; and then the corresponding sequence capacitive admittances are obtained by substituting the sequence capacitances in [35].

The sequence capacitances calculated by the procedure summarized above will be *exactly the same* as those obtained by calculating Maxwell's coefficients and from them the sequence capacitances. The positive-, negative-, and zero-sequence potential coefficients and capacitances are unaffected by the choice of the reference phase; the sequence mutual potential coefficients and capacitances, however, depend upon the phase selected as reference. The sequence capacitances of a three-phase circuit without ground wires will be calculated from Maxwell's coefficients and also by the potential coefficient method in the following problem, before considering three-phase circuits with ground wires.

Problem 1. Given three conductors, each 40 feet above ground with 0.5 inch diameters, spaced 10 feet apart (see Fig. 5(*a*)). Determine:

(*a*) The equivalent three-conductor capacitance circuit.

(*b*) The sequence capacitances of the three-phase circuit using Maxwell's coefficients.

(*c*) Use potential coefficient method for (*b*).

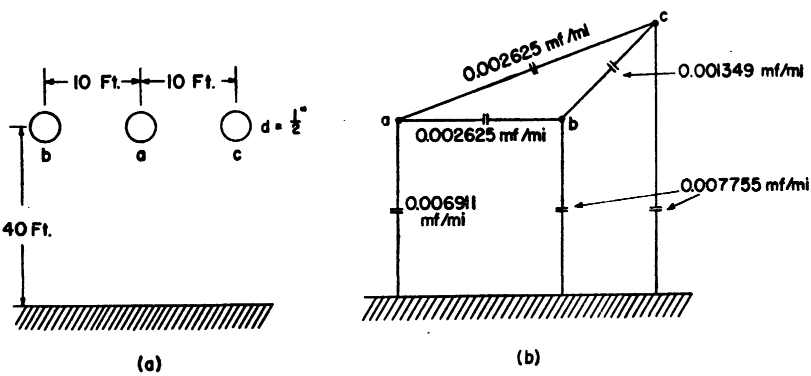


FIG. 5. (*a*) Three conductors parallel to ground. (*b*) Three-conductor capacitance circuit for (*a*).

Solution. Designating the middle conductor as *a* and the two outside conductors as *b* and *c*,

$$\log_{10}\frac{S_{aa}}{r} = \log_{10}\frac{S_{bb}}{r} = \log_{10}\frac{S_{cc}}{r} = \log_{10}\frac{80 \times 12}{0.25} = 3.5843$$

$$\log_{10}\frac{S_{ab}}{s_{ab}} = \log_{10}\frac{S_{ac}}{s_{ac}} = \log_{10}\frac{\sqrt{\overline{80}^2 + \overline{10}^2}}{10} = 0.9064$$

$$\log_{10}\frac{S_{bc}}{s_{bc}} = \log_{10}\frac{\sqrt{\overline{80}^2 + \overline{20}^2}}{20} = 0.6152$$

(*a*) Substituting these logarithms in [31], Maxwell's coefficients in farads per mile are obtained.

$$\Delta = 3.5843[(3.5843)^2 - (0.6152)^2] - 2(0.9064)^2[3.5843 - 0.6152] = 39.813$$

$$C_{aa} = \frac{0.03883 \times 10^{-6}}{\Delta}[(3.5843)^2 - (0.6152)^2] = 0.012161 \times 10^{-6} \text{ farad per mile}$$

$$C_{bb} = C_{cc} = \frac{0.03883 \times 10^{-6}}{\Delta}[(3.5843)^2 - (0.9064)^2]$$

$$= 0.011729 \times 10^{-6} \text{ farad per mile}$$

$$C_{ab} = C_{ba} = C_{ac} = C_{ca} = \frac{0.03883 \times 10^{-6}}{\Delta}[(0.9064)(3.5843) - (0.9064)(0.6152)]$$

$$= 0.002625 \times 10^{-6} \text{ farad per mile}$$

$$C_{bc} = C_{cb} = \frac{0.03883 \times 10^{-6}}{\Delta}[(0.6152)(3.5843) - (0.9064)^2]$$

$$= 0.001349 \times 10^{-6} \text{ farad per mile}$$

The equivalent three-conductor capacitance circuit is obtained by substituting Maxwell's coefficients in Fig. 3(*b*). This circuit is shown in Fig. 5(*b*), where

$$C_{aa} - C_{ab} - C_{ac} = 0.006911 \times 10^{-6} \text{ farad per mile}$$

$$C_{bb} - C_{ab} - C_{bc} = 0.007755 \times 10^{-6} \text{ farad per mile}$$

$$C_{cc} - C_{ac} - C_{bc} = 0.007755 \times 10^{-6} \text{ farad per mile}$$

(*b*) Substituting Maxwell's coefficients in [34], the following sequence capacitances of the three-phase circuit are obtained:

$$C_{11} = C_{22} = 0.014073 \times 10^{-6} \text{ farad per mile}$$

$$C_{00} = 0.007474 \times 10^{-6} \text{ farad per mile}$$

$$C_{12} = C_{21} = 0.000995 \times 10^{-6} \text{ farad per mile}$$

$$C_{10} = C_{02} = C_{20} = C_{01} = -0.000281 \times 10^{-6} \text{ farad per mile}$$

For the flat horizontal configuration assumed, and a the center conductor, the mutual capacitances are reciprocal. The zero-sequence self-capacitance C_{00} is 53% of the positive-sequence self-capacitance C_{11}. The mutual capacitance between the positive- and negative-sequence networks is about 7% of the positive-sequence self-capacitance. The mutual capacitance between the zero- and the positive- or negative-sequence networks is negative and less than 4% of the zero-sequence self-capacitance.

(*c*) Substituting the conductor potential coefficients from [7] in [38], the sequence potential coefficients in daraf-miles are

$$P_{11} = P_{22} = \frac{10^6}{0.03883}[3.5843 - \tfrac{1}{3}(2 \times 0.9064 + 0.6152)] = \frac{10^6}{0.03883}(2.775)$$

$$P_{00} = \frac{10^6}{0.03883}[3.5843 + \tfrac{2}{3}(2 \times 0.9064 + 0.6152)] = \frac{10^6}{0.03883}(5.203)$$

$$P_{12} = P_{21} = \frac{10^6}{0.03883}[\tfrac{2}{3}(-0.9064 + 0.6152)] = \frac{10^6}{0.03883}(-0.1941)$$

$$P_{10} = P_{02} = P_{20} = P_{01} = \frac{10^6}{0.03883}[-\tfrac{1}{3}(-0.9064 + 0.6152)] = \frac{10^6}{0.03883}(0.0971)$$

Substituting the sequence potential coefficients in [40],

$$\Delta = \left(\frac{10^6}{0.03883}\right)^3 [(2.775)^2 \times 5.203 - 2 \times 2.775 \times (0.0971)^2 - 5.203 \times (-0.1941)^2 + 2(-0.1941)(0.0971)^2] = 39.813$$

$$C_{11} = C_{22} = \frac{0.03883 \times 10^{-6}}{39.813}[2.775 \times 5.203 - (0.09707)^2] = 0.014072 \times 10^{-6} \text{ farad per mile}$$

$$C_{00} = \frac{0.03883 \times 10^{-6}}{39.813}[(2.775)^2 - (0.1941)^2] = 0.007475 \times 10^{-6} \text{ farad per mile}$$

$$C_{12} = C_{21} = -\frac{0.03883 \times 10^{-6}}{39.813}[-0.1941 \times 5.203 - (0.09707)^2]$$
$$= 0.000994 \times 10^{-6} \text{ farad per mile}$$

$$C_{10} = C_{02} = C_{20} = C_{01} = -\frac{0.03883 \times 10^{-6}}{39.813} \times [-0.1941 \times 0.09707 - 2.775 \times 0.09707]$$
$$= -0.000281 \times 10^{-6} \text{ farad per mile}$$

The sequence self- and mutual capacitances calculated by the two methods are the same within the degree of precision used in the calculations.

Approximate Values for Positive-, Negative-, and Zero-Sequence Self-Capacitances. By the method here called the potential coefficient method, the positive-, negative-, and zero-sequence self-capacitances and the mutual capacitances between the sequence network can be determined to any desired degree of precision based on the initial assumptions. These assumptions are not exact, as explained at the beginning of the chapter; therefore an *extremely* high degree of precision in calculating capacitances is not worth while. Moreover, the overhead three-phase transmission circuit is an unsymmetrical circuit, and therefore, when treated by the method of symmetrical components, has small mutual capacitances between the sequence networks. In a completely transposed circuit, these mutual capacitances are substantially zero; in an untransposed circuit, they are small and in the usual system problem can be neglected. Because of this coupling, the positive-, negative-, and zero-sequence capacitive self-impedances and their corresponding admittances are not the *exact* reciprocals of each other; but, since the mutual capacitive coupling between the sequence network is small, it is to be expected that one set of values will be very nearly the reciprocals of the other set. And this is the case.

In Problem 1, part (c), P_{00} is greater than $P_{11} = P_{22}$, and P_{12}, P_{21}, P_{10}, and P_{01} are all less than 10% of P_{11}. In both numerator and denominator of the equations of [40] for $C_{11} = C_{22}$ and C_{00}, the mutual potential coefficients occur only as second or higher powers. If P_{00} were equal to P_{11} and all mutual potential coefficients were equal to 10% of P_{11}, the error in neglecting all terms except the first in numerator and denominator of $C_{11} = C_{22}$ and C_{00} would be less than 3%; but with P_{00} greater than P_{11} and the mutual potential coefficients all less than 10% of P_{11}, the error may be less than $\frac{1}{2}$%.

Neglecting terms containing second or higher powers of mutual potential coefficients in $C_{11} = C_{22}$ and in C_{00} in [40],

$$\begin{aligned} C_{11} &= C_{22} = \frac{P_{11}P_{00}}{P_{11}^2P_{00}} = \frac{1}{P_{11}} \\ C_{00} &= \frac{P_{11}^2}{P_{11}^2P_{00}} = \frac{1}{P_{00}} \end{aligned} \tag{41}$$

Substituting calculated values of P_{11} and P_{00} from Problem 1 in [41] and comparing with the calculated values of C_{11} and C_{00} in Problem 1, expressed in farads per mile:

$C_{11} = C_{22}$ (from [41]) $= \dfrac{1}{P_{11}} = 0.01400 \times 10^{-6}$; from Problem 1,

$C_{11} = C_{22} = 0.01407 \times 10^{-6}$

C_{00} (from [41]) $= \dfrac{1}{P_{00}} = 0.00746 \times 10^{-6}$; from Problem 1,

$C_{00} = 0.00747 \times 10^{-6}$

The errors in the approximate equations of [41] are negligible for the given circuit; they are also shown to be negligible with one ground wire (see Problem 3). It can be shown that the errors in [41] are unimportant for any other of the usual configurations of three-phase overhead transmission circuit both with and without ground wires. This has been pointed out and illustrated with examples by Professor W. V. Lyon.[7]

The *influence of circuit configuration* on the sequence potential coefficients for a three-phase circuit without ground wires can be determined by rewriting $P_{11} = P_{22}$ and P_{00} in [38] in terms of radius of the conductors, spacings between the conductors and between conductors and images. Expressed in daraf-miles,

$$P_{11} = P_{22} = \frac{10^6}{0.03883}\left[\log_{10}\frac{\sqrt[3]{s_{ab}s_{ac}s_{bc}}}{r} + \log_{10}\sqrt[3]{\frac{S_{aa}S_{bb}S_{cc}}{S_{ab}S_{ac}S_{bc}}}\right] \qquad [42]$$

$$P_{00} = \frac{10^6}{0.03883}\log_{10}\frac{\sqrt[3]{S_{aa}S_{bb}S_{cc}S_{ab}^2S_{ac}^2S_{bc}^2}}{r\sqrt[3]{s_{ab}^2s_{ac}^2s_{bc}^2}} \qquad [43]$$

Presence of Earth Neglected in Positive- and Negative-Sequence Self-Capacitances. For the usual transmission line, the height above ground is such that the distances between conductors and their own images and the images of other conductors are of the same order of magnitude. If the distances S_{aa}, S_{bb}, S_{cc}, S_{ab}, S_{ac}, S_{bc} are assumed equal, which is equivalent to neglecting the presence of the earth, [42] in daraf-miles becomes

$$P_{11} = P_{22} = \frac{10^6}{0.03883}\log_{10}\frac{\sqrt[3]{s_{ab}s_{ac}s_{bc}}}{r} \qquad [44]$$

For the circuit of Problem 1, [44] gives in daraf-miles,

$$P_{11} = P_{22} = \frac{10^6}{0.03883}\log_{10}\frac{\sqrt[3]{10 \times 10 \times 20}}{\frac{0.5}{24}} = \frac{10^6}{0.03883}(2.7815)$$

Substituting this value in [41],

$$C_{11} = C_{22} = 0.01396 \times 10^{-6} \text{ farad per mile}$$

Compared with 0.01400×10^{-6}, also obtained from [41], the error in neglecting the presence of the earth for the circuit considered is less than one-half per cent.

Capacitive Susceptance Curves for Three-Phase Circuit without Ground Wires. Substituting $P_{11} = P_{22}$ and P_{00} from [44] and [43], respectively, in [41], and multiplying by $2\pi f$, the positive-, negative-, and zero-sequence capacitive susceptances per mile in mhos to a close approximation are

$$b_{11} = b_{22} = 2\pi f C_{11} = \frac{2\pi f}{P_{11}} = \left(\frac{f}{60}\right)\frac{14.64 \times 10^{-6}}{\log_{10}\frac{s_{\overline{ab}}}{r}} \qquad [45]$$

$$b_{00} = 2\pi f C_{00} = \left(\frac{f}{60}\right)\frac{4.88 \times 10^{-6}}{\log_{10}\frac{2H}{\sqrt[3]{r(s_{\overline{ab}})^2}}} \qquad [46]$$

where

$$r = \text{radius of conductor}$$

$$s_{\overline{ab}} = \sqrt[3]{s_{ab}s_{ac}s_{bc}} = \text{geometric mean spacing between conductors}$$

$$H = \tfrac{1}{2}\sqrt[9]{S_{aa}S_{bb}S_{cc}S_{ab}^2S_{ac}^2S_{bc}^2} = \tfrac{1}{2} \text{ (ninth root of nine distances between conductors and images)}$$
$$= \text{average height above ground of conductors, approximately}$$

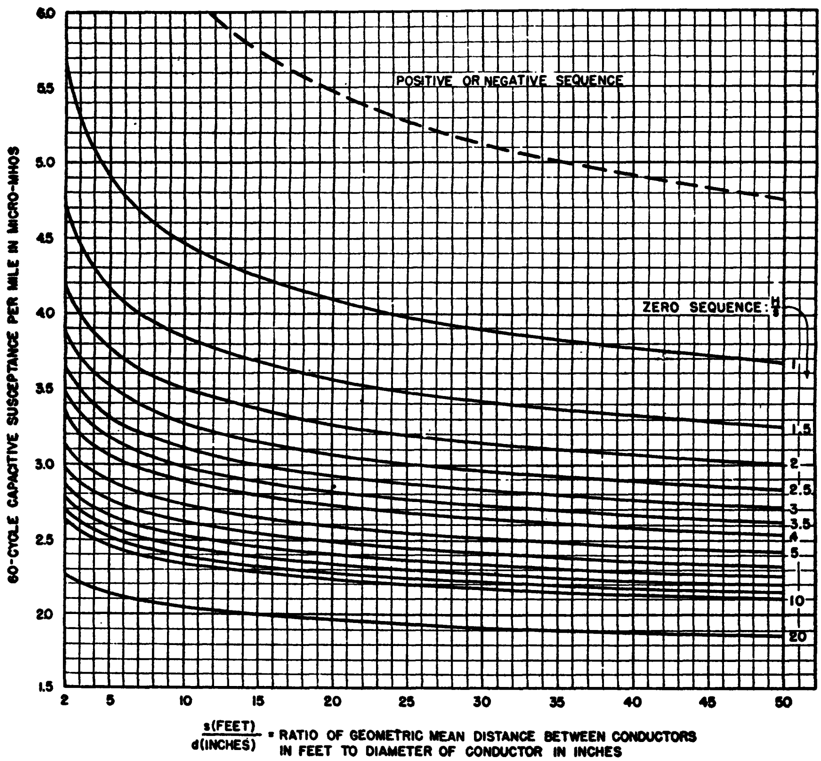


Fig. 6. Capacitive susceptances at 60 cycles of three-phase transmission circuits without ground wires.

H/s = ratio of H (the average height of conductors above ground, approximately), to s (the geometric mean distance between conductors) both in feet

Figure 6 gives the positive-, negative-, and zero-sequence capacitive susceptance at 60 cycles in micromhos per mile for a three-phase circuit without ground wires calculated from [45] and [46]. In these equations r, $s_{\overline{ab}}$, and H are in the same unit of length, but for convenience the curves have been plotted with the ratio s/d as abscissa, where s is the geometric mean spacing between conductors in feet and d is the diameter of the conductors in inches; for the zero-sequence capacitive susceptance, the parameter is the ratio of H to s both in feet, where H represents the average height above ground of the conductors.

Transposed Three-Phase Circuit. Under the assumption of constant voltage along the conductors, the capacitances of a completely transposed three-phase circuit form a symmetrical circuit between terminals in which there is no mutual capacitive coupling between the sequence networks. The sequence capacitances of the symmetrical circuit are equal to the corresponding self-capacitances in any section between transpositions and are determined by the same equations. Equations [41] are approximate to the same degree for the completely transposed circuit as for the untransposed circuit; but the capacitive coupling between the sequence networks in the untransposed circuit (normally neglected) does not exist in the completely transposed circuit.

Problem 2. Find the positive-, negative-, and zero-sequence capacitive susceptances per mile at 60 cycles from Fig. 6 for a three-phase circuit without ground wires in which the conductors are 40 feet above ground, have 0.5 inch diameters and are spaced 10 feet apart. Compare with Problem 1.

Solution.

$$s_{\overline{ab}} = \sqrt[3]{10 \cdot 10 \cdot 20} = 12.6 \text{ feet}; \quad d = 0.5 \text{ inch}; \quad H = 40 \text{ feet}; \quad \frac{s_{\overline{ab}}}{d} = \frac{12.6}{0.5} = 25.2;$$

$$\frac{H}{s_{\overline{ab}}} = \frac{40}{12.6} = 3.2$$

From Fig. 6: With $s/d = 25.2$,

$$b_{11} = b_{22} = 5.28 \times 10^{-6} \text{ mho per mile}$$

With $s/d = 25.2$ and $H/s = 3.2$,

$$b_{00} = 2.82 \times 10^{-6} \text{ mho per mile}$$

From Problem 1:

$$b_{11} = b_{22} = 2\pi f C_{11} = 377 \times 0.01407 \times 10^{-6} = 5.30 \times 10^{-6} \text{ mho per mile}$$

$$b_{00} = 377 C_{00} = 377 \times 0.00747 \times 10^{-6} = 2.82 \times 10^{-6} \text{ mho per mile}$$

In $b_{11} = b_{22}$, read from Fig. 6, the effect of the earth is neglected and C_{11} is assumed equal to $1/P_{11}$. These two assumptions together produce an error of less than $\frac{1}{2}$%. For b_{00}, the check is within the degree of precision obtainable from the curves.

THREE-PHASE CIRCUITS WITH GROUND WIRES

One ground wire will be indicated by w, and two ground wires by w and v.

With *one ground wire* w, [6] becomes

$$\begin{aligned} V_a &= P_{aa}Q_a + P_{ab}Q_b + P_{ac}Q_c + P_{aw}Q_w \\ V_b &= P_{ba}Q_a + P_{bb}Q_b + P_{bc}Q_c + P_{bw}Q_w \\ V_c &= P_{ca}Q_a + P_{cb}Q_b + P_{cc}Q_c + P_{cw}Q_w \\ V_w &= P_{wa}Q_a + P_{wb}Q_b + P_{wc}Q_c + P_{ww}Q_w \end{aligned} \qquad [47]$$

The ground wire w is at zero potential. With $V_w = 0$, Q_w in the last equation of [47] is

$$Q_w = -\frac{1}{P_{ww}}(P_{wa}Q_a + P_{wb}Q_b + P_{wc}Q_c) \qquad [48]$$

Substituting [48] in the first three equations of [47] and replacing P_{ba} by P_{ab}, P_{wa} by P_{aw}, etc.,

$$\begin{aligned} V_a &= \left(P_{aa} - \frac{P_{aw}^2}{P_{ww}}\right)Q_a + \left(P_{ab} - \frac{P_{aw}P_{bw}}{P_{ww}}\right)Q_b + \left(P_{ac} - \frac{P_{aw}P_{cw}}{P_{ww}}\right)Q_c \\ V_b &= \left(P_{ab} - \frac{P_{aw}P_{bw}}{P_{ww}}\right)Q_a + \left(P_{bb} - \frac{P_{bw}^2}{P_{ww}}\right)Q_b \\ &\qquad + \left(P_{bc} - \frac{P_{bw}P_{cw}}{P_{ww}}\right)Q_c \\ V_c &= \left(P_{ac} - \frac{P_{aw}P_{cw}}{P_{ww}}\right)Q_a + \left(P_{cb} - \frac{P_{bw}P_{cw}}{P_{ww}}\right)Q_b + \left(P_{cc} - \frac{P_{cw}^2}{P_{ww}}\right)Q_c \end{aligned} \qquad [49]$$

Replacing the charges in [49] by their symmetrical components of charge and resolving the phase voltages into their symmetrical components of voltage, the coefficients in the resultant equations when equated to the corresponding coefficients of [37] give the sequence potential coefficients. These sequence potential coefficients will consist of two parts. The first part is given by [38], the second part will be treated as a correction. Let ΔP indicate this correction, the subscripts of P indicating the sequence potential coefficient to which the correction applies. An additional subscript w will be added for one ground wire, and two additional subscripts wv for two ground wires. Thus, for use in [37],

$P_{11-w} = P_{11} + \Delta P_{11-w}$ = positive-sequence potential coefficient with one ground wire w

$P_{00-w} = P_{00} + \Delta P_{00-w}$ = zero-sequence potential coefficient with one ground wire w

$P_{12-w} = P_{12} + \Delta P_{12-w}$ = mutual potential coefficient of Q_{a2} in equation for V_{a1} with one ground wire w

. . . .

$P_{11-wv} = P_{11} + \Delta P_{11-wv}$ = positive-sequence potential coefficient with two ground wires w and v

$P_{00-wv} = P_{00} + \Delta P_{00-wv}$ = zero-sequence potential coefficient with two ground wires w and v

$P_{01-wv} = P_{01} + \Delta P_{01-wv}$ = mutual potential coefficient of Q_{a1} in equation for V_{a0} with two ground wires w and v

. . . .

where P_{11}, P_{00}, etc., are the potential coefficients without ground wires, given by [38].

The corrections to the sequence potential coefficients for one ground wire in statdaraf-centimeters are

$$\Delta P_{11-w} = \Delta P_{22-w} = -\frac{1}{3P_{ww}}[(P_{aw}^2 + P_{bw}^2 + P_{cw}^2) - (P_{aw}P_{bw} + P_{aw}P_{cw} + P_{bw}P_{cw})]$$

$$\Delta P_{00-w} = -\frac{1}{3P_{ww}}(P_{aw} + P_{bw} + P_{cw})^2 = -\frac{3(P_{\overline{aw}})^2}{P_{ww}}$$

$$\Delta P_{12-w} = -\frac{1}{3P_{ww}}[(P_{aw}^2 + a^2P_{bw}^2 + aP_{cw}^2) + 2(aP_{aw}P_{bw} + a^2P_{aw}P_{cw} + P_{bw}P_{cw})]$$

$$\Delta P_{21-w} = -\frac{1}{3P_{ww}}[(P_{aw}^2 + aP_{bw}^2 + a^2P_{cw}^2) + 2(a^2P_{aw}P_{bw} + aP_{aw}P_{cw} + P_{bw}P_{cw})] \tag{50}$$

$$\Delta P_{10} = \Delta P_{02} = -\frac{1}{3P_{ww}}[(P_{aw}^2 + aP_{bw}^2 + a^2P_{cw}^2) - (a^2P_{aw}P_{bw} + aP_{aw}P_{cw} + P_{bw}P_{cw})]$$

$$\Delta P_{20} = \Delta P_{01} = -\frac{1}{3P_{ww}}[(P_{aw}^2 + a^2P_{bw}^2 + aP_{cw}^2) - (aP_{aw}P_{bw} + a^2P_{aw}P_{cw} + P_{bw}P_{cw})]$$

where

$$P_{\overline{aw}} = \tfrac{1}{3}(P_{aw} + P_{bw} + P_{cw})$$

Adding the *numerical values* of the corrections given by [50] resulting from one ground wire to the *numerical values* of the sequence potential coefficients without ground wires given by [38] and substituting the resultant values in [40], the sequence capacitances are obtained.

If the ground wire could be assumed equidistant from the phase conductors and their images (as is the case in a completely transposed circuit), the potential coefficients P_{aw}, P_{bw}, and P_{cw} in [50] would be equal and all the ΔP's except ΔP_{00-w} would be zero. Even if this assumption is not made, P_{aw}, P_{bw}, and P_{cw} are of the same order of magnitude and all ΔP's except ΔP_{00-w} are relatively unimportant.

Problem 3. For the conductor arrangement of Problem 1 with one ground wire $\frac{3}{8}$ inch in diameter, 8 feet above the center conductor, find the sequence capacitances using both [40] and [41].

Solution.

$$\log_{10} \frac{S_{ww}}{r_w} = \log_{10} \frac{96 \times 24}{(\frac{3}{8})} = 3.7885$$

$$\log_{10} \frac{S_{aw}}{s_{aw}} = \log_{10} \frac{88}{8} = 1.0414$$

$$\log_{10} \frac{S_{bw}}{s_{bw}} = \log_{10} \frac{S_{cw}}{s_{cw}} = \log_{10} \frac{88.6}{12.8} = 0.8398$$

Replacing P_{ww}, P_{aw}, $P_{bw} = P_{cw}$, respectively, in [50] by the above logarithmic values multiplied by 2×2.3026, and dividing the equations by 0.1788×10^{-6}, the changes in the sequence potential coefficients resulting from one ground wire w expressed in daraf-miles are

$$\Delta P_{11-w} = \Delta P_{22-w} = -\frac{10^6}{0.03883} \times \frac{0.0406}{11.365} = -\frac{10^6}{0.03883}(0.0036)$$

$$\Delta P_{00-w} = -\frac{10^6}{0.03883} \times \frac{7.404}{11.365} = -\frac{10^6}{0.03883}(0.6514)$$

$$\Delta P_{12-w} = \Delta P_{21-w} = -\frac{10^6}{0.03883} \times \frac{0.0406}{11.365} = -\frac{10^6}{0.03883}(0.0036)$$

$$\Delta P_{10-w} = \Delta P_{02-w} = \Delta P_{20-w} = \Delta P_{01-w} = -\frac{10^6}{0.03883} \times \frac{0.5486}{11.365}$$
$$= -\frac{10^6}{0.03883}(0.0483)$$

Adding the corrections for one ground wire to the sequence potential coefficients calculated in Problem 1, the potential coefficients with one ground wire in daraf-miles are

$$P_{11-w} = P_{22-w} = P_{11} + \Delta P_{11-w} = \frac{10^6}{0.03883}(2.775 - 0.004) = \frac{10^6}{0.03883}(2.771)$$

$$P_{00-w} = P_{00} + \Delta P_{00-w} = \frac{10^6}{0.03883}(5.203 - 0.651) = \frac{10^6}{0.03883}(4.552)$$

$$P_{12-w} = P_{21-w} = P_{12} + \Delta P_{12-w} = \frac{10^6}{0.03883}(-0.1941 - 0.0036)$$
$$= \frac{10^6}{0.03883}(-0.198)$$

$$P_{10-w} = P_{20-w} = P_{01-w} = P_{02-w} = P_{10} + \Delta P_{10-w} = \frac{10^6}{0.03883}(0.0971 - 0.0483)$$
$$= \frac{10^6}{0.03883}(0.049)$$

Substituting the above potential coefficients in [40], the sequence capacitances in farads per mile are

$$C_{11-w} = C_{22-w} = \frac{0.03883 \times 10^{-6}}{34.76}(12.61) = 0.01405 \times 10^{-6}$$

$$C_{00-w} = \frac{0.03883 \times 10^{-6}}{34.76}(7.64) = 0.00851 \times 10^{-6}$$

$$C_{12-w} = C_{21-w} = \frac{0.03883 \times 10^{-6}}{34.76}(0.904) = 0.00101 \times 10^{-6}$$

$$C_{10-w} = C_{20-w} = C_{01-w} = C_{02-w} = \frac{0.03883 \times 10^{-6}}{34.76}(-0.145)$$
$$= -0.00016 \times 10^{-6}$$

Substituting P_{11-w} and P_{00-w} in the approximate equations of [41],

$$C_{11-w} = C_{22-w} = \frac{1}{P_{11-w}} = 0.01401 \times 10^{-6} \text{ farad per mile}$$

$$C_{00-w} = \frac{1}{P_{00-w}} = 0.00853 \times 10^{-6} \text{ farad per mile}$$

Comparing the sequence capacitances with those of Problem 1, the zero-sequence self-capacitance is increased 14% by the presence of the ground wire, the positive- and negative-sequence self-capacitances are substantially unchanged, the mutual capacitances between the sequence networks remain relatively unimportant. The errors in $C_{11-w} = C_{22-w}$ and C_{00-w} calculated from [41] are negligible.

With *two ground wires* w and v, [6] becomes

$$\begin{aligned}
V_a &= P_{aa}Q_a + P_{ab}Q_b + P_{ac}Q_c + P_{aw}Q_w + P_{av}Q_v \\
V_b &= P_{ba}Q_a + P_{bb}Q_b + P_{bc}Q_c + P_{bw}Q_w + P_{bv}Q_v \\
V_c &= P_{ca}Q_a + P_{cb}Q_b + P_{cc}Q_c + P_{cw}Q_w + P_{cv}Q_v \\
V_w &= P_{wa}Q_a + P_{wb}Q_b + P_{wc}Q_c + P_{ww}Q_w + P_{wv}Q_v \\
V_v &= P_{va}Q_a + P_{vb}Q_b + P_{vc}Q_c + P_{vw}Q_w + P_{vv}Q_v
\end{aligned} \qquad [51]$$

Since $V_w = V_v = 0$, Q_w and Q_v from the last two equations of [51] are

$$Q_w = -\frac{P_{vv}(P_{aw}Q_a+P_{bw}Q_b+P_{cw}Q_c)-P_{wv}(P_{av}Q_a+P_{bv}Q_b+P_{cv}Q_c)}{P_{ww}P_{vv} - P^2_{wv}}$$
$$Q_v = -\frac{P_{ww}(P_{av}Q_a+P_{bv}Q_b+P_{cv}Q_c)-P_{wv}(P_{aw}Q_a+P_{bw}Q_b+P_{cw}Q_c)}{P_{ww}P_{vv} - P^2_{wv}} \qquad [52]$$

Q_w and Q_v are given by [52] in terms of the charges Q_a, Q_b, and Q_c for the general case of ground wires of different diameters, unsymmetrically spaced with respect to the conductors a, b, and c. As in the case of one ground wire, the changes in sequence potential coefficients because of two ground wires are relatively unimportant except for the change ΔP_{00-wv} in the zero-sequence potential coefficient. Only ΔP_{00-wv} will be given here; if the other sequence potential coefficients are required, they can be obtained by replacing the conductor potential coefficients in [51] and [52] by their numerical values, substituting [52] in [51], and then proceeding as for one ground wire.

The correction ΔP_{00-wv} can be obtained by assuming only zero-sequence charges on the phase conductors. This is analogous to the method used in Chapter VIII for determining zero-sequence self-impedance by assuming only zero-sequence currents flowing in the circuit. With only zero-sequence charges on the conductors, Q_w and Q_v in [52] become

$$Q_w = -Q_{a0}\frac{P_{vv}(3P_{\overline{aw}}) - P_{wv}(3P_{\overline{av}})}{P_{ww}P_{vv} - (P_{wv})^2}$$
$$Q_v = -Q_{a0}\frac{P_{ww}(3P_{\overline{av}}) - P_{wv}(3P_{\overline{aw}})}{P_{ww}P_{vv} - (P_{wv})^2} \qquad [53]$$

where

$$P_{\overline{aw}} = \tfrac{1}{3}(P_{aw} + P_{bw} + P_{cw})$$
$$P_{\overline{av}} = \tfrac{1}{3}(P_{av} + P_{bv} + P_{cv})$$

The voltages in the three phases resulting from the charges on the ground wires, or the changes in these voltages because of the ground wires, from [51] are

$$\Delta V_a = Q_wP_{aw} + Q_vP_{av}$$
$$\Delta V_b = Q_wP_{bw} + Q_vP_{bv}$$
$$\Delta V_c = Q_wP_{cw} + Q_vP_{cv}$$

The change in V_{a0} because of the ground wires is

$$\Delta V_{a0} = \tfrac{1}{3}(\Delta V_a + \Delta V_b + \Delta V_c) = Q_wP_{\overline{aw}} + Q_vP_{\overline{av}} \qquad [54]$$

With only zero-sequence charges on the conductors and no ground wire, from [37],

$$V_{a0} = Q_{a0}P_{00}$$

With two ground wires,

$$V_{a0} + \Delta V_{a0} = Q_{a0}(P_{00} + \Delta P_{00-wv})$$

Substituting [53] in [54], and dividing both sides of the equation by Q_{a0},

$$\Delta P_{00-wv} = \frac{\Delta V_{a0}}{Q_{a0}} = -3\,\frac{P_{vv}(P_{\overline{aw}})^2 + P_{ww}(P_{\overline{av}})^2 - 2P_{wv}P_{\overline{aw}}P_{\overline{av}}}{P_{ww}P_{vv} - (P_{wv})^2} \qquad [55]$$

Dividing numerator and denominator of the fraction on the right-hand side of [55] by $P_{ww}P_{vv}$,

$$\Delta P_{00-wv} = -3\,\frac{\dfrac{(P_{\overline{aw}})^2}{P_{ww}} + \dfrac{(P_{\overline{av}})^2}{P_{vv}} - 2\dfrac{P_{\overline{aw}}}{\sqrt{P_{ww}}}\dfrac{P_{\overline{av}}}{\sqrt{P_{vv}}}\sqrt{\dfrac{P_{wv}}{P_{ww}}}\sqrt{\dfrac{P_{wv}}{P_{vv}}}}{1 - \left(\dfrac{P_{wv}}{P_{ww}}\right)\left(\dfrac{P_{wv}}{P_{vv}}\right)} \qquad [56]$$

Rewriting [56],

$$\Delta P_{00-wv} = \frac{\Delta P_{00-w} + \Delta P_{00-v} - 2\sqrt{(\Delta P_{00-w})(\Delta P_{00-v})}\,\sqrt{K_wK_v}}{1 - K_wK_v} \qquad [57]$$

where

ΔP_{00-w} is the correction for one ground wire w alone, given by [50]

ΔP_{00-v} is the correction for one ground wire v alone, also given by [50] if w is replaced by v

$$K_w = \frac{P_{wv}}{P_{ww}}$$

$$K_v = \frac{P_{wv}}{P_{vv}}$$

K_w and K_v are ratios and therefore without dimensions. ΔP_{00-w} and ΔP_{00-v} are both negative and the sign of the term in [57] involving the square root of their product is opposite to the sign of ΔP_{00-w} and ΔP_{00-v}.

If the ground wires are identical and at equal heights above ground, $K_w = K_v$ and [57] becomes

$$\Delta P_{00-wv} = \frac{\Delta P_{00-w} + \Delta P_{00-v} - 2\sqrt{(\Delta P_{00-w})(\Delta P_{00-v})}K_w}{1 - K_w^2} \qquad [58]$$

If in addition to being identical and at equal heights above ground, the ground wires are symmetrically spaced with respect to the conductors,

$$\Delta P_{00-w} = \Delta P_{00-v}$$

and

$$\Delta P_{00-wv} = \frac{2\Delta P_{00-w}}{1 + K_w} \qquad [59]$$

Equation [59] may also be written

$$\Delta P_{00-wv} = -\frac{3(P_{\overline{aw}})^2}{\frac{1}{2}(P_{ww} + P_{wv})} \qquad [60]$$

Comparing [60] with the equation for ΔP_{00-w} in [50], two identical ground wires equidistant from ground and symmetrically spaced with respect to the conductors are equivalent as far as zero-sequence capacitances are concerned to one ground wire with the same geometric mean spacings between ground wire and conductors and ground wire and images of conductors, but with P_{ww} replaced by $\frac{1}{2}(P_{ww} + P_{wv})$.

60-Cycle Zero-Sequence Capacitive Reactance Curves for One Three-Phase Circuit with Ground Wires. Zero-sequence capacitive susceptances of three-phase transmission circuits with ground wires cannot be simply given by means of charts because of the large number of parameters involved. This is not the case with *zero-sequence capacitive reactances* which are directly proportional to their corresponding potential coefficients and can be given by simple graphs. Figure 6 gives b_{00}, the 60-cycle zero-sequence capacitive susceptance per mile of a three-phase circuit without ground wires based on $C_{00} = 1/P_{00}$ given by [41], and therefore x_{00}, the zero-sequence capacitive reactance of one mile of line, can be obtained by taking the reciprocal of b_{00} read from Fig. 6. The corrections to x_{00} for one, two, or more ground wires can be obtained from Fig. 7 and applied to x_{00} to give the zero-sequence capacitive reactance with ground wires. The zero-sequence capacitive susceptance with ground wires is the reciprocal of the corresponding capacitive reactance to a close approximation, as illustrated in Problem 3.

From [36], for ℓ miles of line, the zero-sequence capacitive self-reactances without ground wires and with one and two ground wires are

$$x_{00} = \frac{P_{00}}{2\pi f\ell} = \frac{1}{2\pi f\ell C_{00}} = \frac{1}{b_{00}}$$

$$x_{00-w} = x_{00} + \Delta x_{00-w} = x_{00} + \frac{\Delta P_{00-w}}{2\pi f\ell}$$

$$x_{00-wv} = x_{00} + \Delta x_{00-wv} = x_{00} + \frac{\Delta P_{00-wv}}{2\pi f\ell}$$

where x_{00} is the reciprocal of b_{00} given by [46] and Fig. 6; ΔP_{00-w} is given by [50], and ΔP_{00-wv} by [57] or [59].

One Ground Wire w. From ΔP_{00-w} in [50],

$$\Delta x_{00-w} = \frac{\Delta P_{00-w}}{2\pi f \ell} = -\frac{0.205 \times 10^6}{\ell}\left(\frac{60}{f}\right)\frac{\left(\log_{10}\dfrac{S_{\overline{aw}}}{s_{\overline{aw}}}\right)^2}{\log_{10}\dfrac{2h_w}{r_w}} \text{ ohms} \qquad [61]$$

where

$S_{\overline{aw}} = \sqrt[3]{S_{aw}S_{bw}S_{cw}}$ = geometric mean distance between ground wire w and images of conductors = height of ground wire plus average height of conductors, approximately

$s_{\overline{aw}} = \sqrt[3]{s_{aw}s_{bw}s_{cw}}$ = geometric mean distance between ground wire w and conductors

r_w, h_w = radius and height above ground of ground wire w, respectively

In [61], $S_{\overline{aw}}$, $s_{\overline{aw}}$, h_w, and r_w are in the same unit of length, but for convenience the curves of Fig. 7(a) are plotted with the ratio $S_{\overline{aw}}/s_{\overline{aw}}$ ($S_{\overline{aw}}$ and $s_{\overline{aw}}$ both in feet) as abscissa and the ratio h_w/d_w as parameter, where h_w, the height above ground of the ground wire, is in feet, and d_w, the diameter of the ground wire, is in inches. Problem 4 illustrates the use of Fig. 7(a).

Two Unsymmetrical Ground Wires w and v. From ΔP_{00-wv} in [57],

$$\Delta x_{00-wv} = \frac{\Delta P_{00-wv}}{2\pi f \ell}$$
$$= \frac{\Delta x_{00-w} + \Delta x_{00-v} - 2\sqrt{(\Delta x_{00-w})(\Delta x_{00-v})}\,\sqrt{K_w K_v}}{1 - K_w K_v} \qquad [62]$$

With two identical ground wires w and v, equidistant from earth and symmetrically spaced with respect to the conductors, from ΔP_{00-wv} in [59],

$$\Delta x_{00-wv} = \frac{2\Delta x_{00-w}}{1 + K_w} \qquad [63]$$

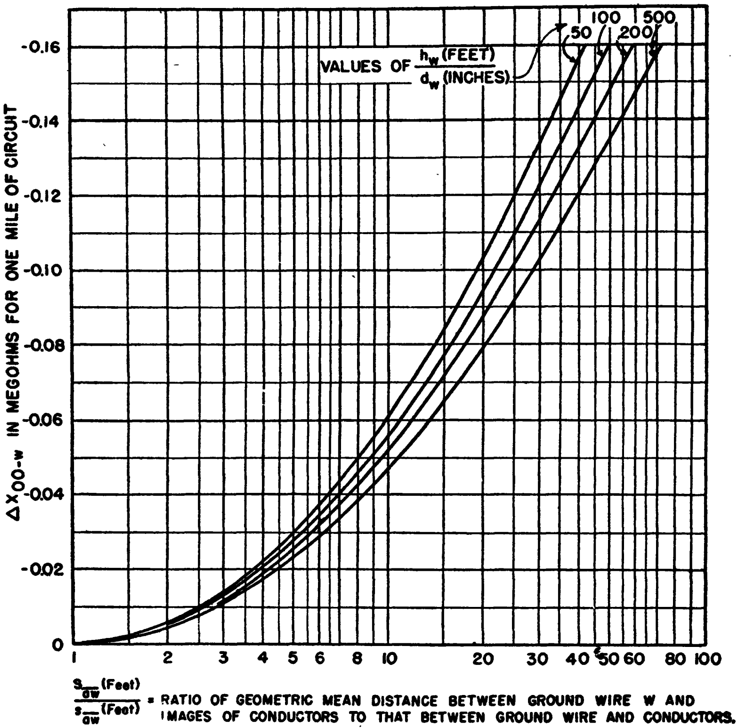


FIG. 7(*a*). 60-cycle correction for one mile of circuit to be added to x_{00} without ground wires to include the effect of one ground wire on the zero-sequence capacitive reactance. (d is diameter and h is height above ground of ground wire.) For ℓ miles of line, the capacitive impedance is $-jx_{00-w} = -j\left[\frac{1}{\ell b_{00}} + \frac{\Delta x_{00-w}}{\ell}\right]$, where b_{00} is read from Fig. 6 for one mile of circuit. Note that Δx_{00-w} is *negative.*

where Δx_{00-w} is given by [61] and Fig. 7(*a*). Δx_{00-v} is also given by [61] and Fig. 7(*a*) if w is replaced by v.

$$K_w = \frac{P_{wv}}{P_{ww}} = \frac{\log_{10} \dfrac{S_{wv}}{s_{wv}}}{\log_{10} \dfrac{2h_w}{r_w}} \qquad [64]$$

$$K_v = \frac{\log_{10} \dfrac{S_{wv}}{s_{wv}}}{\log_{10} \dfrac{2h_v}{r_v}}$$

where

S_{wv} = distance between one ground wire and image of other

s_{wv} = distance between ground wires

r_w, r_v = radii of ground wires, w and v, respectively

h_w, h_v = height above ground of w and v, respectively

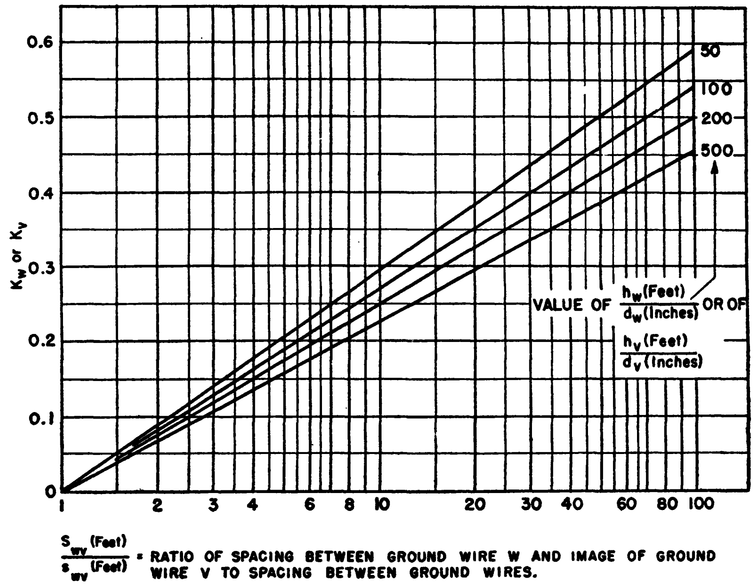


FIG. 7(*b*). K_w and K_v to be substituted in [62], or K_w in [63], to obtain the correction Δx_{00-wv} for two ground wires to be applied to x_{00} without ground wires.

K_w and K_v can be read directly from Fig. 7(*b*). In this chart, the abscissa is ~~the~~ ratio S_{wv}/s_{wv} (both in feet) and the parameter is h_w/d_w or h_v/d_v, where h_w or h_v is in feet, and d_w or d_v is in inches.

For unsymmetrically spaced ground wires, Δx_{00-w}, Δx_{00-v}, K_w, and K_v are substituted in [62], where the third term in the numerator is opposite in sign to Δx_{00-w} and Δx_{00-v}, which are negative. For symmetrical ground wires, [63] may be used. (See Problem 4 for application of Figs. 7(*a*) and (*b*).)

More Than Two Ground Wires. The method given for two unsymmetrically spaced ground wires can be extended by analogy to more than two ground wires if the ground wires are divided into groups. For example, if a three-phase circuit has one (or two) ground wires and another circuit (assumed for the present to be out of service) on the same right-of-way but not on the same towers has one (or two) ground wires, the ground wires can be divided into two groups, the first group consisting of the ground wire or wires on the same towers as the given circuit, the second group on other towers on the same right-of-way. The procedure is to calculate the correction to the zero-sequence capacitive reactance for the ground wire or wires of the first group with the other group disregarded, calling the correction ΔP_{00-w}; then to calculate the correction for the second group with the first group disregarded, calling this correction ΔP_{00-v}. K_w and K_v for the two ground wire groups are given approximately by Fig. 7(*b*) if s_{wv} and S_{wv} are the geometric mean distances between the ground wires of one group and the ground wires and images of the ground wires of the other group, respectively, and h_w and h_v are the geometric mean heights above ground of the ground wires of the two groups. ΔP_{00-w}, ΔP_{00-v}, K_w, and K_v are then substituted in [62] to determine the correction Δx_{00-wv} to be applied to x_{00}.

Three-Phase Circuit with Grounded Neutral Conductor. The effect of a grounded neutral conductor upon the sequence capacitances of a three-phase circuit is similar to that of a ground wire and can be determined from the same formulas.

Problem 4. Determine the 60-cycle zero-sequence capacitive impedance and admittance of ℓ miles of one of the three-phase circuits shown in Fig. 13 of Chapter XI, with the second circuit assumed open

(*a*) With ground wires not installed
(*b*) With one ground wire over the center line of the double circuit towers as indicated by shaded wire
(*c*) With two ground wires as indicated by solid wires.

Solution. (*a*) *No Ground Wires.*

$$s_{\overline{ab}} = 16.04 \text{ feet}, d = 0.953 \text{ inch}; \quad \frac{s_{\overline{ab}} \text{ (feet)}}{d \text{ (inches)}} = 16.85$$

H = 54.9 feet (one-half of ninth root of nine distances between conductors and images)

= 55.3 feet, approximately (average height of conductors above ground)

$$\frac{H}{s_{\overline{ab}}} = 3.42 \text{ (or 3.45, approximately)}$$

From Fig. 6, with $s_{\overline{ab}}$ (feet)/d (inches) = 16.85 and $H/s_{\overline{ab}}$ = 3.42 (or 3.45),

$$b_{00} = 2.86 \times 10^{-6} \text{ mho per mile}$$

$$Y_{00} = j2.86 \times 10^{-6} \times \ell \text{ mhos}$$

$$Z_{00} = -j\frac{0.349 \times 10^6}{\ell} \text{ ohms}$$

(*b*) *One Ground Wire.*

$$S_{\overline{aw}} = 131.5\ (= \text{approximately } 76 + 55.3 = 131.3), \quad s_{\overline{aw}} = 22.9; \quad \frac{S_{\overline{aw}}}{s_{\overline{aw}}} = 5.74$$

$$h_w = 76 \text{ feet}, \quad d_w = \tfrac{1}{2} \text{ inch}, \quad \frac{h_w \text{ (feet)}}{d_w \text{ (inches)}} = 152$$

Read from Fig. 7 (*a*), with $S_{\overline{aw}}/s_{\overline{aw}} = 5.74$ and h_w (feet)/d_w (inches) = 152,

$$\Delta x_{00-w} = -0.031$$

$$Z_{00-w} = -j\left(\frac{0.349 - 0.031}{\ell}\right)10^6 = -j\frac{0.318 \times 10^6}{\ell} \text{ ohms}$$

$$Y_{00-w} = j3.14 \times 10^{-6} \times \ell \text{ mhos}$$

(*c*) *Two Ground Wires.* With nearer ground wire indicated by w and the other by v,

$$S_{\overline{aw}} = 131.1, \quad s_{\overline{aw}} = 17.8; \quad \frac{S_{\overline{aw}}}{s_{\overline{aw}}} = 7.35$$

$$S_{\overline{av}} = 132.8, \quad s_{\overline{av}} = 30.4; \quad \frac{S_{\overline{av}}}{s_{\overline{av}}} = 4.37$$

$$\frac{h_w \text{ (feet)}}{d_w \text{ (inches)}} = \frac{h_v \text{ (feet)}}{d_v \text{ (inches)}} = \frac{76}{0.5} = 152$$

$$\frac{S_{wv}}{s_{wv}} = \frac{153.3}{20} = 7.66$$

Read from Fig. 7 (*a*), with $S_{\overline{aw}}/s_{\overline{aw}} = 7.35$ and h_w (feet)/d_w (inches) = 152,

$$\Delta x_{00-w} = -0.040 \times 10^6$$

and with $S_{\overline{av}}/s_{\overline{av}} = 4.37$ and h_v (feet)/d_v (inches) = 152,

$$\Delta x_{00-v} = -0.022 \times 10^6$$

Read from Fig. 7 (*b*), with $S_{wv}/s_{wv} = 7.66$ and $h_w/d_w = 152$,

$$K_w = K_v = 0.233$$

Substituting Δx_{00-w}, Δx_{00-v}, and $K_w = K_v$ in [62],

$$\Delta x_{00-wv} = \frac{-0.040 - 0.022 + 2\sqrt{0.022 \times 0.040}\,(0.233)}{[1 - (0.233)^2]\,\ell} = -\frac{0.051 \times 10^6}{\ell} \text{ ohms}$$

$$Z_{00-wv} = -j\frac{(0.349 - 0.051)10^6}{\ell} = -j\frac{0.298 \times 10^6}{\ell} \text{ ohms}$$

$$Y_{00-wv} = j3.36 \times 10^{-6} \times \ell \text{ mhos}$$

For *two unsymmetrically spaced identical ground wires equidistant from ground on the same towers*, very nearly the same result is obtained by assuming Δx_{00-w} and Δx_{00-v} equal and using [63] instead of [62], where $s_{\overline{aw}} = s_{\overline{av}}$ = the geometric mean spacing between ground wires and conductors and $S_{\overline{aw}} = S_{\overline{av}}$ = average height of ground wires plus average height of conductors. Using [63] instead of [62] for the circuit of Problem 4, with $K_w = K_v = 0.233$,

$$s_{\overline{aw}} = s_{\overline{av}} = 23.3; \quad S_{\overline{aw}} = S_{\overline{av}} = 76 + 55.3 = 131 \text{ feet}; \quad \frac{S_{\overline{aw}}}{s_{\overline{aw}}} = 5.62$$

From Fig. 7(a),

$$\Delta x_{00-w} = -\frac{0.030}{\ell} \times 10^6 \text{ ohms}$$

and from [63],

$$\Delta x_{00-wv} = -\frac{2(0.030) \times 10^6}{1.233\ell} = -\frac{0.049 \times 10^6}{\ell}$$

$$Z_{00-wv} = -j\frac{(0.349 - 0.049)10^6}{\ell} = -j\frac{0.300 \times 10^6}{\ell} \text{ ohms}$$

$$Y_{00-wv} = j3.33 \times 10^{-6}\, \ell \text{ mhos}$$

PARALLEL THREE-PHASE CIRCUITS

The capacitances associated with transmission circuits are capacitances to neutral in the positive- and negative-sequence networks and capacitances to ground in the zero-sequence network. Capacitances of parallel circuits, therefore, have a common neutral point in the positive- and negative-sequence networks and a common ground point in the zero-sequence network. Neglecting mutual capacitive coupling between the sequence networks because of dissymmetry, the equivalent capacitive circuits for two parallel transmission circuits in the sequence networks are three-terminal circuits which can be represented by either equivalent Y's or equivalent Δ's. If capacitive impedances are used, the equivalent Y's are more convenient; if capacitive admittances, the equivalent Δ's. Since either circuit can be obtained from the other (see Fig. 7, Chapter VI), and the simpler procedure where there are ground wires is to determine the equivalent capacitive impedance Y, the equivalent Y's will be developed to replace the capacitances of parallel circuits, with and without ground wires, in the sequence networks. The equivalent admittance Δ to replace the capacitances of two parallel circuits without ground wires in the zero-sequence network will be determined directly as well as from the corresponding impedance Y. The equivalent capacitive circuits are those to be used in the nominal equivalent circuits of parallel lines with distributed constants (see Fig. 8, Chapter VI).

The equivalent Y to replace two circuits with self-impedances Z_{aa}

and Z_{bb} and mutual impedance Z_{ab} between them when they have one terminal in common is given by Fig. 11(b) of Chapter I. Equivalent circuits of this type can be used to replace the capacitive impedances of two parallel transmission circuits in the sequence networks if Z_{aa} and Z_{bb} are the capacitive self-impedances of a given sequence of the two circuits, each with the other circuit open, and Z_{ab} is the capacitive mutual impedance of the same sequence between the two circuits. The sequence capacitive self-impedance of either circuit with the other circuit open (but with ground wires remaining in place) are determined as for the single three-phase circuit already described. The capacitive mutual impedances between parallel circuits with and without ground wires remain to be evaluated.

It will be shown that the capacitive coupling in the positive- and negative-sequence networks between two parallel transmission circuits is small, depends upon the relative arrangement of the phases in the two circuits, and in most transmission problems can be neglected. The zero-sequence capacitive coupling, however, may be an appreciable part of the zero-sequence self-capacitance of either circuit alone.

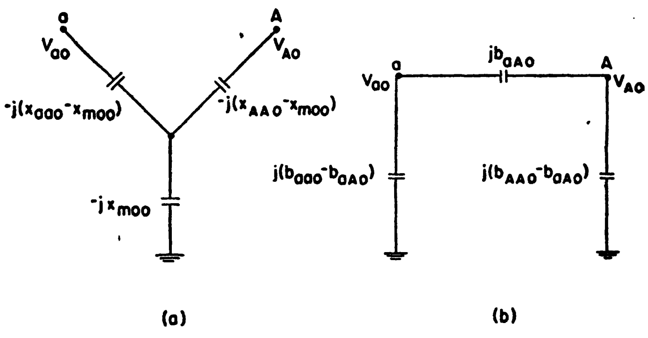


FIG. 8. (a) Capacitive impedance Y, and (b) capacitive admittance Δ to replace two parallel transmission circuits in the zero-sequence network.

Zero-Sequence Capacitive Equivalent Circuits for Two Parallel Transmission Circuits. Figures 8(a) and (b), respectively, show the capacitive impedance Y and the capacitive admittance Δ, either of which may be used to replace the two parallel transmission circuits in the zero-sequence network. In these equivalent circuits, the coupling between the zero-sequence network and the positive- and negative-sequence networks because of dissymmetry is neglected. If the conductors of one circuit are indicated by a, b, c, and of the other circuit by A, B, C (a and A, b and B, c and C being of the same phase), x_{aa0} and x_{AA0} in Fig. 8(a) are the zero-sequence capacitive self-reactances of circuits a, b, c, and A, B, C, respectively, each with the other circuit open, and X_{m00} is the mutual capacitive reactance between the two circuits. In Fig. 8(b), b_{aa0} and b_{AA0} are the total zero-sequence capacitive susceptance of the two circuits, each obtained with the other circuit grounded, and b_{aA0} is the transfer zero-sequence capacitive susceptance between the two circuits. Figures 8(a) and (b) are the same as Figs. 7(b) and (a), respectively, of Chapter VI with different notation; in the former a and A are the two parallel three-phase circuit, in the latter A and B.

Mutual Capacitive Impedances between Parallel Circuits — Two Circuits without Ground Wires. With charging currents I_a, I_b, and I_c flowing in circuit a, b, c, and circuit A, B, C open, there will be voltages to ground on conductors A, B, C of the open circuit but, on the assumption of equal potential along the conductors and no leakance, there can be no charging currents. With no charging currents in circuit A, B, C, there can be no charges if sinusoidal quantities only are considered.

The potentials of conductors A, B, and C due to the charges Q_a, Q_b, and Q_c on the conductors of the closed circuit and the equal and opposite charges on their images in absolute units from [6] are

$$\begin{aligned} V_A &= Q_aP_{aA} + Q_bP_{bA} + Q_cP_{cA} \\ V_B &= Q_aP_{aB} + Q_bP_{bB} + Q_cP_{cB} \\ V_C &= Q_aP_{aC} + Q_bP_{bC} + Q_cP_{cC} \end{aligned} \qquad [65]$$

where $P_{aA} = 2 \log_e S_{aA}/s_{aA}$, $P_{bA} = 2 \log_e S_{bA}/s_{bA}$, etc., s and S representing distances between the conductors of the two circuits and between the conductors of one circuit and the images of the other circuit, respectively, indicated by the subscripts.

Replacing Q_a, Q_b, and Q_c in [65] by their positive-, negative-, and zero-sequence components of charge and resolving V_A, V_B, and V_C into their symmetrical components of voltage, the following equations in absolute units are obtained:

$$\begin{aligned} V_{A1} &= Q_{a1}P_{m11} + Q_{a2}P_{m12} + Q_{a0}P_{m10} \\ V_{A2} &= Q_{a1}P_{m21} + Q_{a2}P_{m22} + Q_{a0}P_{m20} \\ V_{A0} &= Q_{a1}P_{m01} + Q_{a2}P_{m02} + Q_{a0}P_{m00} \end{aligned} \qquad [66]$$

where

$$\begin{aligned} P_{m11} = \tfrac{1}{3}[&(P_{Aa} + P_{Bb} + P_{Cc}) + (a^2P_{Ab} + aP_{Ba}) \\ &+ (a^2P_{Bc} + aP_{Cb}) + (a^2P_{Ca} + aP_{Ac})] \end{aligned}$$

$$\begin{aligned} P_{m22} = \tfrac{1}{3}[&(P_{Aa} + P_{Bb} + P_{Cc}) + (aP_{Ab} + a^2P_{Ba}) \\ &+ (aP_{Bc} + a^2P_{Cb}) + (aP_{Ca} + a^2P_{Ac})] \end{aligned}$$

$$
\begin{aligned}
P_{m00} &= \tfrac{1}{3}[(P_{Aa} + P_{Bb} + P_{Cc}) + (P_{Ab} + P_{Ba}) + (P_{Bc} + P_{Cb}) \\
&\qquad + (P_{Ca} + P_{Ac})] \\
P_{m12} &= \tfrac{1}{3}[(P_{Aa} + a^2 P_{Bb} + aP_{Cc}) + a(P_{Ab} + P_{Ba}) + (P_{Bc} + P_{Cb}) \\
&\qquad + a^2(P_{Ca} + P_{Ac})] \\
P_{m21} &= \tfrac{1}{3}[(P_{Aa} + aP_{Bb} + a^2 P_{Cc}) + a^2(P_{Ab} + P_{Ba}) \\
&\qquad + (P_{Bc} + P_{Cb}) + a(P_{Ca} + P_{Ac})] \\
P_{m10} &= \tfrac{1}{3}[(P_{Aa} + aP_{Bb} + a^2 P_{Cc}) + (P_{Ab} + aP_{Ba}) \\
&\qquad + (aP_{Bc} + a^2 P_{Cb}) + (a^2 P_{Ca} + P_{Ac})] \\
P_{m02} &= \tfrac{1}{3}[(P_{Aa} + aP_{Bb} + a^2 P_{Cc}) + (aP_{Ab} + P_{Ba}) \\
&\qquad + (a^2 P_{Bc} + aP_{Cb}) + (P_{Ca} + a^2 P_{Ac})] \\
P_{m20} &= \tfrac{1}{3}[(P_{Aa} + a^2 P_{Bb} + aP_{Cc}) + (P_{Ab} + a^2 P_{Ba}) \\
&\qquad + (a^2 P_{Bc} + aP_{Cb}) + (aP_{Ca} + P_{Ac})] \\
P_{m01} &= \tfrac{1}{3}[(P_{Aa} + a^2 P_{Bb} + aP_{Cc}) + (a^2 P_{Ab} + P_{Ba}) \\
&\qquad + (aP_{Bc} + a^2 P_{Cb}) + (P_{Ca} + aP_{Ac})]
\end{aligned}
\qquad [67]
$$

P_{m11}, P_{m22}, and P_{m00} are the positive-, negative-, and zero-sequence mutual potential coefficients between the two parallel circuits. P_{m12}, P_{m21}, P_{m10}, P_{m01}, P_{m20}, and P_{m02} are mutual potential coefficients between the sequence networks. For example, V_{A0} in [66] is the sum of three terms: $Q_{a1}P_{m01}$, $Q_{a2}P_{m02}$, and $Q_{a0}P_{m00}$. These three components of zero-sequence voltage on the open circuit are produced by Q_{a1}, Q_{a2}, and Q_{a0}, respectively, the positive-, negative-, and zero-sequence charges on the closed circuit.

If all conductor potential coefficients P_{Aa}, $P_{Ab} \cdots P_{Bc}$, etc., in [67] could be assumed equal, all mutual potential coefficients except P_{m00} would be zero. In high-voltage transmission circuits, where parallel circuits are on opposite sides of the center lines of double circuit towers or on different towers on the same right-of-way, the conductor potential coefficients are of the same order of magnitude and, because of the multipliers 1, a, and a^2, the mutual potential coefficients other than P_{m00} are relatively small. From [67], it is also apparent that except for P_{m00}, the mutual potential coefficients depend upon the relative arrangements of the phases in the two circuits.

Expressed in daraf-miles, the zero-sequence mutual potential coefficient is

$$P_{m00} = \frac{10^6 \times 3}{0.03883} \log_{10} \frac{S_{\overline{aA}}}{s_{\overline{aA}}} \qquad [68]$$

The zero-sequence mutual capacitive impedance Z_{m00} for ℓ miles of line in ohms is

$$Z_{m00} = -jx_{m00} = -j\frac{P_{m00}}{2\pi f\ell} = -j\frac{0.205 \times 10^6}{\ell} \times \left(\frac{60}{f}\right) \log_{10} \frac{S_{\overline{aA}}}{s_{\overline{aA}}} \quad [69]$$

where

$$S_{\overline{aA}} = \sqrt[9]{S_{aA}S_{bA}S_{cA}S_{aB}S_{bB}S_{cB}S_{aC}S_{bC}S_{cC}}$$

= ninth root of the product of the nine distances between the conductors of one circuit and the images of the other circuit

$$s_{\overline{aA}} = \sqrt[9]{s_{aA}s_{bA}s_{cA}s_{aB}s_{bB}s_{cB}s_{aC}s_{bC}s_{cC}}$$

= ninth root of the product of the nine distances between the conductors of the two circuits

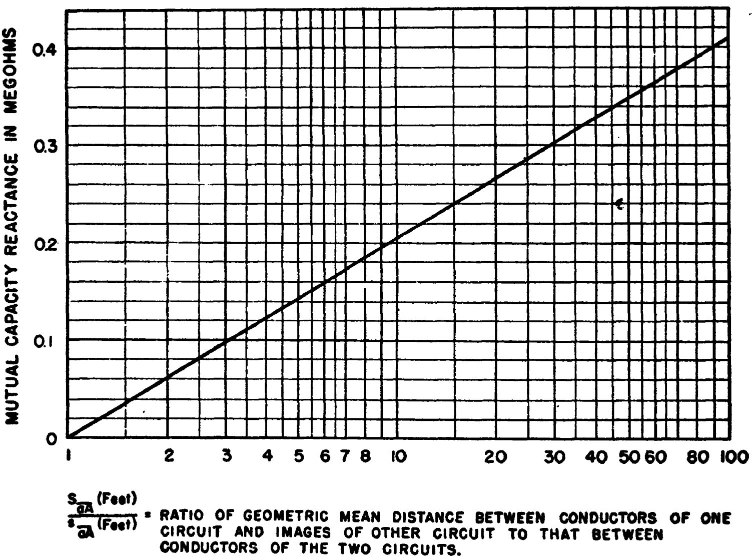


FIG. 9. 60-cycle zero-sequence mutual capacitive reactance between two parallel three-phase circuits without ground wires, calculated from [69] for one mile of circuit.

Figure 9 gives the 60-cycle zero-sequence mutual capacitive reactance x_{m00} between two parallel three-phase circuits without ground wires for one mile of line, plotted from [69] with the ratio $S_{\overline{aA}}/s_{\overline{aA}}$ as abscissa and x_{m00} as ordinate. The value read from Fig. 9 is to be divided by ℓ for lines ℓ miles in length and the numerical value of x_{m00} then inserted in the equivalent circuit of Fig. 8(*a*).

Two Parallel Circuits with Ground Wires. The zero-sequence mutual impedances Z_{m-w} and Z_{m-wv} between two parallel circuits with one ground wire w and two ground wires w and v, respectively, are given by the following equations:

$$Z_{m-w} = Z_{m00} + \Delta Z_{m-w} = -j(x_{m00} + \Delta x_{m-w})$$
$$= -j\left(\frac{P_{m00} + \Delta P_{m-w}}{2\pi f\ell}\right) \qquad [70]$$

$$Z_{m-wv} = Z_{m00} + \Delta Z_{m-wv} = -j(x_{m00} + \Delta x_{m-wv})$$
$$= -j\left(\frac{P_{m00} + \Delta P_{m-wv}}{2\pi f\ell}\right) \qquad [71]$$

where x_{m00} is the mutual impedance without ground wires given by [69] and Fig. 9, and Δx_{m-w} and Δx_{m-wv} are the corrections to the mutual impedance without ground wire for one and two ground wires, respectively.

The corrections ΔP_{m-w} and ΔP_{m-wv} to the zero-sequence mutual potential coefficient P_{m00} because of one or two ground wires can be determined by assuming only zero-sequence charges on the conductors a, b, c of the closed circuit and calculating the change in zero-sequence voltage of the open circuit resulting from the presence of the ground wire or wires, the ratio of this change ΔV_{A0} to the zero-sequence charge Q_{a0} giving the correction to be applied to P_{m00}. A similar procedure was used to determine the correction to be applied with two ground wires to the zero-sequence potential coefficient without ground wires. Following this procedure, it is found that the corrections Δx_{m-w} and Δx_{m-wv} to be inserted in [70] and [71] can be expressed in terms of the corrections to the zero-sequence capacitive reactance of each of the circuits alone with one ground wire w or v defined by [61] and given by Fig. 7(*a*).

One Ground Wire. For the general case of one ground wire w unsymmetrical with respect to the two circuits,

$$\Delta x_{m-w} = \frac{\Delta P_{m-w}}{2\pi f\ell} = \sqrt{(\Delta x_{00-w})_a(\Delta x_{00-w})_A} \qquad [72]$$

where Δx_{00-w} is given by [61] and Fig. 7(*a*) and subscripts a and A outside the parentheses refer to circuits abc and ABC, respectively, and Δx_{m-w} has the sign of Δx_{00-w}, which is negative.

If the ground wire is symmetrically spaced with respect to the two circuits, $(\Delta x_{00-w})_a = (\Delta x_{00-w})_A$, and

$$\Delta x_{m-w} = \Delta x_{00-w} \qquad [73]$$

Two Ground Wires. For the general case of two ground wires w and v, unsymmetrical with respect to the two circuits,

$$\Delta x_{m-wv} = \frac{\sqrt{(\Delta x_{00-w})_a(\Delta x_{00-w})_A} + \sqrt{(\Delta x_{00-v})_a(\Delta x_{00-v})_A} - \sqrt{K_wK_v} \times [\sqrt{(\Delta x_{00-w})_a(\Delta x_{00-v})_A} + \sqrt{(\Delta x_{00-w})_A(\Delta x_{00-v})_a}]}{1 - K_wK_v} \quad [74]$$

where Δx_{00-w} and Δx_{00-v} are obtained separately from Fig. 7(a) for each of the circuits a, b, c and A, B, C as indicated by subscripts a and A, respectively, outside the parentheses. K_w and K_v are read from Fig. 7(b), and the sign of the third term in the numerator of [74] is opposite to that of the first two terms, which have the sign of Δx_{00-w} and Δx_{00-v} and are negative.

For two identical circuits on the same towers and ground wires symmetrical about the center line of the towers, as in Fig. 13 of Chapter XI, $(\Delta x_{00-w})_A = (\Delta x_{00-v})_a$, $(\Delta x_{00-v})_A = (\Delta x_{00-w})_a$, and $K_v = K_w$, giving

$$\Delta x_{m-wv} = \frac{2\sqrt{(\Delta x_{00-w})(\Delta x_{00-v})} - K_w(\Delta x_{00-w} + \Delta x_{00-v})}{1 - K_w^2} \quad [75]$$

where x_{00-w} and x_{00-v} are read from Fig. 7(a) for circuit a, b, c, K_w from 7(b), and the sign of the first term is the same as that of Δx_{00-w} and Δx_{00-v}, which are negative.

With *more than two ground wires*, the ground wires can be divided into two groups and the effect of each group determined as though acting alone; then, using [74] or [75], the change in x_{m00} resulting from both ground wire groups can be obtained.

The *capacitive susceptances of two parallel circuits without ground wire for use in Fig.* 8(b) can be obtained from the equivalent impedance Y, or directly as follows: Comparing b_{00} in [46] for the three-conductor circuit with b in 4(a), for a single conductor, it will be noted that b_{00} is equivalent to one-third the capacitive susceptance of a single conductor with a geometric mean radius of $\sqrt[3]{r(s_{\overline{ab}})^2}$ and height above ground equal to one-half the geometric mean distance between conductors and images. Equation [46] was determined from the reciprocal of the capacitive reactance; it could as well have been developed on the assumption that the zero-sequence capacitive susceptance of an unsymmetrical circuit is the same as that of a symmetrical circuit with all distances between conductors and between conductors and images equal to the corresponding geometric mean distances. Under this assumption, there is no mutual coupling between the sequence networks, and the *zero-sequence admittances* in the equivalent circuit of Fig. 8(*b*) can be obtained by analogy from the two conductor equivalent circuit of Fig. 2 and equations [14] if the three conductors of each circuit are replaced by one equivalent conductor with geometric mean radius $(\sqrt[3]{r(s_{\bar{a}\bar{b}})^2}$, distances between conductors and between conductors and images being replaced by corresponding geometric mean distances. Zero-sequence admittances, which are admittances per phase, are one-third those calculated by using two such equivalent conductors. Multiplying equation [14] by $\frac{1}{3}(2\pi f)$ and replacing radii and spacings for the two parallel conductors of Fig. 2(*a*) by the geometric mean values calculated from the two parallel circuits, b_{aa0}, b_{AA0}, and b_{aA0} for insertion in Fig. 8(*b*) can be obtained as illustrated in the following problem.

Problem 5. Determine the 60-cycle zero-sequence equivalent capacitive impedance Y's and susceptance Δ's for 50 miles of the two parallel three-phase transmission circuits shown in Fig. 13 of Chapter XI, (*a*) with ground wires not installed, (*b*) with two ground wires as indicated.

Solution. The zero-sequence self-capacitive reactances x_{aa0} and x_{AA0} to be substituted in Fig. 8(*a*) have already been determined in Problem 4 for either of the two circuits with the other open.

(*a*) $Z_{aa0} = Z_{AA0} = Z_{00}$ (Problem 4) $= -j\ (0.349 \times 10^6)/50$ ohms $= -j6980$ ohms. $S_{a\bar{A}}$ and $s_{a\bar{A}}$ defined under [69] are 113 feet and 28.3 feet, respectively.

From [69] or Fig. 9, with $S_{a\bar{A}}/s_{a\bar{A}} = 3.99$, the zero-sequence mutual impedance between the two circuits is

$$Z_{m00} = -j\frac{0.123}{50} \times 10^6 \text{ ohms} = -j2460 \text{ ohms}$$

The capacitive impedance Y determined by substituting $Z_{aa0} = Z_{AA0}$ and Z_{m00} in Fig. 8(*a*) is shown in Fig. 10(*a*). The equivalent admittance Δ calculated from Fig. 10(*a*) by using the reciprocals of equations [40] of Chapter I is shown in Fig. 10(*b*).

The capacitive admittance equivalent circuit of Fig. 8(*b*) will be calculated directly as outlined above. Referring to Problem 4, the geometric mean values to be substituted in [14], are

$$r_a = r_b = \sqrt[3]{(s_{\bar{a}\bar{b}})^2 r} = \sqrt[3]{(16.04)^2 \times \frac{0.953}{24}} = 2.17 \text{ feet}$$

$$S_{aa} = S_{bb} = 2H = 109.8 \text{ feet};\ \log_{10}\frac{S_{aa}}{r_a} = 1.704$$

$$\frac{S_{ab}}{s_{ab}} = \frac{S_{a\bar{A}}}{s_{a\bar{A}}} = 3.99;\ \log\frac{S_{ab}}{s_{ab}} = 0.601$$

From [14] multiplied by $2\pi f/3$ and 50 (for 50 miles of line),

$$b_{aa0} = b_{AA0} = \frac{4.88 \times 10^{-6}}{2.542}(1.704) \times 50 = 163.6 \times 10^{-6} \text{ mhos}$$

$$b_{aA0} = \frac{4.88 \times 10^{-6}}{2.542}(0.601) \times 50 = 57.6 \times 10^{-6} \text{ mhos}$$

Substituting these values in Fig. 8(*b*), the circuit of Fig. 10(*b*) is obtained.

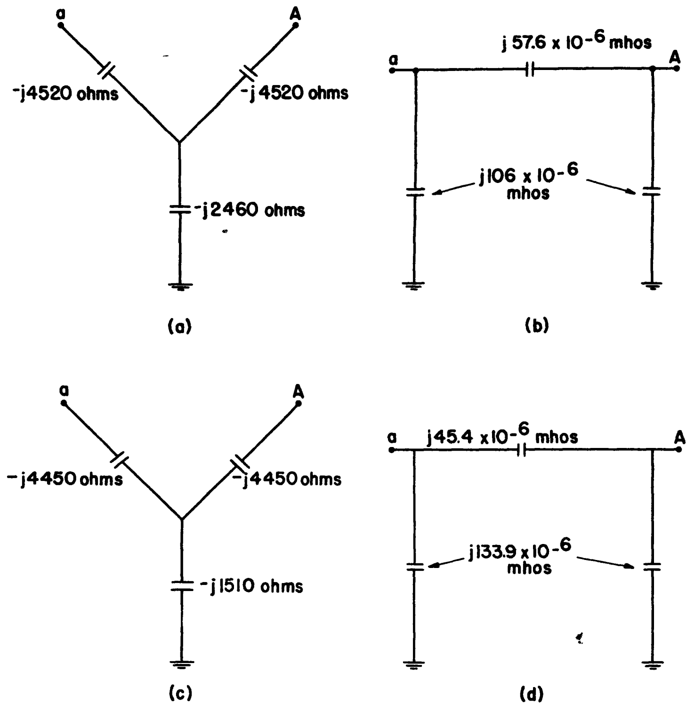


FIG. 10. Capacitive impedance Y's and admittance Δ's developed in Problem 5 for 50 miles of the two three-phase, 60-cycle, parallel transmission lines shown in Fig. 13 of Chapter XI. (*a*) and (*b*) are without ground wire. (*c*) and (*d*) are with two ground wires.

(*b*) $Z_{aa0} = Z_{AA0} = Z_{00-wv}$ (Problem 4) $= -j(0.298 \times 10^6)/50 = -j5960$ ohms.

$$K_w = 0.233$$

$$\Delta x_{00-w} = -\frac{0.040 \times 10^6}{50} = -800 \text{ ohms}$$

$$\Delta x_{00-v} = -\frac{0.022 \times 10^6}{50} = -440 \text{ ohms}$$

From [75],

$$\Delta x_{m-wv} = \frac{-2\sqrt{800 \times 440} - 0.233(-1240)}{1 - (0.233)^2} = -950$$

From [71],

$$Z_{m-wv} = -j(x_{m00} + \Delta x_{m-wv}) = -j(2460 - 950) = -j1510$$

Replacing both Z_{aa0} and Z_{AA0} in Fig. 8(*a*) by Z_{00-wv}, and Z_{m00} by Z_{m-wv}, the equivalent capacitive impedance circuit of Fig. 10(*c*) is obtained and from that the equivalent admittance circuit of Fig. 10(*d*).

Two Parallel Three-Phase Circuits Operated at Different Voltages. It is shown in Chapter XI that, with different base voltages in two parallel circuits, the self-impedance of each circuit is based on system base kva and the base voltage of the circuit, while mutual impedance is based on system base kva and the square root of the product of the base voltages in the two circuits. This applies to capacitive impedances as well as inductive impedances and to admittances as well as impedances.

SINGLE-PHASE AND TWO-PHASE CIRCUITS

When positive- and zero-sequence symmetrical components (defined in Chapter IX) are used in the analyses of single-phase and two-phase systems, the positive- and zero-sequence self-capacitances and the mutual capacitance between the positive- and zero-sequence networks are given by [21]–[23] in terms of Maxwell's coefficients. For the single-phase circuit without ground wires, Maxwell's coefficients in [21]–[23] are defined in [14]; for the single-phase circuit with one ground wire, or for the three-wire single-phase or two-phase circuit with ground neutral conductor, Maxwell's coefficients in [21]–[23] are defined in [31].

The potential coefficient method of determining the sequence capacitive impedances and from them the capacitive admittances, used for the three-phase circuit, can also be applied to single-phase and two-phase circuits treated as two-vector systems. In Chapter IX, the relations between the positive- and zero-sequence components of currents and voltages are given by [21] in terms of sequence self- and mutual impedances and in [52] in terms of sequence self- and mutual admittances; the sequence admittances are given in terms of the sequence impedance in [54]. To apply these equations to the calculation of sequence capacitances, the sequence potential coefficients are determined and from them the sequence capacitive impedances [$Z = -jx = -j(P/2\pi f\ell)$]; if the capacitive admittances are required, they are obtained by substituting the capacitive impedances in [54] of Chapter IX for a rigorous solution based on initial assumptions. As for the three-phase system, it can be shown that the positive- and zero-sequence capacitive susceptances are the reciprocals of the positive- and zero-sequence capacitive self-reactances to close approximation.

$$b_{11} = \frac{1}{x_{11}} \quad \text{and} \quad b_{00} = \frac{1}{x_{00}} \text{ (approximately)}$$

Single-Phase Circuit without Ground Wires. Expressed in terms of conductor potential coefficients, the sequence potential coefficients

P_{11}, P_{00}, and $P_{10} = P_{01}$ in 1/(statfarads per centimeter) are

$$\begin{aligned} P_{11} &= \tfrac{1}{2}(P_{aa} + P_{bb} - 2P_{ab}) \\ P_{00} &= \tfrac{1}{2}(P_{aa} + P_{bb} + 2P_{ab}) \\ P_{10} &= P_{01} = \tfrac{1}{2}(P_{aa} - P_{bb}) \end{aligned} \qquad [76]$$

where the factor to convert to daraf-miles is $10^6/0.03883$ if $\log_{10}$ instead of $2 \log_e$ is used to determine the conductor potential coefficients.

Neglecting the coupling $P_{10} = P_{01}$ between the sequence networks and the presence of the earth, C_{11} is given by [26]. The positive-sequence capacitive susceptance b_{11}, with the presence of the earth neglected, obtained by multiplying C_{11} in [26] by $2\pi f$ is the same as b_{11} for the three-phase circuit given by [45] if $s_{\overline{ab}}$ is replaced by s_{ab}. The positive-sequence capacitive susceptance curve of Fig. 6 will give the positive-sequence capacitive susceptance for the single-phase circuit at 60 cycles if spacing between the two conductors replaces the geometric mean spacing between the three conductors.

Based on the assumption that the capacitive impedance coupling between the sequence networks can be neglected, $C_{00} = 1/P_{00}$, and the zero-sequence capacitive susceptance in mhos per mile is

$$b_{00} = 2\pi f C_{00} = \frac{1}{x_{00}} = \left(\frac{f}{60}\right)\frac{7.32 \times 10^{-6}}{\log_{10}\dfrac{2H}{\sqrt{r s_{ab}}}} \qquad [77]$$

where

$H = \frac{1}{2}\sqrt[4]{S_{aa}S_{bb}S_{ab}^2}$ = average height above ground of conductors, approximately

Single-Phase Circuit with One Ground Wire, or Three-Wire Single-Phase or Two-Phase Circuit with Grounded Neutral Conductor. Let w indicate the ground wire or ground neutral conductor, Z_{11-w} and Z_{00-w} the positive- and zero-sequence self-capacitive impedances, and $Z_{10-w} = Z_{01-w}$ the mutual capacitive impedances between the positive- and zero-sequence networks. Then,

$$\begin{aligned} Z_{11-w} &= -jx_{11-w} = -j(x_{11} + \Delta x_{11-w}) = -j\frac{P_{11} + \Delta P_{11-w}}{2\pi f l} \\ Z_{00-w} &= -jx_{00-w} = -j(x_{00} + \Delta x_{00-w}) = -j\frac{P_{00} + \Delta P_{00-w}}{2\pi f l} \\ Z_{01-w} &= -jx_{01-w} = -j(x_{01} + \Delta x_{01-w}) = -j\frac{P_{01} + \Delta P_{01-w}}{2\pi f l} \end{aligned} \qquad [78]$$

where the sequence impedances are in ohms if the sequence potential coefficients P_{11}, P_{00}, and $P_{10} = P_{01}$ given by [76] and their corrections are expressed in daraf-miles.

The corrections are

$$\begin{aligned} \Delta P_{11-w} &= -\frac{1}{2P_{ww}}(P_{aw} - P_{bw})^2 \\ \Delta P_{00-w} &= -\frac{1}{2P_{ww}}(P_{aw} + P_{bw})^2 = -\frac{2(P_{\overline{aw}})^2}{P_{ww}} \\ \Delta P_{01-w} &= \Delta P_{10-w} = \frac{1}{2P_{ww}}(P_{aw}^2 - P_{bw}^2) \end{aligned} \qquad [79]$$

where

$$P_{\overline{aw}} = \tfrac{1}{2}(P_{aw} + P_{bw})$$

For a ground wire symmetrical with respect to the two conductors, $P_{aw} = P_{bw}$ and $\Delta P_{11-w} = \Delta P_{01-w} = \Delta P_{10-w} = 0$. For an unsymmetrical ground wire, P_{aw} and P_{bw} are of the same order of magnitude and all corrections except ΔP_{00-w} are small.

In ohms,

$$\Delta x_{00-w} = \frac{\Delta P_{00-w}}{2\pi f l} = -\frac{0.1366 \times 10^6}{l}\left(\frac{60}{f}\right)\frac{\left(\log_{10}\dfrac{S_{\overline{aw}}}{s_{\overline{aw}}}\right)^2}{\log_{10}\dfrac{2h_w}{r_w}} \qquad [80]$$

where

$$S_{\overline{aw}} = \sqrt{S_{aw}S_{bw}} \quad \text{and} \quad s_{aw} = \sqrt{s_{aw}s_{bw}}$$

h_w and r_w = height and radius of ground wire, respectively

Comparing [80] with [61], Δx_{00-w} for the single-phase or two-phase circuit is two-thirds of Δx_{00-w} for the three-phase circuit and can be obtained from Fig. 7(*a*) by taking two-thirds of the reading corresponding to $S_{\overline{aw}}/s_{\overline{aw}}$ and h_w(feet)/d_w(inches).

For two ground wires w and v, the correction Δx_{00-wv} is given by [62] for unsymmetrical ground wires and by [63] for symmetrical ground wires, where Δx_{00-w} is given by [80] and two-thirds the value read from Fig. 7(*a*) for the ground wire w; Δx_{00-v} is the corresponding value for ground wire v, and K_w and K_v are read from Fig. 7(*b*).

Two Parallel Single-Phase Circuits. Let the conductors of one circuit be a and b and of the other circuit A and B, a and A, and b and B being of the same phase. Proceeding as for two parallel three-phase circuits, without ground wire the equations for the single-phase circuit corresponding to [66] for the three-phase circuit are

$$\begin{aligned} V_{A1} &= Q_{a1}P_{m11} + Q_{a0}P_{m10} \\ V_{A0} &= Q_{a1}P_{m01} + Q_{a0}P_{m00} \end{aligned} \qquad [81]$$

where

$$\begin{aligned} P_{m11} &= \tfrac{1}{2}[(P_{Aa} + P_{Bb}) - (P_{Ab} + P_{Ba})] \\ P_{m00} &= \tfrac{1}{2}[(P_{Aa} + P_{Bb}) + (P_{Ab} + P_{Ba})] \\ P_{m10} &= \tfrac{1}{2}[(P_{Aa} - P_{Bb}) + (P_{Ab} - P_{Ba})] \\ P_{m01} &= \tfrac{1}{2}[P_{Aa} - P_{Bb}) - (P_{Ab} - P_{Ba})] \end{aligned} \qquad [82]$$

P_{m11}, P_{m10}, and P_{m01} given by [82] depend upon the relative arrangement of the phases in the two circuits. For any given case, they can be calculated.

$$Z_{m00} = -jx_{m00} = -j\frac{P_{m00}}{2\pi f\ell} = -j\frac{0.1366}{\ell}\left(\frac{60}{f}\right)\log_{10}\frac{S_{\overline{aA}}}{s_{\overline{aA}}} \qquad [83]$$

where

$S_{\overline{aA}} = \sqrt[4]{S_{aA}S_{aB}S_{bA}S_{bB}}$ = geometric mean spacing between the conductor of one circuit and the images of the other circuit

$s_{\overline{aA}} = \sqrt[4]{s_{aA}s_{aB}s_{bA}s_{bB}}$ = geometric mean spacing between the conductors of the two circuits

Comparing [83] with [69], x_{m00} for two parallel single-phase circuits is two-thirds of x_{m00} for two parallel three-phase circuits and can be obtained by taking two-thirds of x_{m00} read from Fig. 9, corresponding to the ratio of the geometric mean spacings $S_{\overline{aA}}$ and $s_{\overline{aA}}$ of the two parallel single-phase circuits.

The equivalent capacitive impedance Y *for two parallel single-phase circuits with ground wires or two parallel three-wire single-phase or two-phase circuits with grounded neutral conductors* can be determined by analogy from two parallel three-phase circuits with ground wires. The correction to the mutual impedance Z_{m00} between two parallel single-phase circuits defined in [83] is given by [72] for one ground wire in terms of $(\Delta x_{00-w})_a$ and $(\Delta x_{00-w})_A$ which can be obtained from [80] for circuit a, b and A, B, respectively. For two parallel single-phase circuits with two ground wires or for two parallel three-wire single-phase or two-phase circuits, each with a grounded neutral conductor, equations [74] or [75] give the correction to be applied to Z_{m00}, defined in [83]. In these equations $(\Delta x_{00-w})_a$, $(\Delta x_{00-w})_A$, $(\Delta x_{00-v})_a$, and

$(\Delta x_{00-v})_A$ refer to the correction to the zero-sequence self-capacitive reactance of a single circuit with one ground wire calculated from (80) for each of the circuits a, b and A, B with each of the ground wires w and v.

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APPENDIX A

DETERMINANTS

By the use of determinants the solution of linear simultaneous equations can be greatly simplified. It is often convenient to express the unknowns in terms of the ratio of two determinants which can be evaluated for specific problems when numerical values are given.

Expansion of Determinants. The simplest determinant consists of two rows and two columns. Thus,

$$\Delta_2 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 \qquad \text{[A-1]}$$

This determinant is merely a symbolic way of writing the difference of the diagonal products, the diagonal product starting from the upper left-hand corner being the positive one.

A determinant consists of the same number of rows as columns. The value of a determinant is not altered by changing the columns into rows and the rows into columns. Thus,

$$\Delta_2 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

All laws true for the rows of a determinant, therefore, hold for the columns, and vice versa.

A third and n^{th} order determinant, respectively, are written:

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_n = \begin{vmatrix} a_1 & b_1 & c_1 & \cdots & n_1 \\ a_2 & b_2 & c_2 & \cdots & n_2 \\ a_3 & b_3 & c_3 & \cdots & n_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_n & b_n & c_n & \cdots & n_n \end{vmatrix}$$

A determinant is expanded by adding algebraically all the products formed by taking one element from each row, with no two from the same column. Various rules* which have been formulated for evalu-

* *Higher Mathematics for Students of Chemistry and Physics*, by J. W. Mellor, Longmans, Green and Company, 1926.

ating determinants can be used to advantage when the elements of the determinants are numerical values. When the elements are letters to which numerical values will be assigned later, the determinant can be simplified by expansion into the products of elements and minors of the next lower order. The minor associated with any element is the determinant remaining when both the row and the column in which the element appears are omitted. If the elements are taken in order from the first column (or the first row) the signs of the terms will be alternately positive and negative. Thus, for the third-order determinants,

$$\Delta_3 = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$$

A minor with proper sign is called a cofactor.

Expanding the second-order determinants,

$$\begin{aligned} \Delta_3 &= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) \\ &= a_1b_2c_3 - a_1b_3c_2 - a_2b_1c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 \end{aligned}$$

A particular minor is indicated by reference to its element. Given the fourth-order determinant,

$$\Delta_4 = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} \qquad \text{[A–2]}$$

The minor of the element c_3 is

$$\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_4 & b_4 & d_4 \end{vmatrix}$$

The sign associated with the product of c_3 and its minor can be determined from the number of interchanges of rows and columns required to bring c_3 into the first row and the first column, the order of the constituents of the minor remaining unchanged. If the number of changes is even, the sign is positive; if odd, negative. Interchanging the third row of the given determinant in [A–2], first with the second row then with the first row, and afterwards interchanging the third column first with the second column and then with the first column, will make a total of four interchanges to place c_3 in the first row and the first column; therefore the sign of the product of c_3 and its minor is positive. In [A–3] the interchanges to bring c_3 into the first row and first column have been made.

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = \begin{vmatrix} c_3 & a_3 & b_3 & d_3 \\ c_1 & a_1 & b_1 & d_1 \\ c_2 & a_2 & b_2 & d_2 \\ c_4 & a_4 & b_4 & d_4 \end{vmatrix} \qquad \text{[A-3]}$$

Rules for Solving Linear Simultaneous Equations. 1. Write the equations with the unknowns on the left of the equality signs and the given quantities on the right, the unknowns being in the same order in all equations. Thus, for three simultaneous equations,

$$\begin{aligned} a_1x + b_1y + c_1z &= k_1 \\ a_2x + b_2y + c_2z &= k_2 \\ a_3x + b_3y + c_3z &= k_3 \end{aligned} \qquad \text{[A-4]}$$

2. Express each unknown as the ratio of two determinants in the form of a fraction. The denominator of the fractions will be the determinant formed from the coefficients of the unknowns. The numerators will be the determinants obtained from the denominator by replacing the column consisting of the coefficients of the unknown by the quantities on the right of the equality signs. For the three simultaneous equations of [A-4] the solution is

$$x = \frac{\begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & c_3 \end{vmatrix}}{\Delta} \qquad y = \frac{\begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & c_3 \end{vmatrix}}{\Delta}$$

$$z = \frac{\begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}}{\Delta} \qquad \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

By expanding the determinants, the value of any of the unknowns can be obtained. For the three simultaneous equations of [A-4] the solution is given in convenient form for numerical calculations by the following equations:

$$\begin{aligned} x &= \frac{k_1}{\Delta}(b_2c_3 - b_3c_2) - \frac{k_2}{\Delta}(b_1c_3 - b_3c_1) + \frac{k_3}{\Delta}(b_1c_2 - b_2c_1) \\ y &= -\frac{k_1}{\Delta}(a_2c_3 - a_3c_2) + \frac{k_2}{\Delta}(a_1c_3 - a_3c_1) - \frac{k_3}{\Delta}(a_1c_2 - a_2c_1) \qquad \text{[A-5]} \\ z &= \frac{k_1}{\Delta}(a_2b_3 - a_3b_2) - \frac{k_2}{\Delta}(a_1b_3 - a_3b_1) + \frac{k_3}{\Delta}(a_1b_2 - a_2b_1) \end{aligned}$$

where

$$\Delta = a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$$

Two simultaneous equations are written

$$a_1x + b_1y = k_1$$

$$a_2x + b_2y = k_2$$

The solution is

$$x = \frac{\begin{vmatrix} k_1 & b_1 \\ k_2 & b_2 \end{vmatrix}}{\Delta} = \frac{k_1b_2 - k_2b_1}{a_1b_2 - a_2b_1}$$

$$y = \frac{\begin{vmatrix} a_1 & k_1 \\ a_2 & k_2 \end{vmatrix}}{\Delta} = \frac{k_2a_1 - k_1a_2}{a_1b_2 - a_2b_1} \qquad \text{[A-6]}$$

where

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

APPENDIX B

TABLES AND CHARTS FOR OVERHEAD TRANSMISSION CIRCUITS

Characteristics of Copper, Aluminum, and Copperweld Conductors

Tables I–V, compiled by Mr. G. K. Carter, give conductor diameters, d-c resistances at 25°C, and — for frequencies of 25, 50, and 60 cycles — a-c resistances at 25°C, internal reactances, and conductor equivalent geometric mean radii.

The copper tables are based[1] on an average (commercial) conductivity of 97.3% of that of annealed copper; for wires of any other conductivity, the resistance will be in inverse ratio to the per cent conductivity. Skin effect and the effect of spiraling are taken into account. The temperature coefficients are given at 25°C; indicating this coefficient by k_{25}, the resistance R_t at temperature t is

$$R_t = R_{25}[1 + k_{25}(t - 25)] \qquad \text{[B–1]}$$

The equivalent geometric mean radius[2] is related to the conductor radius and the conductor internal reactance by the expression

$$x_i = 0.0046565 f \log_{10} \frac{r}{gmr}, \qquad \text{[B–2]}$$

where

x_i = internal reactance, in ohms per mile
f = frequency, in cycles per second
gmr = geometric mean radius
r = conductor radius, in same unit as gmr

From [B–2], x_i can be calculated from gmr or the gmr from x_i, depending upon which is the more readily obtained. In cases where the positive-sequence reactance of magnetic or partly magnetic conductors at given spacings and a specified current are obtained by test, the internal reactance of the conductors can be obtained by subtracting the positive-sequence reactance external to the conductors from the total positive-sequence reactance. (See [4], Chapter XI.)

Table I. For the solid copper conductors in this table, skin effect is negligible at frequencies of 60 cycles or less.

Table II. Resistance for stranded copper conductors is on the same basis as that for solid conductors of the same copper cross section, except that 2% is added to allow for spiraling.[1] To change a conductor of given cross section from a given stranding to the next higher number of strands, the following table may be used:

To change stranding	Multiply *gmr* by	Increase x_i by			Multiply *r* by
		25~	50~	60~	
from 3 to 7	0.976	0.003	0.007	0.008	0.9115
7 to 19	1.056	0.002	0.004	0.005	1.0117
19 to 37	1.017	0.001	0.001	0.002	1.0033
37 to 61	1.007	.....	0.001	0.001	1.0013
61 to 91	1.004	.....	.....	.....	1.0007
91 to 127	1.002	.....	.....	.....	1.0004
127 to 169	1.001	.....	.....	.....	1.0002

Skin effect is calculated on the assumption that the per cent change in resistance or reactance is the same as that for a solid conductor of the same copper cross section[2]; the skin effect ratios were taken from reference 3.

Table III. Characteristics are given for hollow cylindrical copper conductors of specified outside diameter and ratios of inside to outside diameters. Skin effect at 60 cycles and less is negligible. Resistance and reactance are calculated as for the other copper conductors, the *gmr* being obtained as described in reference 2.

Table IV. Characteristics of aluminum cable, steel reinforced (A.C.S.R.) were derived from handbook data[11] and checked against the files of the Aluminum Company of America.

Table V. The Copperweld tables include a wide enough variety of types, so that practically any of the usual copperweld conductors can be found either directly or by interpolation. These tables were supplied by the Copperweld Steel Company. As an approximation, the resistance temperature coefficient for copper, 0.00375, may be applied to this table.

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Various other papers are referred to in the foregoing books and articles.

TABLE I

Solid Copper

Hard Drawn (97.3% Conductivity)

Temperature Coefficient = 0.00375 per °C

A.W.G.	Diameter, Inches	D-c and A-c Resistance, 25°C, Ohms/Mile	Internal Reactance, Ohms/Mile			Equivalent Geometric Mean Radius Feet
			25~	50~	60~	
0000	0.460	0.271	0.013	0.025	0.030	0.0149
000	0.410	0.342				0.0133
00	0.365	0.431				0.0119
0	0.325	0.543				0.0105
1	0.289	0.686				0.0094
2	0.258	0.864				0.0084
3	0.229	1.090				0.0074
4	0.204	1.373				0.0066
5	0.1819	1.733				0.0059
6	0.1620	2.185				0.0053
7	0.1443	2.75				0.0047
8	0.1285	3.47				0.0042
9	0.1144	4.38				0.0037
10	0.1019	5.52				0.0033
11	0.0907	6.96				0.0029
12	0.0808	8.78				0.00262
13	0.0720	11.08				0.00234
14	0.0641	13.97				0.00208
15	0.0571	17.62				0.00185
16	0.0508	22.2				0.00165
17	0.0453	28.0				0.00147
18	0.0403	35.3				0.00131
19	0.0359	44.5				0.00116
20	0.0320	56.1				0.00104
21	0.0285	70.8				0.00092

TABLE II. Stranded Copper. Hard Drawn (97.3% Conductivity). Temperature Coefficient = 0.00375 per °C

A.W.G. or Circular Mils	Strands	Conductor Diameter, Inches	D-c Resistance, 25°C, Ohms/Mile	A-c Resistance, 25°C, Ohms/Mile			Internal Reactance, Ohms/Mile			Equivalent Geometric Mean Radius, Feet		
				25~	50~	60~	25~	50~	60~	25~	50~	60~
2,000,000	91	1.630	0.029	0.031	0.035	0.038	0.013	0.023	0.027	0.0530	0.0539	0.0545
1,750,000	91	1.526	0.033	0.035	0.039	0.041	0.013	0.024	0.028	0.0495	0.0502	0.0506
1,500,000	61	1.411	0.039	0.040	0.044	0.046	0.013	0.025	0.029	0.0456	0.0461	0.0464
1,250,000	61	1.288	0.047	0.048	0.051	0.053	0.013	0.025	0.029	0.0415	0.0418	0.0421
1,000,000	61	1.157	0.059	0.060	0.062	0.064	0.013	0.025	0.030	0.0373	0.0375	0.0376
900,000	61	1.093	0.065	0.066	0.068	0.070	0.013	0.026	0.030	0.0352	0.0354	0.0355
800,000	61	1.031	0.073	0.074	0.076	0.077	0.013	0.026	0.031	0.0332	0.0333	0.0334
750,000	61	0.997	0.078	0.079	0.081	0.082	0.013	0.026	0.031	0.0321	0.0322	0.0323
700,000	61	0.964	0.084	0.084	0.086	0.087	0.013	0.026	0.031	0.0310	0.0311	0.0311
650,000	61	0.929	0.090	0.091	0.092	0.093	0.013	0.026	0.031	0.0299	0.0300	0.0300
600,000	37	0.891	0.098	0.098	0.100	0.101	0.013	0.026	0.032	0.0285	0.0285	0.0286
550,000	37	0.853	0.106	0.107	0.109	0.109	0.013	0.026	0.032	0.0273	0.0273	0.0273
500,000	37	0.813	0.117	0.118	0.119	0.120	0.013	0.027	0.032	0.0260	0.0260	0.0261
450,000	37	0.772	0.130	0.131	0.132	0.133	0.013	0.027	0.032	0.0247	0.0247	0.0247
400,000	19	0.729	0.147	0.147	0.148	0.149	0.014	0.028	0.034	0.0230	0.0230	0.0230
350,000	19	0.679	0.167	0.168	0.169	0.169	0.014	0.028	0.034	0.0214	0.0214	0.0214
300,000	19	0.628	0.195	0.196	0.197	0.197	0.014	0.028	0.034	0.0199	0.0199	0.0199
250,000	19	0.574	0.234	0.234	0.235	0.235	0.014	0.028	0.034	0.0182	0.0182	0.0182
0000	19	0.528	0.276	0.276	0.276	0.276	0.014	0.028	0.034	0.0167	0.0167	0.0167

TABLE II_3 (*Continued*)

A.W.G. or Circular Mils	Strands	Conductor Diameter, Inches	D-c Resistance, 25°C, Ohms/Mile	A-c Resistance, 25°C, Ohms/Mile			Internal Reactance, Ohms/Mile			Equivalent Geometric Mean Radius, Feet		
				25~	50~	60~	25~	50~	60~	25~	50~	60~
0000	7	0.522	0.276	0.276	0.276	0.276	0.016	0.033	0.039	0.0158	0.0158	0.0158
000	7	0.464	0.349	0.349	0.349	0.349	0.016	0.033	0.039	0.0141	0.0141	0.0141
00	7	0.414	0.440	0.440	0.440	0.440	0.016	0.033	0.039	0.0125	0.0125	0.0125
0	7	0.368	0.554	0.554	0.554	0.554	0.016	0.033	0.039	0.0112	0.0112	0.0112
1	7	0.328	0.700	0.700	0.700	0.700	0.016	0.033	0.039	0.0099	0.0099	0.0099
2	7	0.292	0.881	0.881	0.881	0.881	0.016	0.033	0.039	0.0088	0.0088	0.0088
3	7	0.260	1.112	1.112	1.112	1.112	0.016	0.033	0.039	0.0079	0.0079	0.0079
4	7	0.232	1.400	1.400	1.400	1.400	0.016	0.033	0.039	0.0070	0.0070	0.0070
5	7	0.206	1.768	1.768	1.768	1.768	0.016	0.033	0.039	0.0062	0.0062	0.0062
6	7	0.184	2.23	2.23	2.23	2.23	0.016	0.033	0.039	0.0056	0.0056	0.0056
1	3	0.360	0.700	0.700	0.700	0.700	0.020	0.040	0.047	0.0102	0.0102	0.0102
2	3	0.321	0.881	0.881	0.881	0.881	0.020	0.040	0.047	0.0091	0.0091	0.0091
3	3	0.286	1.112	1.112	1.112	1.112	0.020	0.040	0.047	0.0081	0.0081	0.0081
4	3	0.254	1.400	1.400	1.400	1.400	0.020	0.040	0.047	0.0072	0.0072	0.0072
5	3	0.227	1.768	1.768	1.768	1.768	0.020	0.040	0.047	0.0064	0.0064	0.0064
6	3	0.202	2.23	2.23	2.23	2.23	0.020	0.040	0.047	0.0057	0.0057	0.0057
7	3	0.180	2.81	2.81	2.81	2.81	0.020	0.040	0.047	0.0051	0.0051	0.0051
8	3	0.160	3.54	3.54	3.54	3.54	0.020	0.040	0.047	0.0045	0.0045	0.0045

TABLE III

HOLLOW COPPER

Hard Drawn (97.3% Conductivity)

Temperature Coefficient = 0.00375 per °C

Diameter, Inches	$r_2/r_1 = 0.8$					$r_2/r_1 = 0.9$				
	D-c and A-c Resistance, 25°C, Ohms/Mile	Internal Reactance, Ohms/Mile			Equivalent Geometric Mean Radius, Feet	D-c and A-c Resistance, 25°C, Ohms/Mile	Internal Reactance, Ohms/Mile			Equivalent Geometric Mean Radius, Feet
		25~	50~	60~			25~	50~	60~	
1	0.1594	0.003	0.007	0.008	0.0390	0.302	0.002	0.003	0.004	0.0403
1¼	0.1020	0.003	0.007	0.008	0.0487	0.1933	0.002	0.003	0.004	0.0504
1½	0.0708	0.003	0.007	0.008	0.0585	0.1343	0.002	0.003	0.004	0.0605
1¾	0.0520	0.003	0.007	0.008	0.0682	0.0985	0.002	0.003	0.004	0.0705
2	0.0398	0.003	0.007	0.008	0.0780	0.0755	0.002	0.003	0.004	0.0806

TABLE IV. ALUMINUM CABLE STEEL REINFORCED (A.C.S.R.)*

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
1,590,000	54/19	1.545	0.0587	0	0.0588	0.0591	0.011	0.026	0.0521	0.0521
				600	0.0592	0.0607				
				1200	0.0606	0.0662				
1,510,500	54/19	1.506	0.0618	0	0.0619	0.0622	0.011	0.026	0.0508	0.0508
				600	0.0623	0.0637				
				1200	0.0636	0.0691				
1,431,000	54/19	1.465	0.0652	0	0.0653	0.0656	0.011	0.026	0.0494	0.0494
				600	0.0657	0.0671				
				1200	0.0670	0.0724				
1,351,500	54/19	1.424	0.0691	0	0.0692	0.0695	0.011	0.026	0.0480	0.0480
				600	0.0695	0.0709				
				1200	0.0708	0.0761				
1,272,000	54/19	1.382	0.0734	0	0.0735	0.0738	0.011	0.026	0.0466	0.0466
				600	0.0738	0.0752				
				1200	0.0751	0.0802				
1,192,500	54/19	1.338	0.0783	0	0.0784	0.0788	0.011	0.026	0.0452	0.0452
				600	0.0787	0.0801				
				1200	0.0799	0.0849				
1,113,000	54/19	1.293	0.0839	0	0.0840	0.0844	0.011	0.026	0.0437	0.0437
				600	0.0843	0.0857				
				1200	0.0855	0.0903				

* Data on A.C.S.R. conductors furnished by the Aluminum Company of America.

TABLE IV. (*Continued*)

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
1,033,500	54/7	1.246	0.0903	0	0.0905	0.0909	0.011	0.026	0.0421	0.0421
				600	0.0908	0.0922				
				1200	0.0919	0.0965				
954,000	54/7	1.196	0.0979	0	0.0980	0.0982	0.011	0.026	0.0402	0.0402
				600	0.0983	0.0997				
				1200	0.0994	0.104				
900,000	54/7	1.162	0.104	0	0.104	0.104	0.011	0.026	0.0392	0.0392
				600	0.104	0.106				
				1200	0.105	0.110				
874,000	54/7	1.146	0.107	0	0.107	0.108	0.011	0.026	0.0388	0.0388
				600	0.107	0.109				
				1200	0.108	0.113				
795,000	54/7	1.093	0.117	0	0.118	0.119	0.011	0.026	0.0369	0.0369
				600	0.118	0.119				
				1200	0.119	0.123				
795,000	26/7	1.108	0.117	0	0.117	0.117	0.011	0.025	0.0375	0.0375
				600	0.117	0.117				
				1200	0.117	0.117				
795,000	30/19	1.140	0.117	0	0.117	0.117	0.010	0.023	0.0394	0.0394
				600	0.117	0.117				
				1200	0.117	0.117				
715,500	54/7	1.036	0.131	0	0.131	0.132	0.011	0.026	0.0349	0.0349
				600	0.131	0.133				
				1200	0.132	0.136				

TABLE IV. (*Continued*)

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
715,500	26/7	1.051	0.131	0	0.131	0.131	0.010	0.025	0.0357	0.0357
				600	0.131	0.131				
				1200	0.131	0.131				
715,500	30/19	1.081	0.131	0	0.131	0.131	0.010	0.023	0.0374	0.0374
				600	0.131	0.131				
				1200	0.131	0.131				
666,600	54/7	1.000	0.140	0	0.140	0.141	0.011	0.026	0.0336	0.0336
				600	0.141	0.142				
				1200	0.142	0.146				
636,000	54/7	0.977	0.147	0	0.147	0.148	0.011	0.026	0.0329	0.0329
				600	0.147	0.149				
				1200	0.149	0.154				
636,000	26/7	0.990	0.147	0	0.147	0.147	0.010	0.025	0.0336	0.0336
				600	0.147	0.147				
				1200	0.147	0.147				
636,000	30/19	1.019	0.147	0	0.147	0.147	0.010	0.023	0.0351	0.0351
				600	0.147	0.147				
				1200	0.147	0.147				
605,000	54/7	0.953	0.154	0	0.155	0.155	0.011	0.026	0.0321	0.0321
				600	0.155	0.157				
				1200	0.156	0.162				
605,000	26/7	0.966	0.154	0	0.154	0.154	0.010	0.025	0.0328	0.0328
				600	0.154	0.154				
				1200	0.154	0.154				

TABLE IV. (*Continued*)

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
556,500	26/7	0.927	0.168	0	0.168	0.168	0.010	0.025	0.0315	0.0315
				600	0.168	0.168				
				1200	0.168	0.168				
556,500	30/7	0.953	0.168	0	0.168	0.168	0.009	0.023	0.0329	0.0329
				600	0.168	0.168				
				1200	0.168	0.168				
500,000	30/7	0.904	0.187	0	0.187	0.187	0.009	0.023	0.0312	0.0312
				600	0.187	0.187				
				1200	0.187	0.187				
477,000	26/7	0.858	0.196	0	0.196	0.196	0.010	0.025	0.0291	0.0291
				600	0.196	0.196				
				1200	0.196	0.196				
477,000	30/7	0.883	0.196	0	0.196	0.196	0.009	0.023	0.0305	0.0305
				600	0.196	0.196				
				1200	0.196	0.196				
397,500	26/7	0.783	0.235	0	0.235	0.235	0.010	0.025	0.0266	0.0266
				600	0.235	0.235				
				1200	0.235	0.235				
397,500	30/7	0.806	0.235	0	0.235	0.235	0.009	0.023	0.0280	0.0280
				600	0.235	0.235				
				1200	0.235	0.235				
336,400	26/7	0.721	0.278	0	0.278	0.278	0.010	0.025	0.0245	0.0245
				600	0.278	0.278				
				1200	0.278	0.278				

TABLE IV. (*Continued*)

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
				0	0.278	0.278	0.010	0.023	0.0256	0.0256
336,400	30/7	0.741	0.278	600	0.278	0.278				
				1200	0.278	0.278				
				0	0.311	0.311	0.010	0.025	0.0231	0.0231
300,000	26/7	0.680	0.311	600	0.311	0.311				
				1200	0.311	0.311				
				0	0.311	0.311	0.009	0.023	0.0242	0.0242
300,000	30/7	0.700	0.311	600	0.311	0.311				
				1200	0.311	0.311				
				0	0.350	0.350	0.010	0.025	0.0218	0.0218
266,800	26/7	0.642	0.350	600	0.350	0.350				
				1200	0.350	0.350				
				0	0.350	0.351	0.010	0.024	0.0215	0.0215
266,800	6/7	0.633	0.350	600	0.353	0.363	0.018	0.044	0.0186	0.0186
				1200	0.378	0.463	0.051	0.123	0.00962	0.00962
				0	0.442	0.445	0.028	0.068	0.0134	0.0134
0000	6/1	0.563	0.441	600	0.447	0.464	0.037	0.089	0.0113	0.0113
				1200	0.463	0.530	0.048	0.115	0.00906	0.00906
				0	0.557	0.560	0.029	0.071	0.0117	0.0117
000	6/1	0.502	0.556	600	0.562	0.579	0.035	0.085	0.0104	0.0104
				1200	0.574	0.628	0.048	0.115	0.00815	0.00815
				0	0.702	0.706	0.030	0.071	0.0104	0.0104
00	6/1	0.447	0.702	600	0.706	0.718	0.034	0.081	0.00957	0.00957
				1200	0.715	0.756	0.046	0.110	0.00754	0.00754

TABLE IV. (*Continued*)

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
0	6/1	0.398	0.885	0	0.885	0.888	0.029	0.070	0.00928	0.00928
				600	0.887	0.893	0.032	0.077	0.00875	0.00875
				1200	0.893	0.920	0.043	0.103	0.00713	0.00713
1	6/1	0.355	1.12	0	1.12	1.12	0.029	0.069	0.00840	0.00840
				600	1.12	1.12	0.031	0.075	0.00800	0.00800
				1200	1.12	1.15	0.040	0.095	0.00677	0.00677
2	6/1	0.316	1.41	0	1.41	1.41	0.028	0.067	0.00758	0.00758
				600	1.41	1.41	0.030	0.072	0.00731	0.00731
				1200	1.41	1.43	0.036	0.086	0.00650	0.00650
3	6/1	0.281	1.78	0	1.78	1.78	0.027	0.065	0.00687	0.00687
				600	1.78	1.78	0.028	0.068	0.00672	0.00672
				1200	1.78	1.79	0.032	0.076	0.00625	0.00625
4	6/1	0.250	2.24	0	2.24	2.24	0.026	0.063	0.00620	0.00620
				600	2.24	2.24	0.027	0.065	0.00612	0.00612
				1200	2.24	2.24	0.028	0.068	0.00595	0.00595
5	6/1	0.223	2.82	0	2.82	2.82	0.026	0.062	0.00557	0.00557
				600	2.82	2.82	0.026	0.064	0.00550	0.00550
				1200	2.82	2.82	0.029	0.070	0.00520	0.00520
6	6/1	0.198	3.56	0	3.56	3.56	0.025	0.061	0.00500	0.00500
				600	3.56	3.56	0.026	0.063	0.00490	0.00490
				1200	3.56	3.56	0.029	0.069	0.00468	0.00468
203,000	8/7	0.584	0.460	0	0.460	0.461	0.014	0.031	0.0185	0.0188
				600	0.462	0.466	0.019	0.044	0.0166	0.0170
				1200	0.471	0.500	0.046	0.090	0.00988	0.0116

TABLE IV. *(Continued)*

Circular Mils or A.W.G.	Stranding, Al/Steel	Cond. Diam., In.	D-c Resistance, 25°C, Ohms/Mile	A-c Amp. per square inch	Effective Resistance, Ohms/Mile, 25°C		Internal Reactance, Ohms/Mile		Geometric Mean Radius, Feet	
					25~	60~	25~	60~	25~	60~
203,200	16/19	0.714	0.460	0	0.460	0.461	0.014	0.032	0.0228	0.0228
				600	0.462	0.466	0.019	0.043	0.0203	0.0209
				1200	0.471	0.500	0.045	0.090	0.0122	0.0142
211,300	12/7	0.663	0.442	0	0.442	0.443	0.014	0.031	0.0211	0.0214
				600	0.444	0.448	0.019	0.042	0.0190	0.0195
				1200	0.453	0.481	0.046	0.090	0.0112	0.0132
190,800	12/7	0.631	0.490	0	0.490	0.491	0.014	0.031	0.0200	0.0203
				600	0.492	0.496	0.019	0.042	0.0181	0.0186
				1200	0.502	0.533	0.046	0.090	0.0106	0.0126
176,900	12/7	0.607	0.528	0	0.528	0.529	0.014	0.031	0.0192	0.0196
				600	0.530	0.535	0.019	0.042	0.0173	0.0179
				1200	0.540	0.574	0.046	0.090	0.0102	0.0121
159,000	12/7	0.576	0.587	0	0.587	0.588	0.013	0.032	0.0185	0.0185
				600	0.589	0.594	0.019	0.043	0.0164	0.0169
				1200	0.601	0.639	0.045	0.090	0.00978	0.0115
134,600	12/7	0.530	0.694	0	0.694	0.695	0.014	0.032	0.0168	0.0170
				600	0.696	0.703	0.019	0.043	0.0151	0.0155
				1200	0.710	0.755	0.045	0.090	0.00900	0.0106
110,800	12/7	0.481	0.843	0	0.843	0.845	0.014	0.032	0.0154	0.0154
				600	0.846	0.854	0.019	0.043	0.0137	0.0141
				1200	0.863	0.917	0.045	0.090	0.00820	0.00956
101,800	12/7	0.461	0.917	0	0.917	0.918	0.014	0.032	0.0146	0.0148
				600	0.920	0.929	0.019	0.042	0.0132	0.0136
				1200	0.939	0.998	0.045	0.090	0.00783	0.00920
80,000	8/1	0.367	1.170	0	1.170	1.170	0.030	0.071	0.00850	0.00850
				600	1.170	1.170	0.035	0.08	0.00772	0.00772
				1200	1.177	1.197	0.047	0.113	0.00600	0.00600

TABLE V. COPPERWELD CABLES. 30% Conductivity (Copperweld Cables)

Nominal Designation	No. and A.W.G. Size of Wires	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C)			Internal Reactance Ohms/Mile			Equivalent Geometric Mean Radius, Feet	
					25~	50~	60~	25~	50~	60~	25~	50~ 60~
21/32"	19#8	0.642	0.613	0	0.623	0.633	0.638	0.096	0.163	0.195	0.00401	0.00535
				100	0.643	0.673	0.687					
				425	0.708	0.804	0.843					
9/16"	19#9	0.572	0.773	0	0.783	0.793	0.798				0.00358	0.00477
				100	0.808	0.844	0.859					
				375	0.877	0.984	1.026					
5/8"	7#4	0.613	0.656	0	0.664	0.672	0.676				0.00383	0.00511
				100	0.676	0.696	0.705					
				400	0.713	0.770	0.792					
9/16"	7#5	0.546	0.827	0	0.835	0.843	0.847				0.00341	0.00455
				100	0.850	0.874	0.883					
				350	0.889	0.951	0.975					
1/2"	7#6	0.486	1.043	0	1.051	1.059	1.063				0.00304	0.00405
				100	1.070	1.098	1.109					
				300	1.109	1.176	1.201					
7/16"	7#7	0.433	1.315	0	1.323	1.331	1.335				0.00271	0.00361
				100	1.348	1.380	1.393					
				250	1.384	1.454	1.480					
3/8"	7#8	0.385	1.658	0	1.666	1.674	1.678	0.096	0.163	0.195	0.00241	0.00321
				100	1.697	1.736	1.751					
				225	1.736	1.813	1.843					

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 100°C temperature rise.

Data for Copperweld conductors furnished by the Copperweld Steel Company.

TABLE V. (*Continued*)

30% Conductivity (Copperweld Cables)

Nominal Desig-nation	No. and A.W.G. Size of Wires	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C)			Internal Reactance Ohms/Mile			Equivalent Geometric Mean Radius, Feet	
					25~	50~	60~	25~	50~	60~	25~	50~ 60~
11/32"	7#9	0.343	2.09	0 100 200	2.10 2.14 2.18	2.11 2.19 2.27	2.11 2.20 2.30	0.096	0.163	0.195	0.00214	0.00286
5/16"	7#10	0.306	2.64	0 100 175	2.65 2.70 2.74	2.66 2.76 2.83	2.66 2.77 2.86	0.096	0.163	0.195	0.00191	0.00255
3#5	3#5	0.392	1.926	0 100 225	1.931 1.944 1.961	1.936 1.962 1.996	1.938 1.969 2.01	0.081	0.129	0.154	0.00327	0.00457
3#6	3#6	0.349	2.43	0 100 200	2.44 2.46 2.47	2.44 2.47 2.51	2.44 2.48 2.52				0.00291	0.00407
3#7	3#7	0.311	3.06	0 100 175	3.07 3.09 3.11	3.07 3.11 3.14	3.07 3.13 3.16				0.00259	0.00363
3#8	3#8	0.277	3.86	0 100 150	3.87 3.90 3.91	3.87 3.92 3.95	3.87 3.94 3.97				0.00231	0.00323
3#9	3#9	0.247	4.87	0 50 125	4.88 4.90 4.92	4.88 4.91 4.96	4.88 4.92 4.98				0.00206	0.00288
3#10	3#10	0.220	6.14	0 50 100	6.15 6.17 6.19	6.15 6.19 6.23	6.15 6.20 6.25	0.081	0.129	0.154	0.00183	0.00257

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 100°C temperature rise.

TABLE V_8 (*Continued*)

40% Conductivity (Copperweld Cables)

Nominal Desig- nation	No. and A.W.G. Size of Wires	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C)			Internal Reactance Ohms/Mile			Equivalent Geometric Mean Radius, Feet	
					25~	50~	60~	25~	50~	60~	25~	50~ 60~
				0	0.470	0.480	0.485					
21/32"	19#8	0.642	0.460	100	0.485	0.510	0.521	0.070	0.118	0.142	0.00669	0.00829
				475	0.541	0.624	0.657					
				0	0.590	0.600	0.605					
9/16"	19#9	0.572	0.580	100	0.609	0.638	0.651				0.00596	0.00739
				425	0.670	0.762	0.799					
				0	0.500	0.508	0.512					
5/8"	7#4	0.613	0.492	100	0.509	0.526	0.534				0.00639	0.00792
				450	0.541	0.591	0.610					
				0	0.628	0.636	0.640					
9/16"	7#5	0.546	0.620	100	0.640	0.659	0.668				0.00569	0.00705
				400	0.674	0.729	0.750					
				0	0.790	0.798	0.802					
1/2"	7#6	0.486	0.782	100	0.805	0.827	0.837				0.00506	0.00628
				350	0.841	0.900	0.923					
				0	0.994	1.002	1.006					
7/16"	7#7	0.433	0.986	100	1.012	1.039	1.050				0.00451	0.00559
				300	1.049	1.112	1.137					
				0	1.252	1.260	1.264					
3/8"	7#8	0.385	1.244	100	1.275	1.306	1.319				0.00401	0.00497
				250	1.310	1.376	1.401					
				0	1.576	1.584	1.588					
11/32"	7#9	0.343	1.568	100	1.605	1.642	1.657	0.070	0.118	0.142	0.00357	0.00443
				225	1.642	1.716	1.744					

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 100°C temperature rise.

TABLE V. (*Continued*)

40% Conductivity (Copperweld Cables)

Nominal Desig-nation	No. and A.W.G. Size of Wires	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C)			Internal Reactance Ohms/Mile			Equivalent Geometric Mean Radius, Feet	
					25~	50~	60~	25~	50~	60~	25~	50~ 60~
5/16"	7#10	0.306	1.978	0 100 200	1.986 2.02 2.06	1.994 2.07 2.14	1.998 2.09 2.17	0.070	0.118	0.142	0.00319	0.00395
3#5	3#5	0.392	1.445	0 100 250	1.450 1.460 1.475	1.455 1.475 1.505	1.457 1.481 1.517	0.061	0.098	0.117	0.00490	0.00621
3#6	3#6	0.349	1.821	0 100 225	1.826 1.839 1.854	1.831 1.856 1.887	1.833 1.863 1.901				0.00436	0.00553
3#7	3#7	0.311	2.30	0 100 200	2.31 2.33 2.34	2.31 2.34 2.37	2.31 2.35 2.39				0.00389	0.00492
3#8	3#8	0.277	2.90	0 100 175	2.91 2.93 2.94	2.91 2.95 2.98	2.91 2.96 2.99				0.00346	0.00439
3#9	3#9	0.247	3.65	0 100 150	3.66 3.69 3.70	3.66 3.71 3.74	3.66 3.73 3.76				0.00309	0.00391
3#10	3#10	0.220	4.61	0 100 125	4.62 4.65 4.66	4.62 4.68 4.70	4.62 4.69 4.71				0.00275	0.00348
3#12	3#12	0.174	7.32	0 50 85	7.33 7.36 7.37	7.33 7.38 7.42	7.33 7.40 7.44	0.061	0.098	0.117	0.00218	0.00276

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 100°C temperature rise.

TABLE V. (*Continued*) — Copperweld-Copper Cables

Nominal Designation	Size of Conductor: Number and Diameter of Wires: Copperweld	Copper	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C) 25~	50~	60~	Internal Reactance Ohms/Mile 25~	50~	60~	Equivalent Geometric Mean Radius, Feet 25~ 50~ 60~
2A	1 x 0.1699″	2 x 0.1699″	0.366	0.883	0 100 250	0.886 0.891 0.899	0.888 0.899 0.916	0.889 0.902 0.921	0.035	0.070	0.084	0.00763
3A	1 x 0.1513″	2 x 0.1513″	0.326	1.114	0 100 200	1.117 1.124 1.130	1.119 1.133 1.147	1.120 1.136 1.153	0.035	0.070	0.084	0.00679
4A	1 x 0.1347″	2 x 0.1347″	0.290	1.404	0 100 175	1.407 1.415 1.421	1.409 1.427 1.440	1.410 1.431 1.446	0.035	0.070	0.084	0.00604
5A	1 x 0.1200″	2 x 0.1200″	0.258	1.771	0 100 150	1.774 1.784 1.790	1.776 1.799 1.810	1.777 1.803 1.816	0.035	0.070	0.084	0.00538
6A	1 x 0.1068″	2 x 0.1068″	0.230	2.23	0 100 125	2.24 2.25 2.26	2.24 2.27 2.28	2.24 2.27 2.28	0.035	0.070	0.084	0.00479
7A	1 x 0.1266″	2 x 0.08949″	0.223	2.82	0 100 125	2.82 2.84 2.84	2.82 2.86 2.87	2.82 2.86 2.87	0.038	0.075	0.090	0.00441
8A	1 x 0.1127″	2 x 0.07969″	0.199	3.55	0 50 100	3.56 3.57 3.58	3.56 3.58 3.61	3.56 3.58 3.61	0.038	0.075	0.090	0.00394
8C	1 x 0.08081″	2 x 0.08336″	0.179	3.55	0 50 100	3.56 3.57 3.58	3.56 3.58 3.61	3.56 3.58 3.61	0.035	0.070	0.084	0.00373
9-1/2 D	2 x 0.08081″	1 x 0.08081″	0.174	4.93	0 50 75	4.94 4.96 4.96	4.94 4.97 4.99	4.94 4.98 5.00	0.048	0.095	0.114	0.00283

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 50°C temperature rise.

TABLE V. (*Continued*) — Copperweld-Copper Cables

Nominal Designation	Size of Conductor: Number and Diameter of Wires, Copperweld	Size of Conductor: Number and Diameter of Wires, Copper	Size of Conductor: Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C), 25~	50~	60~	Internal Reactance Ohms/Mile, 25~	50~	60~	Equivalent Geometric Mean Radius, Feet, 25~ 50~ 60~
					0	0.1011	0.1031	0.1041				
1.020″E	7 x 0.2043″	12 x 0.2043″	1.020	0.0981	100	0.1020	0.1048	0.1061	0.020	0.041	0.049	0.0285
					850	0.1085	0.1176	0.1213				
					0	0.1267	0.1287	0.1297				
0.910″E	7 x 0.1819″	12 x 0.1819″	0.910	0.1237	100	0.1278	0.1309	0.1322	0.020	0.041	0.049	0.0254
					750	0.1349	0.1451	0.1488				
					0	0.1590	0.1610	0.1620				
0.810″E	7 x 0.1620″	12 x 0.1620″	0.810	0.1560	100	0.1604	0.1638	0.1652	0.020	0.041	0.049	0.0226
					650	0.1679	0.1789	0.1829				
					0	0.1997	0.202	0.203				
0.721″E	7 x 0.1443″	12 x 0.1443″	0.721	0.1967	100	0.201	0.205	0.207	0.020	0.041	0.049	0.0201
					550	0.209	0.221	0.225				
					0	0.251	0.253	0.254				
0.642″E	7 x 0.1285″	12 x 0.1285″	0.642	0.248	100	0.253	0.257	0.259	0.020	0.041	0.049	0.0179
					500	0.262	0.275	0.280				
					0	0.283	0.285	0.286				
4/0 E	7 x 0.1225″	12 x 0.1225″	0.613	0.280	100	0.285	0.290	0.292	0.020	0.041	0.049	0.0171
					450	0.294	0.307	0.312				
					0	0.316	0.318	0.319				
0.572″E	7 x 0.1144″	12 x 0.1144″	0.572	0.313	100	0.319	0.324	0.325	0.020	0.041	0.049	0.0160
					400	0.327	0.340	0.345				
					0	0.356	0.358	0.359				
3/0 E	7 x 0.1091″	12 x 0.1091″	0.545	0.353	100	0.359	0.364	0.366	0.020	0.041	0.049	0.0152
					400	0.368	0.383	0.388				
					0	0.397	0.399	0.400				
0.509″E	7 x 0.1019″	12 x 0.1019″	0.509	0.394	100	0.400	0.406	0.409	0.020	0.041	0.049	0.0142
					350	0.409	0.423	0.429				

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 50°C temperature rise.

TABLE V. (*Continued*) — Copperweld-Copper Cables

Nominal Designation	Size of Conductor: Number and Diameter of Wires, Copperweld	Size of Conductor: Number and Diameter of Wires, Copper	Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C) 25~	50~	60~	Internal Reactance Ohms/Mile 25~	50~	60~	Equivalent Geometric Mean Radius, Feet 25~ 50~ 60~
4/0 F	1 x 0.1833″	6 x 0.1833″	0.550	0.280	0	0.283	0.285	0.286	0.020	0.039	0.047	0.0156
					100	0.285	0.289	0.291				
					450	0.293	0.304	0.308				
3/0 F	1 x 0.1632″	6 x 0.1632″	0.490	0.353	0	0.356	0.357	0.359	0.020	0.039	0.047	0.0139
					100	0.359	0.362	0.365				
					400	0.367	0.378	0.384				
2/0 F	1 x 0.1454″	6 x 0.1454″	0.436	0.445	0	0.448	0.450	0.451	0.020	0.039	0.047	0.0124
					100	0.451	0.457	0.459				
					350	0.460	0.473	0.478				
1/0 F	1 x 0.1294″	6 x 0.1294″	0.388	0.561	0	0.564	0.566	0.567	0.020	0.039	0.047	0.0110
					100	0.568	0.574	0.577				
					300	0.577	0.591	0.597				
1 F	1 x 0.1153″	6 x 0.1153″	0.346	0.708	0	0.711	0.713	0.714	0.020	0.039	0.047	0.00980
					100	0.717	0.723	0.726				
					250	0.725	0.739	0.745				
2 F	1 x 0.1026″	6 x 0.1026″	0.308	0.892	0	0.895	0.897	0.898	0.020	0.039	0.047	0.00873
					100	0.902	0.910	0.914				
					200	0.909	0.923	0.930				
3/0 J	3 x 0.1851″	4 x 0.1851″	0.555	0.353	0	0.357	0.361	0.363	0.035	0.070	0.084	0.0116
					100	0.363	0.372	0.377				
					400	0.381	0.407	0.418				
2/0 J	3 x 0.1648″	4 x 0.1648″	0.494	0.445	0	0.449	0.453	0.455	0.035	0.070	0.084	0.0103
					100	0.456	0.467	0.472				
					350	0.475	0.503	0.516				
1/0 J	3 x 0.1467″	4 x 0.1467″	0.440	0.561	0	0.565	0.569	0.571	0.035	0.070	0.084	0.00917
					100	0.574	0.587	0.593				
					300	0.593	0.623	0.637				

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 50°C temperature rise.

TABLE V. (*Continued*) — Copperweld-Copper Cables

Nominal Designation	Size of Conductor: Number and Diameter of Wires: Copperweld	Size of Conductor: Number and Diameter of Wires: Copper	Size of Conductor: Outside Diameter, Inch	D-c Resistance 25°C Ohms/Mile	A-c Current in Amperes*	Effective Resistance Ohms per Conductor per Mile (Conductor Temp. 25°C): 25~	50~	60~	Internal Reactance Ohms/Mile: 25~	50~	60~	Equivalent Geometric Mean Radius, Feet: 25~ 50~ 60~
1 J	3 x 0.1307″	4 x 0.1307″	0.392	0.708	0 100 250	0.712 0.724 0.742	0.716 0.739 0.773	0.718 0.745 0.787	0.035	0.070	0.084	0.00817
4/0 S	2 x 0.1906″	12 x 0.1258″	0.633	0.280	0 100 450			0.285 0.290 0.309	0.015	0.031	0.037	0.0195
3/0 S	2 x 0.1697″	12 x 0.1120″	0.563	0.353	0 100 400			0.358 0.365 0.385	0.015	0.031	0.037	0.0174
2/0 S	2 x 0.1511″	12 x 0.09975″	0.502	0.445	0 100 350			0.450 0.458 0.480	0.015	0.031	0.037	0.0155
250 V	3 x 0.1480″	9 x 0.1600″	0.637	0.237	0 100 500			0.244 0.249 0.270	0.017	0.033	0.040	0.0191
4/0 V	3 x 0.1361″	9 x 0.1472″	0.586	0.280	0 100 450			0.287 0.293 0.315	0.017	0.033	0.040	0.0176
3/0 V	3 x 0.1212″	9 x 0.1311″	0.522	0.353	0 100 400			0.360 0.368 0.392	0.017	0.033	0.040	0.0157
2/0 V	3 x 0.1080″	9 x 0.1167″	0.465	0.445	0 100 350			0.452 0.462 0.487	0.017	0.033	0.040	0.0140

* The currents tabulated cover a range from zero to the approximate current carrying capacity of the conductor based on a 50°C temperature rise.

Characteristics of Steel and Iron Wires

Steel wires are used extensively as ground wires to protect overhead transmission circuits against lightning. Steel and iron wires have long had a place as power conductors in branch circuits where the current to be transmitted is small, and copper conductors of the required conductivity would not have the necessary mechanical strength. Improved types of steel conductors which combine more efficient a-c characteristics and high tensile strength have been developed in recent years. Their distinctive electrical properties, as compared with older types of iron and steel wires, are relatively low magnetic permeability, low 60-cycle resistance, and low internal inductance over a wide range of current. The importance of these wires as power conductors is increased during copper and aluminum shortages.

Because of their higher permeability, iron and steel wires have higher internal inductances than non-magnetic conductors. Also, the ratios of their effective a-c to d-c resistances are higher than those of non-magnetic conductors of the same diameter because of hysteresis losses and higher skin effect ratios. The a-c resistance and internal inductance vary with current and frequency. For a given conductor, operated at constant ambient temperature and frequency, a-c resistance and internal reactance increase as the current is increased from zero. If the permeability is high, each of these increases reaches a maximum at a relatively small current, beyond which the resistance and reactance decrease. When the permeability is low, however, the rates of increase are less, and maximum a-c resistance and reactance may not occur until the current is above the current rating of the conductor. Maximum resistance and maximum reactance occur at approximately the same value of current — that corresponding to the maximum permeability of the conductor.

Iron wires are designated by the letters B.B. (Best Best) and E.B.B. (Extra Best Best). Steel wires have many designations and fall in two general classes: (1) for mechanical use or (2) as power conductors. Steel wires for mechanical use are described as Common, Siemens-Martin, High Strength, and Extra High Strength. The characteristics of steel conductors of the same outside diameter but of different types of steel vary widely, as pointed out in the following papers:

1. "Use of Steel Wire for Overhead Line Extensions," by S. O. Miller, *Edison Electric Institute Bulletin*, May, 1942, Vol. 10, page 197.

2. "Galvanized Steel as a Power Conductor," by Frederick M. Crapo, *Electrical World*, September 19, 1942, page 50.

Table VI–A gives approximate characteristics of the older types of iron and steel wires read from curves prepared by Professor H. B. Dwight, based on test data available in 1919. Table VI–A is given here to indicate the order of magnitude of the a-c resistance and internal reactance of conductors long in service. It is not representative of the steel conductors manufactured at present for power transmission.

Characteristics of new types of steel conductor, manufactured for electrical purposes, are given in Table VI–B. This table, furnished by the Indiana Steel and Wire Company from their 1943 manual, "Crapo Steel Conductors . . . Manual of Engineering Data and Construction Practices," gives a-c resistances and internal reactances at 60 cycles for Crapo HTC–130 and HTC–80 Steel Conductors at various current loadings.

It is pointed out in Chapter XI that the effect of steel ground wires on the zero-sequence impedance of overhead transmission circuits is slight. Approximate values of a-c resistance and internal reactance of ground wires are, in general, adequate. But, with steel or iron wires as power conductors, a higher degree of precision in these constants is required for correct determination of voltage drop and power loss under specified operating conditions. Where possible, these characteristics should be obtained directly from the manufacturer.

The geometric mean radius of any conductor in Table VI–A or VI–B may be calculated by substituting the internal reactance and conductor radius in [B–2].

TABLE VI–A. Approximate Characteristics of Iron and Steel Wires

(From Curves in "Resistance and Reactance of Commercial Steel Conductors," *Elec. Jour.*, page 25, January, 1919, by H. B. Dwight.)

Size B.W.G.	Type (Dia., In.)	Amps.	Ohms/Mile				Size B.W.G.	Type (Dia., In.)	Amps.	Ohms/Mile.			
			r_a		x_i					r_a		x_i	
			60~	25~	60~	25~				60~	25~	60~	25~
						Solid Conductors							
	Ordinary	1	26	25.5	4.0	2.0		Ordinary	1	11.5	11.5	2.5	1.2
10	Steel	12	36	31	13.5	7.0	6	Steel	13	21	16	10.0	4.5
	(0.134)	20	33	29	11.5	5.8		(0.203)	20	23	17	9.5	4.5
	B.B.	1	24	22.5	5	2.5		B.B.	1	10.5	10	3.2	1.5
10	Iron	8	38	30	14.5	7.5	6	Iron	12	24	17	11	5.0
		20	31	26.5	12	6			20	22.5	16	10	4.8
	E.B.B.	1	22	20.5	6	3		E.B.B.	1	11	9.6	4.5	2
10	Iron	7	39.5	29.5	19	9	6	Iron	8	26	17	13	6
		20	28	24	14	7			20	20	14.2	10.5	5
	Ordinary	1	17	17	3.2	1.5		Ordinary	1	8.5	8.5	2.0	0.9
8	Steel	13	27.5	22	12.0	6	4	Steel	13	16.5	12	8.0	3.9
	(0.165)	20	26.5	21.8	10.5	5.3		(0.238)	20	20	14	8.5	4.0
	B.B.	1	16	15	4	2.0		B.B.	1	7.5	7	2.8	1.0
8	Iron	10	31	23	12.7	6.2	4	Iron	15	21	14	9.0	4.5
		20	25.5	20	11	5.2			20	20.5	13.5	9.0	4.0
	E.B.B.	1	15	14	5	2.5		E.B.B.	1	8	7	3.5	1.5
8	Iron	7	32.5	22.5	15.5	8	4	Iron	10	22	14.2	11	5.3
		20	23	18	12	6			20	18	12.5	9.5	4
						7-Strand Conductors							
*	Siemens-	1	12.5		0.6		*	B.B.	1	3.9	3.9	0.5	0.3
¼ in.	Martin	15	12.8		1.4		⅜ in.	Iron	40	7.3	5.7	2.3	1.2
	(0.249)	30	13.2		2.2				60	6.2	5.2	2.0	1.0
	Ordinary	1	9.5	9.5	0.8	0.6		E.B.B.	1	3.4	3.4	0.7	0.3
¼ in.	Steel	40	11.7	10.7	3.4	1.8	⅜ in.	Iron	30	7.8	5.3	2.8	1.4
		60	11.3	10.3	3.0	1.6			60	6.0	4.8	2.3	1.0
	B.B.	1	8.2	8.2	1.0	0.7		Siemens-	1	3.3		0.3	
¼ in.	Iron	40	12	10	4.0	2.0	½ in.	Martin	15	3.6		0.5	
		60	10.3	9.2	3.0	1.7		(0.495)	30	4.2		0.7	
	E.B.B.	1	7.2	7.2	1.5	0.8		Ordinary	1	2.5	2.3	0.3	0.2
¼ in.	Iron	30	12.7	9.8	4.7	2.5	½ in.	Steel	40	4.9	3.8	1.2	0.6
		60	10.7	9	3.5	1.7			60	5.0	3.8	1.0	0.6
	Siemens-	1	6.0		0.4			B.B.	1	2.1	2.1	0.3	0.2
⅜ in.	Martin	15	6.3		0.7		½ in.	Iron	40	5.3	3.8	1.5	0.8
	(0.360)	30	6.7		1.0				60	5.0	3.6	1.5	0.8
	Ordinary	1	4.5	4.5	0.5	0.3		E.B.B.	1	2.0	1.9	0.4	0.2
⅜ in.	Steel	40	7.0	5.8	2.0	1.0	½ in.	Iron	40	5.8	4.0	2.0	0.8
		60	6.7	5.8	1.5	0.9			60	4.7	3.5	1.8	0.8

* Nominal diameter.

TABLE VI-B. 60-CYCLE RESISTANCE AND INTERNAL REACTANCE OF CRAPO HTC-130 AND HTC-80 STEEL CONDUCTORS
Ambient Temperature 20° C

Current Amperes	Size B.W.G. (Dia. In.)	HTC-130 Max. Resist., Ohms/Mile	HTC-130 Int. Reac., Ohms/Mile	HTC-80 Max. Resist., Ohms/Mile	HTC-80 Int. Reac., Ohms/Mile	Size B.W.G. (Dia., In.)	HTC-130 Max. Resist., Ohms/Mile	HTC-130 Int. Reac., Ohms/Mile	HTC-80 Max. Resist., Ohms/Mile	HTC-80 Int. Reac., Ohms/Mile
	Stranded (3-Wire) Conductors					Solid Conductors				
1.0		8.07	0.72	7.17	1.05		8.25	1.47	7.48	2.13
2.5	4 (0.297)	8.20	0.73	7.20	1.06	4 (0.238)	8.44	1.65	7.59	2.33
5.0		8.39	0.77	7.25	1.07		8.77	2.01	7.92	2.73
7.5		8.60	0.80	7.32	1.08		9.18	2.49		
10.0		8.83	0.85	7.40	1.10		9.83	3.01	9.23	3.90
15.0		9.53	0.90	7.62	1.15		12.1	4.48	11.80	5.38
20.0		10.05	1.07	7.98	1.25		14.3	5.59	13.82	6.29
25.0		10.72	1.51	8.68	1.60		15.3	6.29	15.08	6.73
1.0		11.29	0.72	9.97	1.06		11.37	1.47	10.15	2.21
2.5	6 (0.252)	11.31	0.73	10.01	1.07	6 (0.203)	11.48	1.65	10.42	2.47
5.0		11.36	0.77	10.09	1.09		11.76	2.01	10.79	2.97
7.5		11.43	0.80	10.18	1.12		12.26	2.49		
10.0		11.53	0.85	10.28	1.15		12.87	3.03	12.47	4.46
15.0		11.81	0.96	10.56	1.23		15.65	4.80	15.60	6.21
20.0		12.20	1.15	11.00	1.35		18.13	6.01	18.08	7.23
25.0		13.01	1.58	11.82	1.80		19.25	6.61	19.33	7.71
1.0		16.96	0.75	14.84	1.07		17.06	1.53	15.06	2.19
2.5	8 (0.207)	17.05	0.77	14.89	1.08	8 (0.165)	17.22	1.75	15.26	2.45
5.0		17.17	0.85	14.99	1.12		17.65	2.27	15.71	2.99
7.5		17.23	0.96	15.12	1.25		18.35	2.99		
10.0		17.49	1.15	15.30	1.45		19.75	4.08	18.35	5.18
15.0		18.25	1.29	15.77	1.65		23.86	6.47	23.18	8.36
20.0		19.20	1.45	16.48	1.70		26.02	7.63	25.17	8.94

Resistance, Internal Reactance, and Equivalent Diameter of Rails

In Chapter XI, rails which parallel transmission lines are treated as ground wires of zero height above ground in determining zero-sequence impedances of overhead transmission lines. The information required concerning rails is therefore equivalent diameter, a-c resistance, and internal reactance at operating frequency.

Table VII and Figs. 1 and 2 were supplied by Mr. D. R. MacLeod.

Table VII gives d-c resistance at 20°C and average equivalent radius, where

$$\text{Radius} = \frac{\text{perimeter}}{2\pi}$$

Rails of a given weight per yard do not necessarily have the same perimeter.

TABLE VII

D-C Resistance and Average Equivalent Radius of Rails of Specified Weight

Weight, Pounds/Yard	D-c Resistance, Ohms/Mile at 20°C	Equivalent Radius, Inches
60	0.0877	2.94
85	0.0615	3.55
90	0.0584	3.40
100	0.0526	3.92
130	0.0420	4.10

Positive-Sequence Reactances of Overhead Transmission Lines

Solid Conductors. Figure 3 gives the positive-sequence reactance to neutral at 25, 50, and 60 cycles of a three-phase, single-phase, or two-phase circuit of solid non-magnetic conductors, calculated from [7] or [4] of Chapter XI. The internal reactance included in Fig. 3 is

$$x_i = \frac{f}{60}\,(0.03034) \text{ ohms per mile}$$

By subtracting the internal reactance from the total reactance, the reactance external to the conductors is obtained.

Hollow Conductors. Figure 4 gives the internal reactances at 60 cycles of hollow non-magnetic conductors calculated from [1e] of Chapter XI. By adding internal reactance to external reactance, obtained from Fig. 3 as explained above, total reactance to neutral is obtained.

circuits, calculated from [45] of Chapter XII. At any other frequency f, values read from Fig. 7 are multiplied by $f/60$.

Corona Starting Voltage

Figure 8 gives approximate voltages at which corona starts in fair weather on symmetrical three-phase circuits of copper or A.C.S.R. conductors, calculated from *Dielectric Phenomena in High Voltage*

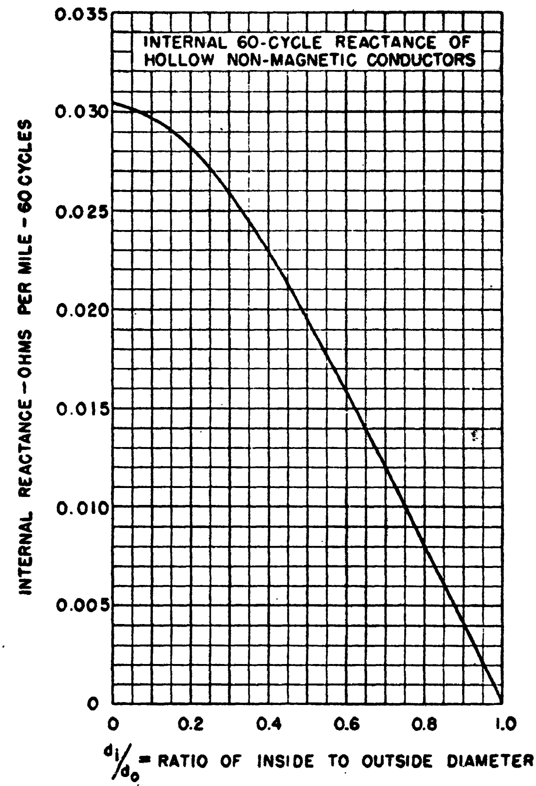


FIG. 4.

Engineering, by F. W. Peek, McGraw-Hill Book Company, 3rd Edition, 1929. With horizontal or vertical arrangement of conductors, corona starting voltage read from Fig. 8 corresponding to the equivalent Δ spacing between conductors should be raised 6% for the two outside conductors and lowered 4% for the inside conductor. Corona starting voltage varies with type and condition of conductors. With mutilated conductors, corona starting voltages may be as low as 80% of the values given in Fig. 8.

The corona power loss P per mile of a balanced three-phase circuit in

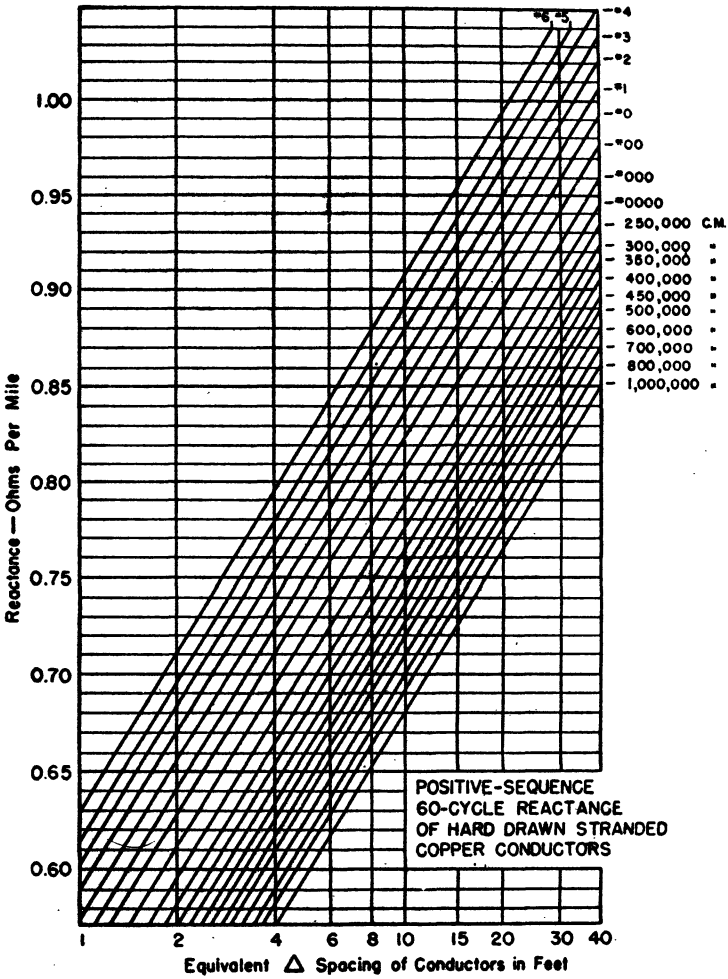

#6
#5
-#4
-#3
-#2
-#1
-#0
-#00
-#000
-#0000
- 250,000 C.M.
- 300,000 "
- 350,000 "
- 400,000 "
- 450,000 "
- 500,000 "
- 600,000 "
- 700,000 "
- 800,000 "
- 1,000,000 "
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
Reactance—Ohms Per Mile
POSITIVE-SEQUENCE
60-CYCLE REACTANCE
OF HARD DRAWN STRANDED
COPPER CONDUCTORS
1
2
4
6
8
10
15
20
30
40
Equivalent Δ Spacing of Conductors in Feet

Fig. 5.

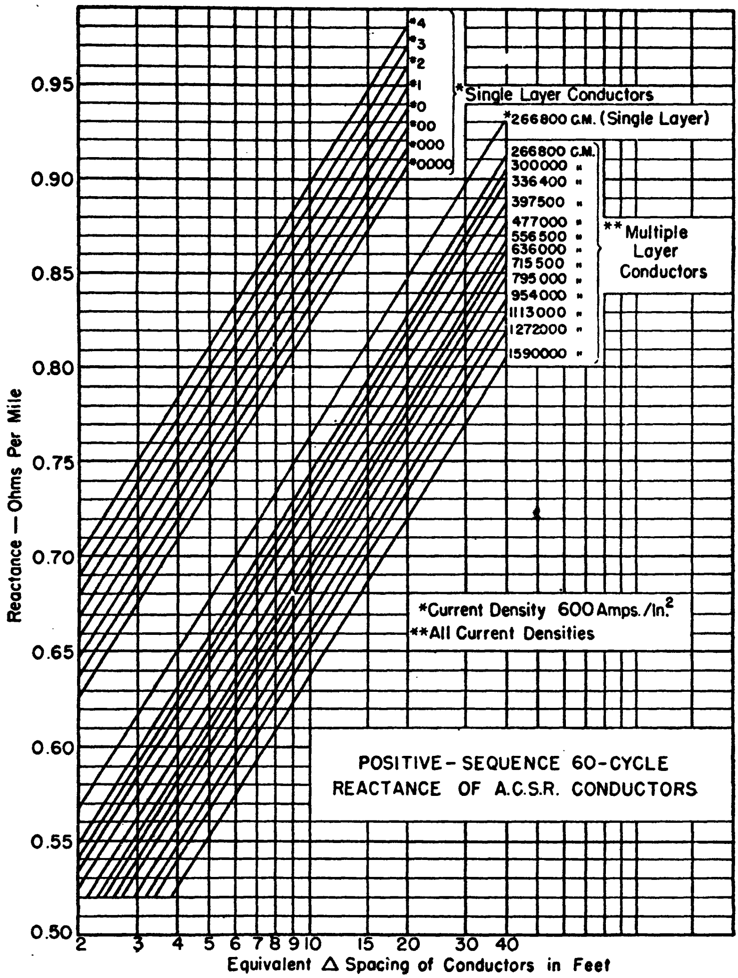

*4
*3
*2
*1
*0
*00
*000
*0000
*Single Layer Conductors
*266800 C.M. (Single Layer)
266800 C.M.
300000 "
336400 "
397500 "
477000 "
556500 "
636000 "
715500 "
795000 "
954000 "
1113000 "
1272000 "
1590000 "
** Multiple Layer Conductors
*Current Density 600 Amps./In.^{2}
**All Current Densities
POSITIVE-SEQUENCE 60-CYCLE REACTANCE OF A.C.S.R. CONDUCTORS
Reactance — Ohms Per Mile
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
2 3 4 5 6 7 8 9 10 15 20 30 40
Equivalent Δ Spacing of Conductors in Feet

FIG. 6

fair weather, operated with corona, is approximately

$$P = 0.05(e - e_0)^2 \left(\frac{f + 25}{85}\right) \qquad \text{[B-3]}$$

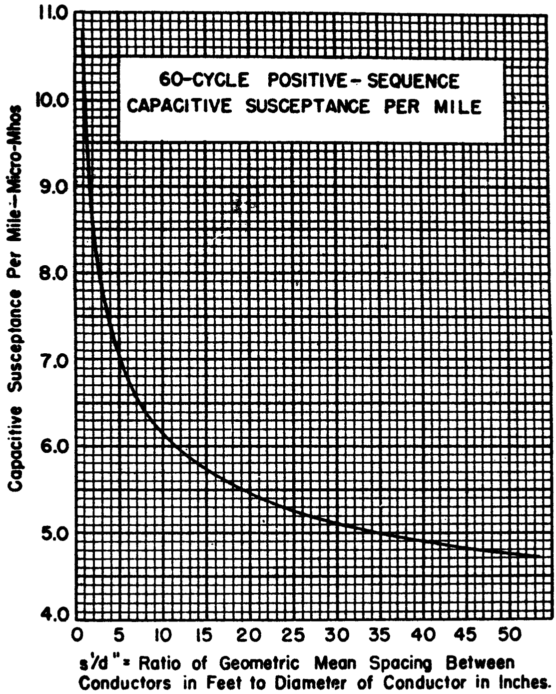


FIG. 7.

where

$$e = \text{kilovolt line-to-neutral operating voltage}$$

$$e_0 = \frac{1}{\sqrt{3}} \text{ (chart line-to-line kilovolts)}$$

For storm loss, use $0.8e_0$ in [B–3].

Temperature-Rise Curves

Figures 9 and 10 give temperature rise in degrees centigrade above ambient temperature for bare copper and A.C.S.R. conductors. These figures were taken from "Capacitors for Higher Line Thermal Loading," by J. W. Butler, *Electrical World*, February 7, 1942, page 76. Wire manufacturers and engineers generally feel that neither hard drawn copper nor A.C.S.R. conductors should be operated continu-

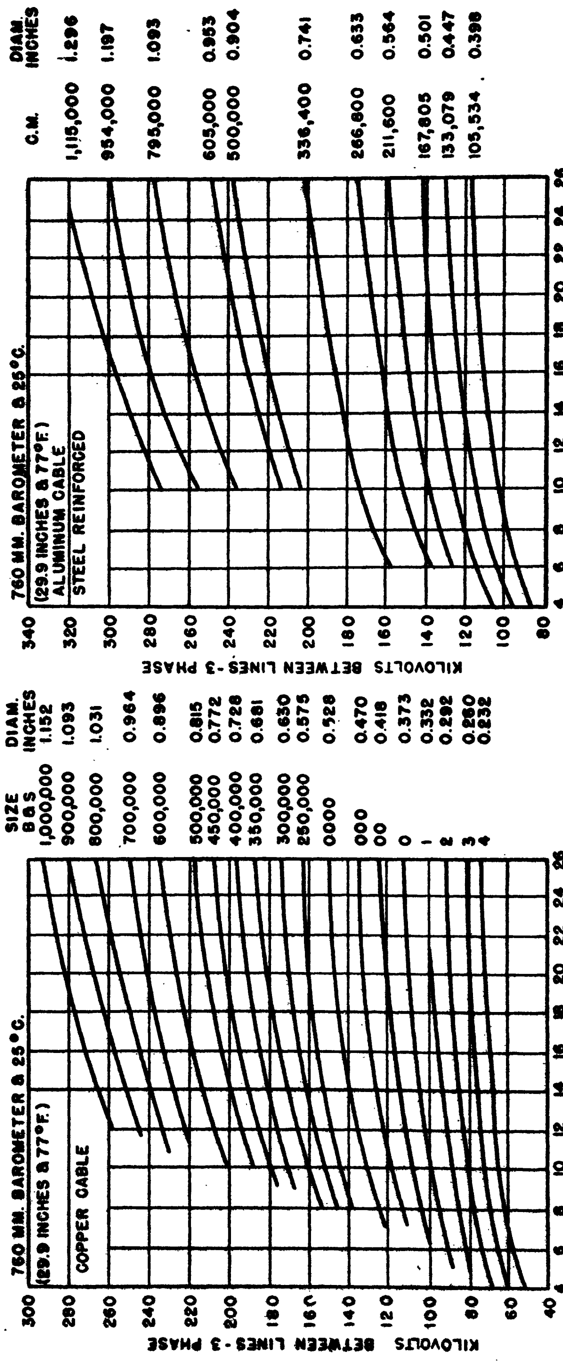


FIG. 8. Corona starting voltage as affected by conductor diameter and spacing.

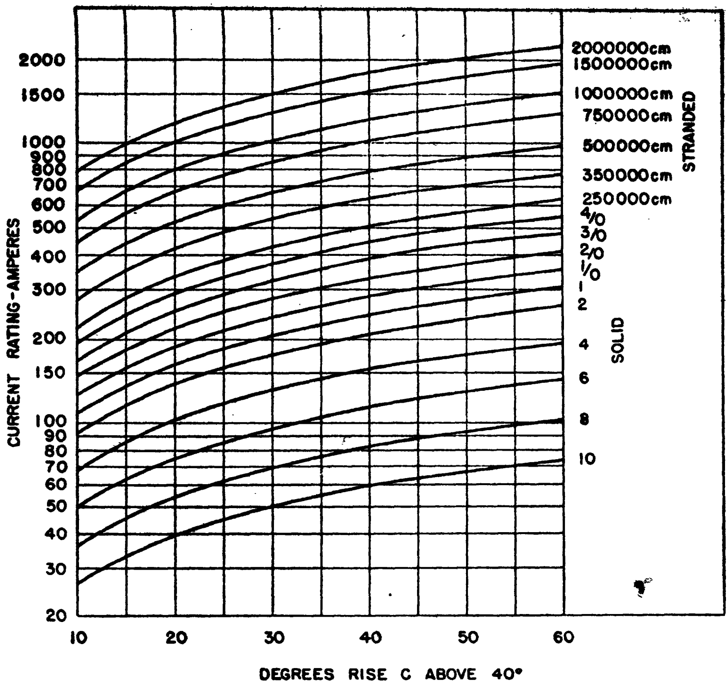
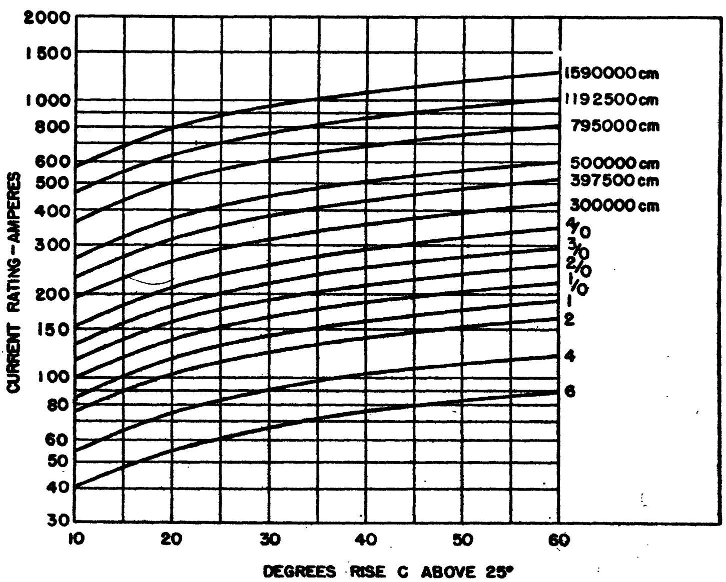


FIG. 9. Outdoor current ratings of bare copper conductors at 60 cycles with cross wind of 2 feet per second. (Plotted from data supplied by the Anaconda Wire and Cable Company.)



IG. 10. Temperature-rise curves for A.C.S.R. conductors at 60 cycles in still air. (Plotted from data supplied by the Aluminum Company of America.)

ously above 100°C. At temperatures slightly above approximately 100°C, both copper and aluminum tend to anneal in time, if operated other than for very short emergency periods. As a conductor carrying no current may reach a temperature of approximately 40°C in strong sunlight, Figs. 9 and 10 give current ratings only up to 60°C rise. The allowable temperature rise in conductors should not exceed the allowable temperature rise in connected equipment. As resistance increases with temperature, it may be found uneconomical to operate conductors continuously at temperatures as high as 100°C, except in special cases.

In Fig. 9, temperature rise is given for 4/0 and smaller solid conductors; for stranded conductors, the current rating is between 1% and 5% higher for the same temperature rise. In Fig. 10, the A.C.S.R. conductor sizes in cir mils or A.W.G. correspond to those listed in Table IV. The curves of Fig. 10 are based on degrees rise above still air at 25°C, but they may be used to determine approximate temperature rise above 40°C, in view of the fact that there is always some motion, even if but slight, in out-of-doors air.

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